tion in which the crystal is regarded as isotropic. Availing ourselves of the data[9] for F bands, and of the expressions (40) and (41), we can determine μ_0/m and a approximately:

$$\mu_0/m \approx 0.9; \quad |a| \approx 7 \text{ eV}.$$

From these parameters we estimated the polaron and condenson terms in (19) and (44). It turned out that for the polaro-condenson the magnitude of the condenson term amounted to about 10% of the total energy, while for the F-center, to about 25%.

The values of the effective masses of the bound electron and the current carrier change appreciably in comparison with the corresponding values obtained without regard to the condenson effect. Thus μ_0/m appeared to be two times smaller than the value in Ref. 1, while $M/m = 7.63$ instead of 9.69 as in Ref. 1.

[1] S. I. Pekar, *Studies on the Electronic Theory of Crystals*, GITTL, 1951.

[2] Iu. E. Perlin, J. Exptl. Theoret. Phys. (U.S.S.R.) **21**, 547 (1951).

[3] H. A. Bethe and A. Sommerfeld, *Electron Theory of Metals*, ONTI, 1938 (Russ. transl.).

[4] M. F. Deigen and S. I. Pekar, J. Exptl. Theoret. Phys. (U.S.S.R.) **21**, 803 (1951).

[5] M. F. Deigen, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 504 (1956); Soviet Phys. JETP **4**, 424 (1957).

[6] K. B. Tolpygo, Trudy, Physical Inst. Academy of Sciences, Ukrainian SSR **6**, (1954).

[7] S. I. Pekar, J. Exptl. Theoret. Phys. (U.S.S.R.) **27**, 411 (1954).

[8] M. A. Krivoglaz and S. I. Pekar, Trudy, Physical Inst., Academy of Sciences, Ukrainian SSR **4**, 37 (1953).

[9] B. Schönwald, Ann. Phys. **15**, 395 (1932).

[10] W. Shockley, *Theory of Electronic Semiconductors*, IIL, 1955 (Russ. transl.).

Translated by R. T. Beyer
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Dispersion Relations for Photoproduction of Pions on Nucleons*

A. A. LOGANOV AND A. N. TAVKHELIDZE

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Dispersion relations are derived for pion photoproduction reactions on nucleons. The spin and isotopic structure of the reaction amplitude is established, and the unobservable energy range is separated. It is shown that the dispersion relations are inhomogeneous.

FORMAL SCATTERING THEORY, based on the unitarity and asymmetry of the scattering matrix, leads to an expression for the amplitude in terms of phase shifts. The values of the phase shifts depend on the dynamics of the collision process. Since the character of the dynamics of the process is not taken into account in the formulation of the formal scattering theory, it is natural that the values of the phases remain unknown in such an analysis. The determination of phase shifts from experimental data is of great interest, for it permits a deeper study of the character of the meson-nucleon collisions. Using the charge-independence hypothesis in the s- and p-wave approximation it becomes possible to find several possible sets of phase shifts for the meson-nucleon collisions. This ambiguity can be eliminated using the causality principle. In fact, Goldberger[1] has found the Hermitian and anti-Hermitian portions of the forward-scattering amplitude to be connected by dispersion relations that lead to a correct choice of phase shifts for the meson-nucleon collision processes.

N. N. Bogoliubov* developed general principles for the derivation of dispersion relations for a great variety of scattering processes. His method is essentially based on analyticity theorems that follow from the principle of causality.

In the present article we shall use the Bogoliubov method to derive dispersion relations for the photoproduction of mesons on nucleons.[3]

*Paper delivered at the All-Union Conference on Physics of High Energy Particles on May 15, 1956.

*Reported by N. N. Bogoliubov to many seminars on theoretical physics in the V. A. Steklov Mathematics Institute in January 1956.

1. AMPLITUDE OF THE PHOTOPRODUCTION REACTION

Let us denote by $\varphi_\rho^{(-)}(q)$, the operator for the annihilation of a meson of type ρ (ρ = 1, 2, 3), and by $a_\nu^{(+)}(k)$ the operator of photon production (ν = 0, 1, 2, 3). The matrix element for the photoproduction reaction can then be written with the aid of the S-matrix formalism in the form

$$S(\alpha, k; \omega, q) = (2\pi)^3 < \Phi^*_{p's'} \varphi_\rho^{(-)}(q) S a_\nu^{(+)}(k) \Phi_{ps} > . \tag{1.1}$$

Here Φ denotes the amplitude of the state of the scatterer, and the indices α and ω, which pertain to the initial and final states respectively, include all the quantum numbers that characterize the system with the exception of the momenta of the photon and meson.

Since

$$A_\nu(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\mathbf{k}}{\sqrt{2k^0}} \{e_{\nu\lambda}(\mathbf{k}) a_\lambda^{(+)}(\mathbf{k}) e^{ikx} + e_{\nu\lambda}(k) a_\lambda^{(-)}(\mathbf{k}) e^{-ikx}\},$$
$$\varphi_\rho(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\mathbf{q}}{\sqrt{2q^0}} \{\varphi_\rho^{(+)}(\mathbf{q}) e^{iqx} + \varphi_\rho^{(-)}(\mathbf{q}) e^{-iqx}\}, \tag{1.2}$$

we have

$$[\varphi_\rho^{(-)}(\mathbf{q}), \varphi_\rho(x)] = \frac{(2\pi)^{-3/2}}{\sqrt{2q^0}} e^{iqx}, \quad [A_\mu(x), a_\nu^{(+)}(\mathbf{k})] = \frac{(2\pi)^{-3/2}}{\sqrt{2k_0}} e_\mu^\nu e^{-ikx} \tag{1.3}$$

and consequently

$$[\varphi_\rho^{(-)}(\mathbf{q}), S] = \frac{(2\pi)^{-3/2}}{\sqrt{2q^0}} \int e^{iqx} \frac{\delta S}{\delta\varphi_\rho(x)} dx, \quad [S, a_\nu^{(+)}(\mathbf{k})] = \frac{(2\pi)^{-3/2}}{\sqrt{2k^0}} e_\mu^\nu \int e^{-iky} \frac{\delta S}{\delta A_\mu(y)} dy. \tag{1.4}$$

Transposing the creation operators in (1.1) to the extreme left position, and the annihilation operators to the extreme right, we obtain with the aid of (1.4)

$$S(\alpha, k; \omega, q) = \sum_\mu e_\mu^\nu S_{\mu\rho}. \quad S_{\mu\rho} = \frac{1}{\sqrt{4k^0 q^0}} \int e^{i(qx-ky)} \left\langle \Phi^*_{p's'} \frac{\delta^2 S}{\delta\varphi_\rho(x) \delta A_\mu(y)} \Phi_{ps} \right\rangle dx\, dy. \tag{1.5}$$

We shall assume that Φ_{ps} describes a scatterer in the lower energy state. No spontaneous processes can occur therefore, and

$$S\Phi_{ps} = \Phi_{ps}, \tag{1.6}$$

or from the unitarity of the S-matrix

$$S^+\Phi_{ps} = \Phi_{ps}. \tag{1.7}$$

Taking (1.7) into account, we can rewrite (1.5) as

$$S_{\mu\rho} = \frac{1}{\sqrt{4k^0 q^0}} \int e^{i(qx-ky)} \langle \Phi^*_{p's'} \frac{\delta^2 S}{\delta\varphi_\rho(x) \delta A_\mu(y)} S^+ \Phi_{ps} \rangle. \tag{1.8}$$

Let us introduce the operators

$$j_\rho(x) = i \frac{\delta S}{\delta\varphi_\rho(x)} S^+, \quad i_\mu(y) = i \frac{\delta S}{\delta A_\mu(y)} S^+ \tag{1.9}$$

and call them respectively the meson-current $j_\rho(x)$ and electromagnetic-current $i_\mu(y)$ operators. These operators are Hermitian. In fact

$$j_\rho^+(x) = -iS\delta S^+ / \delta\varphi_\rho(x),$$

but since $S^+S = 1$,

$$S\frac{\delta S^+}{\delta\varphi_\rho(x)} = -\frac{\delta S}{\delta\varphi_\rho(x)}S^+$$

and consequently

$$j_\rho^+(x) = j_\rho(x). \tag{1.10}$$

Analogously

$$i_\mu^+(x) = i_\mu(x). \tag{1.11}$$

Variation of (1.9) leads to

$$\frac{\delta^2 S}{\delta A_\mu(y)\,\delta\varphi_\rho(x)}S^+ = -j_\rho(x)\,i_\mu(y) - i\frac{\delta j_\rho(x)}{\delta A_\mu(y)},\quad \frac{\delta^2 S}{\delta A_\mu(y)\,\delta\varphi_\rho(x)}S^+ = -i_\mu(y)\,j_\rho(x) - i\frac{\delta i_\mu(y)}{\delta\varphi_\rho(x)}. \tag{1.12}$$

Subtracting one equation from the other, we get

$$\delta i_\mu(y)/\delta\varphi_\rho(x) - \delta j_\rho(x)/\delta A_\mu(y) = -i\,[j_\rho(x)\,i_\mu(y) - i_\mu(y)\,j_\rho(x)]. \tag{1.13}$$

Changing to the Heisenberg representation, we write

$$\langle\Phi^*_{p's'}\,j_\rho(x)\,i_\mu(y)\,\Phi_{ps}\rangle = \langle\Psi^*_{p's'}\,J_\rho(x)\,I_\mu(y)\,\Psi_{ps}\rangle. \tag{1.14}$$

Here J and I are the current operators in the real-particle representation, and Ψ_{ps} – the amplitude of state of the nucleon in the same representation. Let us assume that the energy-momentum operator in this system has a complete set of eigenfunctions, denoted by $\Psi_{n,\,k_n}$. The index $\mathbf{k}_n$ denotes the momentum of the system, and n represents the remaining quantum numbers that determine the state of the system. The energy of the nth state of the system will be

$$E_n = \sqrt{M_n^2 + \mathbf{k}_n^2}. \tag{1.15}$$

From the assumption of the completeness of the system we can write

$$\langle\Psi^*_{p's'}\,J_\rho(x)\,I_\mu(y)\,\Psi_{ps}\rangle = \frac{1}{(2\pi)^3}\sum_n\int d\mathbf{k}_n\,\langle\Psi^*_{p's'}J_\rho(x)\,\Psi_{n,\,k_n}\rangle\;\langle\Psi^*_{n,\,k_n}I_\mu(y)\,\Psi_{ps}\rangle. \tag{1.16}$$

Translational-invariance considerations permit representation of the operators J_ρ and I_μ in the following form

$$J_\rho(x) = e^{i\hat{P}x}\,J_\rho(0)\,e^{-i\hat{P}x},\quad I_\mu(y) = e^{i\hat{P}y}\,I_\mu(0)\,e^{-i\hat{P}y}. \tag{1.17}$$

Inserting (1.17) into (1.16) and recalling that $\Psi_{n,\,k_n}$ are eigenfunctions of the operator $\hat{P}$ we get

$$\langle\Psi^*_{p's'}\,J_\rho(x)\,I_\mu(y)\,\Psi_{ps}\rangle = (2\pi)^{-3}\sum_n\int d\mathbf{k}_n\,\langle\Psi^*_{p's'}\,J_\rho(0)\,\Psi_{n,\,k_n}\rangle\langle\Psi^*_{n,\,k_n}I_\mu(0)\,\Psi_{ps}\rangle$$
$$\times\exp\left\{-ik_n(x-y) + \frac{i}{2}(p+p')(x-y) + \frac{i}{2}(p'-p)(x+y)\right\},\quad k_n\cdot x = x^0\sqrt{M_n^2+\mathbf{k}_n^2} - \mathbf{k}_n\mathbf{x}. \tag{1.18}$$

Introducing the notation

$$F_{\mu\rho}^{ps;\,p's'}(x-y) = (2\pi)^{-3}\sum_n \int d\mathbf{k}_n \langle \Psi^*_{p's'} J_\rho(0)\,\Psi_{n,\,k_n}\rangle\langle \Psi^*_{n,\,k_n} I_\mu(0)\,\Psi_{ps}\rangle \times \exp\left\{-ik_n(x-y) + \frac{i}{2}(p+p')(x-y)\right\}, \tag{1.19}$$

we can rewrite (1.18) as

$$\langle \Psi^*_{p's'} J_\rho(x)\, I_\mu(y)\,\Psi_{ps}\rangle = e^{i(p'-p)(x+y)/2}\, F_{\mu\rho}^{ps;\,p's'}(x-y). \tag{1.20}$$

Analogously, translational-invariance considerations lead to

$$\langle \Phi^*_{p's'} \frac{\delta^2 S}{\delta A_\mu(y)\,\delta\varphi_\rho(x)} S^+ \Phi_{ps}\rangle = e^{i(p'-p)(x+y)/2}\, V_{\mu\rho}^{ps;\,p's'}(x-y), \tag{1.21a}$$

$$\langle \Phi^*_{p's'} \frac{\delta i_\mu(y)}{\delta\varphi_\rho(x)}\Phi_{ps}\rangle = e^{i(p'-p)(x+y)/2}\, W_{\mu\rho}^{ps;\,p's'}(x-y), \tag{1.21b}$$

$$\langle \Phi^*_{p's'}\, i_\mu(y)\, j_\rho(x)\,\Phi_{ps}\rangle = e^{i(p'-p)(x+y)/2}\, F_{\rho\mu}^{ps;\,p's'}(y-x). \tag{1.21c}$$

Let us establish certain relationships between the functions F, V, and W. It follows from (1.20) and (1.21) that

$$F_{\mu\rho}^{*ps;\,p's'}(x) = F_{\rho\mu}^{p's';\,ps}(-x). \tag{1.22}$$

Using (1.12) and (1.13) we get

$$V_{\mu\rho}^{ps;\,p's'}(x) = -iW_{\mu\rho}^{ps;\,p's'}(x) - F_{\rho\mu}^{ps;\,p's'}(-x), \tag{1.23a}$$

$$W_{\mu\rho}^{ps;\,p's'}(x) - W_{\rho\mu}^{ps;\,p's'}(-x) = -i\left[F_{\mu\rho}^{ps;\,p's'}(x) - F_{\rho\mu}^{ps;\,p's'}(-x)\right], \tag{1.23b}$$

$$W_{\mu\rho}^{*ps;\,p's'}(x) = W_{\mu\rho}^{p's';\,ps}(x). \tag{1.23c}$$

According to (1.21a), Eq. (1.8) can be written as

$$S_{\mu\rho} = (4k^0q^0)^{-1/2}\int dx\,dy\exp\left\{\frac{i}{2}(k+q)(x-y) + \frac{i}{2}(q-k)(x+y) + \frac{i}{2}(p'-p)(x+y)\right\} V_{\mu\rho}^{ps;\,p's'}(x-y), \tag{1.24}$$

from which we obtain after integrating over $(x + y)$

$$S_{\mu\rho} = i\,(2\pi)^4(4k^0q^0)^{-1/2}\,\delta(p'-p+q-k)\,T_{\mu\rho}^{ps;\,p's'}\left(\frac{k+q}{2}\right), \tag{1.25}$$

where

$$T_{\mu\rho}^{ps;\,p's'}\left(\frac{k+q}{2}\right) = -i\int e^{i(k+q)x/2}\,V_{\mu\rho}^{ps;\,p's'}(x)\,dx.$$

2. AUXILIARY REACTION AMPLITUDE AND ITS PROPERTIES

The expression (1.25) is too complicated for investigation. Let us introduce the amplitude

$$\mathfrak{M}(\alpha, k;\omega, q) = -i\,(4k^0q^0)^{-1/2}\sum_\mu e^\nu_\mu \int e^{i(qx-ky)}\langle \Phi^*_{p's'}\frac{\delta i_\mu(y)}{\delta\varphi_\rho(x)}\Phi_{ps}\rangle\,dx\,dy, \tag{2.1}$$

which, on the basis of (1.21c), can be rewritten

$$\mathfrak{M}(\alpha, k; \omega, q) = -i\,(4k^0q^0)^{-1/2} \sum_{\mu} e^{\nu}_{\mu} \int dx\,dy \cdot \exp\Big\{\frac{i}{2}(x+y)(q-k) + \frac{i}{2}(k+q)(x-y) + \frac{i}{2}(p'-p)(x+y)\Big\}\, W^{ps;\,p's'}_{\mu\rho}(x-y). \tag{2.2}$$

Integrating over $(x + y)$ we get

$$\mathfrak{M}(\alpha, k; \omega, q) = i\,(2\pi)^4 (4k^0q^0)^{-1/2}\,\delta(p'-p+q-k) \sum_{\mu} e^{\nu}_{\mu} M^{ps;\,p's'}_{\mu\rho}\Big(\frac{k+q}{2}\Big),$$
$$M^{ps;\,p's'}_{\mu\rho}\Big(\frac{k+q}{2}\Big) = -\int e^{i(k+q)x/2}\, W^{ps;\,p's'}_{\mu\rho}(x)\,dx. \tag{2.3}$$

We shall show that in the observed region (real momenta, positive energies) the amplitudes T and M are equal. Subtracting (2.3) from (1.25) and taking (1.23a) into account, we get

$$T^{ps;\,p's'}_{\mu\rho}\Big(\frac{k+q}{2}\Big) - M^{ps;\,p's'}_{\mu\rho}\Big(\frac{k+q}{2}\Big) = i\int e^{i(k+q)x/2}\, F^{ps;\,p's'}_{\rho\mu}(-x)\,dx. \tag{2.4}$$

Inserting the expression (1.19) for F into (2.4) we get by elementary computations

$$T^{ps;\,p's'}_{\mu\rho}\Big(\frac{k+q}{2}\Big) - M^{ps;\,p's'}_{\mu\rho}\Big(\frac{k+q}{2}\Big)$$
$$= 2\pi i \sum_{n} \langle \Psi^{*}_{p's'} J_{\mu}(0)\,\Psi_{n,\,\mathbf{k}_n}\rangle \langle \Psi^{*}_{n,\,\mathbf{k}_n} J_{\rho}(0)\,\Psi_{ps}\rangle\, \delta\Big(\sqrt{M_n^2 + \mathbf{k}_n^2} + \frac{k^0+q^0}{2} - \frac{p^0+p^{0\prime}}{2}\Big),$$
$$\mathbf{k}_n = (\mathbf{p} + \mathbf{p}' - \mathbf{q} - \mathbf{k})/2. \tag{2.5}$$

It can be shown that the argument of the δ-function is greater than zero in the observed region.

Actually, using the energy-momentum conservation laws

$$k^0 - p^{0\prime} = q^0 - p^0, \quad \mathbf{p}' - \mathbf{k} = \mathbf{p} - \mathbf{q},$$

we get

$$\sqrt{M_n^2 + \mathbf{k}_n^2} + \frac{k^0+q^0}{2} - \frac{p^0+p^{0\prime}}{2} = \sqrt{M_n^2 + (\mathbf{p}-\mathbf{q})^2} + \sqrt{\mu^2 + \mathbf{q}^2} - \sqrt{M^2 + \mathbf{p}^2}.$$

Since $M_n \gg M$, to prove the above statement it is necessary to show that

$$\sqrt{M^2 + (\mathbf{p}-\mathbf{q})^2} > \sqrt{M^2 + \mathbf{p}^2} - \sqrt{\mu^2 + \mathbf{q}^2}.$$

The correctness of this inequality is readily proved by squaring both sides. Thus, T and M are equal in the real-amplitude region.

We shall operate hereinafter in a coordinate system in which

$$\mathbf{p} + \mathbf{p}' = 0. \tag{2.6}$$

Using the energy and momentum conservation laws

$$\mathbf{k}^2 = \mu^2 + \mathbf{q}^2; \quad \mathbf{k} - \mathbf{q} = -2\mathbf{p},$$

we get

$$^1/_2(\mathbf{k} - \mathbf{q}) = -\mathbf{p}; \quad ^1/_2(\mathbf{k} + \mathbf{q}) = \lambda\mathbf{e} - (\mu^2 / 4\mathbf{p}^2)\,\mathbf{p}, \tag{2.7}$$

where λ is arbitrary. The vector $\mathbf{e}$ is orthogonal to the vector $\mathbf{p}$. Hence

$$\mathbf{k} = \lambda\mathbf{e} - (1 + \mu^2 / 4\mathbf{p}^2)\,\mathbf{p}, \quad \mathbf{q} = \lambda\mathbf{e} + (1 - \mu^2 / 4\mathbf{p}^2)\,\mathbf{p}. \tag{2.8}$$

The quantity λ can be expressed in terms of the meson energy E as follows:

$$E^2 = \lambda^2 + (1 + \mu^2 / 4\mathbf{p}^2)^2\,\mathbf{p}^2. \tag{2.9}$$

Having made the above remarks concerning the chosen coordinate system, we turn to the study of the properties of the amplitude M. Let us represent the amplitude M in the form

$$M_{\mu\rho}^{ps;\,p's'} = D_{\mu\rho}^{ps;\,p's'} + iA_{\mu\rho}^{ps;\,p's'}, \tag{2.10}$$

where

$$D_{\mu\rho}^{ps;\,p's'} = \frac{1}{2}(M_{\mu\rho}^{ps;\,p's'} + M_{\rho\mu}^{*p's';\,ps}), \quad A_{\mu\rho}^{ps;\,p's'} = \frac{1}{2i}(M_{\mu\rho}^{ps;\,p's'} - M_{\rho\mu}^{*p's';\,ps}). \tag{2.10a}$$

Inserting the expression (2.3) for M into (2.10a) and taking the properties (1.23) into account, we get

$$A_{\mu\rho}^{ps;\,p's'}\left(\frac{k+q}{2}\right) = {}^1/_2\int e^{i(k+q)x/2}\,\{F_{\mu\rho}^{ps;\,p's'}(x) - F_{\rho\mu}^{ps;\,p's'}(-x)\}, \tag{2.11}$$

$$D_{\mu\rho}^{ps;\,p's'}\left(\frac{k+q}{2}\right) = -{}^1/_2\int e^{i(k+q)x/2}\,\{W_{\mu\rho}^{ps;\,p's'}(x) + W_{\rho\mu}^{ps;\,p's'}(-x)\}. \tag{2.12}$$

Let us introduce functions X and Y such that

$$X_{\nu\rho}^{ps;\,p's'}\left(\frac{k+q}{2}\right) = \sum_{\mu} e_{\mu}^{\nu}\,D_{\mu\rho}^{ps;\,p's'}\left(\frac{k+q}{2}\right), \tag{2.13}$$

$$Y_{\nu\rho}^{ps;\,p's'}\left(\frac{k+q}{2}\right) = \sum_{\mu} e_{\mu}^{\nu}\,A_{\mu\rho}^{ps;\,p's'}\left(\frac{k+q}{2}\right). \tag{2.14}$$

Using (2.11) and (2.12), as well as the invariance of the reaction amplitude under reflection of momenta, we can obtain the following important relations

$$P_{ss'}\,X_{\nu\rho}^{*ps;\,p's'}\left(\frac{k^0+q^0}{2}, \frac{\mathbf{k}+\mathbf{q}}{2}\right) = X_{\nu\rho}^{ps;\,p's'}\left(-\frac{k^0+q^0}{2}, \frac{\mathbf{k}+\mathbf{q}}{2}\right), \tag{2.15}$$

$$P_{ss'}\,Y_{\nu\rho}^{*ps;\,p's'}\left(\frac{k^0+q^0}{2}, \frac{\mathbf{k}+\mathbf{q}}{2}\right) = -Y_{\nu\rho}^{ps;\,p's'}\left(-\frac{k^0+q^0}{2}, \frac{\mathbf{k}+\mathbf{q}}{2}\right). \tag{2.16}$$

$P_{ss'}$ is the spin commutation operator. In fact

$$Y_{\nu\rho}^{*ps;\,p's'}\left(\frac{k+q}{2}\right) = +{}^1/_2\int e^{-i(k+q)x/2}\,\{F_{\mu\rho}^{*ps;\,p's'}(x) - F_{\rho\mu}^{*ps;\,p's'}(-x)\}\,e_{\mu}^{\nu}\,dx.$$

taking (1.22) into account, this expression can be rewritten as

$$Y^{*ps;\,p's'}_{\nu\rho}\left(\frac{k+q}{2}\right) = -\,{}^1/_2\int e^{-i(k+q)x/2}\,\{F^{p's';\,ps}_{\mu\rho}(x) - F^{p's';\,ps}_{\rho\mu}(-x)\}\,e^{\nu}_{\mu}\,dx. \tag{2.17}$$

Comparing (2.17) with (2.14) and taking into account the invariance of the reaction amplitude under reflection of the momenta,

$$\mathbf{k}\to-\mathbf{k},\quad \mathbf{q}\to-\mathbf{q},\quad \mathbf{p}\to-\mathbf{p},\quad \mathbf{p}'\to-\mathbf{p}'\quad(\mathbf{p}=-\mathbf{p}'),$$

one can readily check the correctness of (2.16) in the chosen coordinate system. Analogously, one can prove (2.15).

Equations (2.15) and (2.16) can be used to make up the combinations

$$(1\pm P)\,X^{ps;\,p's'}_{\nu\rho} = X^{ps;\,p's'}_{\nu\rho} \pm P_{ss'}\,X^{*ps;\,p's'}_{\nu\rho}, \tag{2.18}$$

$$(1\pm P)\,Y^{ps;\,p's'}_{\nu\rho} = Y^{ps;\,p's'}_{\nu\rho} \pm P_{ss'}\,Y^{*ps;\,p's'}_{\nu\rho}, \tag{2.19}$$

which have the following parity properties relative to E:

$$\begin{aligned}&(1+P)\,X^{ps;\,p's'}_{\nu\rho} \text{ — even},\\ &(1-P)\,X^{ps;\,p's'}_{\nu\rho} \text{ — odd},\\ &(1+P)\,Y^{ps;\,p's'}_{\nu\rho} \text{ — odd},\\ &(1-P)\,Y^{ps;\,p's'}_{\nu\rho} \text{ — even},\end{aligned} \tag{2.20}$$

3. ROLE OF BOUND STATES IN PHOTOPRODUCTION PROCESSES

It is important to study the role of the bound states in photoproduction processes, since this problem is related to the analysis of the unobservable energy region in the dispersion relations. In our chosen reference system $\mathbf{p}+\mathbf{p}'=0$, the photoproduction threshold is $|\mathbf{p}| + \mu^2/4|\mathbf{p}|$. Let us consider the energy region in which $E < |\mathbf{p}| + \mu^2/4|\mathbf{p}|$ and where consequently bound states can be produced. Substituting (1.19) into (2.11) and integrating with respect to x, we get

$$Y^{ps;\,p's'}_{\nu\rho}\left(\frac{k+q}{2}\right) = \pi\sum_{\mu,n} e^{\nu}_{\mu}\,\langle\Psi^{*}_{p',\,s'}J_{\rho}(0)\,\Psi_{n,\,\lambda e-\varepsilon p}\rangle\langle\Psi^{*}_{n,\,\lambda e-\varepsilon p}I_{\mu}(0)\,\Psi_{ps}\rangle \tag{3.1}$$

$$\times\,\delta(\sqrt{M^2+\mathbf{p}^2} + E - \sqrt{M^2+\lambda^2+\varepsilon^2\mathbf{p}^2}),$$

$$-\,\pi\sum_{\mu,n} e^{\nu}_{\mu}\,\langle\Psi^{*}_{p',\,s'}I_{\mu}(0)\,\Psi_{n,\,-\lambda e+\varepsilon p}\rangle\langle\Psi^{*}_{n;\,-\lambda e+\varepsilon p}J_{\rho}(0)\,\Psi_{ps}\rangle\,\delta(\sqrt{M^2+\mathbf{p}^2} - E - \sqrt{M_n^2+\lambda^2+\varepsilon^2\mathbf{p}^2}),$$

($\varepsilon = \mu^2/4\mathbf{p}^2$). By virtue of the smallness of the coupling constant e (electric charge), the energy of the interaction with the electromagnetic field can be considered as a perturbation. This makes possible an expansion in terms of the eigenstates of the energy-momentum vector of the meson-nucleon system.

Bearing the above approximation in mind and taking it into account that there are no bound states whatever in the meson-nucleon system between M and $M+\mu$, it is possible to show (for small scatterer momenta $\mathbf{p}^2 < M\mu/2$) that in the energy range

$$0 < E < \frac{M\mu + \mu^2/4 - \mathbf{p}^2}{\sqrt{M^2+\mathbf{p}^2}} \tag{3.2}$$

the quantity $Y^{ps;\,p's'}_{\nu\rho}\left(\frac{k+q}{2}\right)$ can be written as

$$Y_{\nu\rho}^{ps;\,p's'}(E, \lambda\mathbf{e}) = -\pi\frac{M^2-\mu^2/4}{M^2+p^2}\sum_{\mu s''} e^{\nu}_{\mu}\langle\Psi^{*}_{p's'}I_{\mu}(0)\Psi_{-\lambda\mathbf{e}+\varepsilon\mathbf{p},\,s''}\rangle \times \langle\Psi^{*}_{-\lambda\mathbf{e}+\varepsilon\mathbf{p},\,s''}J_{\rho}(0)\Psi_{ps}\rangle\,\delta\Big(E-\frac{\mathbf{p}^2+\mu^2/4}{\sqrt{M^2+\mathbf{p}^2}}\Big). \tag{3.3}$$

In fact, let us consider the δ-function of expression (3.1). It is easy to show that the δ-singularities of these functions at $M_n = M$ will occur at the energies

$$E_1 = -\frac{\mathbf{p}^2+\mu^2/4}{\sqrt{M^2+\mathbf{p}^2}}\ \text{and}\ E_2 = \frac{\mathbf{p}^2+\mu^2/4}{\sqrt{M^2+\mathbf{p}^2}}$$

for the first and second δ-functions respectively. If $M_n \neq M$, *i.e.*, $M_n \geq M + \mu$, the δ-singularities of these functions will occur at energies

$$E_1' \geqslant \frac{M\mu+\mu^2/4-\mathbf{p}^2}{\sqrt{M^2+\mathbf{p}^2}}\ \text{and}\ E_2' \leqslant -\frac{M\mu+\mu^2/4-\mathbf{p}^2}{\sqrt{M^2+\mathbf{p}^2}}$$

for the first and second δ-functions respectively.

If $\mathbf{p}^2 < M\mu/2$ (small scatterer momenta), it is easy to see that $E_2 < E_1'$ and consequently we have (3.3) in the region (3.2).

Let us consider the mean values of the currents contained in (2.13). By way of example let us calculate the mean value of the meson-current $J_\rho(0)$, and analogously compute the mean value of the electromagnetic-current. We have

$$\langle\Psi^{*}_{p's'}J_{\rho}(x)\Psi_{ps}\rangle = e^{i(p'-p)x}\langle\Psi^{*}_{p's'}J_{\rho}(0)\Psi_{ps}\rangle$$

and on the other hand

$$\langle\Psi^{*}_{p's'}J_{\rho}(x)\Psi_{ps}\rangle = i\langle\Phi^{*}_{p's'}\frac{\delta S}{\delta\varphi_\rho(x)}\Phi_{ps}\rangle.$$

Introducing the Fourier transform

$$\varphi_\rho(x) = (2\pi)^{-4}\int e^{iqx}\varphi_\rho(q)\,dq$$

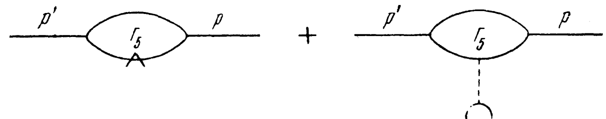
and considering that

$$\frac{\delta S}{\delta\varphi_\rho(q)} = (2\pi)^{-4}\int e^{iqx}\frac{\delta S}{\delta\varphi_\rho(x)}\,dx,$$

we get

$$i\langle\Phi^{*}_{p's'}\frac{\delta S}{\delta\varphi_\rho(q)}\Phi_{ps}\rangle = \langle\Psi^{*}_{p's'}J_\rho(0)\Psi_{ps}\rangle\,\delta(q-p+p').$$

The latter expression can be represented symbolically in the form of the following sum of diagrams:



Here Γ_5 is the strongly-bound portion. Had there been three meson lines in place of the symbol Γ, the sum under consideration would have been

$$\mathfrak{G}_5^{\rho}(p, p', q)\,\Delta_c^{\rho}(q).$$

Since in our case there is no free meson line, the resulting expression must be multiplied in addition by $(\mu^2 - q^2)$. Using the normalization condition for the Green function at $p'^2 = M^2$, $p^2 = M^2$, $q^2 = \mu^2$, and also the energy-momentum conservation law, we obtain as a final expression

$$\langle\Psi^{*}_{p's'}J_\rho(0)\Psi_{ps}\rangle = g\,\langle\bar{u}_{s'}(p')\gamma^5\tau_\rho u_s(p)\rangle, \tag{3.4}$$

where g is the renormalized pseudoscalar coupling constant between the meson and nucleon fields. In the case of the electromagnetic field

$$\langle\Psi^{*}_{p's'}I_\mu(0)\Psi_{ps}\rangle = \langle\bar{u}_{s'}(p')\Big\{e\frac{1+\tau_3}{2}\gamma^\mu + \frac{1}{2}\hat{\mu}\,[(k\cdot\gamma), \gamma^\mu]\Big\}u_s(p)\rangle. \tag{3.5}$$

Here e is the renormalized electron charge and

$$\hat{\mu} = \mu_p\frac{1+\tau_3}{2} + \mu_n\frac{1-\tau_3}{2}, \tag{3.6}$$

μ_p and μ_n are the anomalous magnetic moments of the proton and neutron respectively. Using (3.4) to (3.6), we can rewrite (3.3) as

$$Y_{\nu\rho}^{ps;\,p's'}(E,\lambda e) = -\pi \frac{M^2-\mu^2/4}{M^2+p^2} g \sum_{s'',\,\mu} e_\mu^\nu \langle \bar{u}_{s'}(-\mathbf{p}) \Big\{ e \frac{1+\tau_3}{2} \gamma^\mu + \frac{1}{2}\hat{\mu}\,[k\gamma),\gamma^\mu] \Big\} u_{s''}(p'') \rangle \langle \bar{u}_{s''}(p'')\,\gamma_5\tau_\rho u_s(p) \rangle\, \delta(E-E_2),$$

$$\mathbf{p}'' = -\lambda \mathbf{e} + \varepsilon \mathbf{p}.$$

4. MATRIX STRUCTURE OF THE PHOTOPRODUCTION AMPLITUDE

From consideration of the relativistic and gradient invariances, the photoproduction reaction amplitude can be rewritten as:[4]

$$R_{\nu\rho} = \sum_s \hat{\Omega}_\rho^s \bar{u}(p')\, P_{\alpha\beta}^s(k,p,p',\gamma)\, u(p)\, e_\alpha^\nu k_\beta,$$

$$R_{\nu\rho} = X_{\nu\rho} + iY_{\nu\rho}, \qquad (4.1)$$

where $P_{\alpha\beta}^s$ is an anti-symmetric (relative to α and β) tensor of second rank. In the pseudoscalar theory one can write the following four linearly independent operators

$$\hat{P}_{\alpha\beta}^s = \begin{cases} \gamma_5(p_\alpha p'_\beta - p_\beta p'_\alpha), & \gamma_5(\gamma_\alpha p_\beta - \gamma_\beta p_\alpha), \\ \gamma_5(p'_\alpha \gamma_\beta - \gamma_\alpha p'_\beta). & \gamma_5(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha). \end{cases} \qquad (4.2)$$

$\hat{\Omega}^s$ stands for the isotopic spin operators. In a system consisting of nucleon, pion, and electromagnetic fields, the only matrices that participate in the description of the interaction are the matrices τ_ρ in the coupling between the nucleon and pion fields, and the electric-charge matrix $\frac{1}{2}(1+\tau_3)$ in the coupling between the nucleon and electromagnetic fields. Since the initial state of a photoproduction process is the nucleon plus electromagnetic field and the final state is the nucleon plus meson field, the most general expression for the reaction amplitude in isotopic space can be written as

$$\hat{\Omega}_\rho^s = \Omega_1^s \delta_{3\rho} + \Omega_2^s \tau_\rho + \Omega_3^s \frac{i}{2}[\tau_3,\tau_\rho]. \qquad (4.3)$$

Inserting (4.2) into (4.1), the expression for the photoproduction amplitude, for transverse polarization and for the system of coordinates chosen by us ($\mathbf{p}+\mathbf{p}'=0$), can be written as

$$\hat{R}_{\nu\rho} = (\boldsymbol{\sigma}\boldsymbol{\pi}_\nu)\,\hat{R}_\rho^1 + \boldsymbol{\sigma}(\mathbf{k}-\mathbf{q})\,\hat{R}_\rho^2 + \boldsymbol{\sigma}\,[[\mathbf{k}\boldsymbol{\pi}^\nu]\,\mathbf{q}]\,\hat{R}_\rho^3 + [\mathbf{k}\mathbf{q}]\,\boldsymbol{\pi}_\nu \hat{R}_\rho^4, \qquad (4.4)$$

where the $\hat{R}_\rho^i$ have a matrix structure in isotopic space, analogous to (4.3). Since $\mathbf{k}$ and $\mathbf{q}$ can be expressed in terms of the vectors $\mathbf{e}$ and $\mathbf{p}$ [see (2.8)], the reaction amplitude can be represented as

$$\hat{R}_{\nu\rho} = (\boldsymbol{\sigma}\boldsymbol{\pi}^\nu)\,\hat{L}_\rho^1 + (\boldsymbol{\sigma}\mathbf{p})\,(\mathbf{p}\boldsymbol{\pi}^\nu)\,L_\rho^2 + \lambda\,(\boldsymbol{\sigma}\mathbf{e})\,(\mathbf{p}\boldsymbol{\pi}^\nu)\,\hat{L}_\rho^3 + \lambda\,[\mathbf{p}\mathbf{e}]\,\boldsymbol{\pi}^\nu \hat{L}_\rho^4, \qquad (4.5)$$

where the $\hat{L}_\rho^i$ have a matrix structure analogous to (4.3).

5. DISPERSION RELATIONS

It was shown in Ref. 2 that

$$S_e \hat{R}_{\nu\rho} = \hat{R}_{\nu\rho}(\lambda \mathbf{e}) + \hat{R}_{\nu\rho}(-\lambda \mathbf{e}), \quad \mathfrak{A}_e \hat{R}_{\nu\rho} = \frac{1}{\lambda}(\hat{R}_{\nu\rho}(\lambda \mathbf{e}) - \hat{R}_{\nu\rho}(-\lambda \mathbf{e})) \qquad (5.1)$$

are analytic functions in the upper half-plane. To use the generalized Cauchy theorem and to write down the dispersion relations, it is necessary to make certain assumptions concerning the rate of growth of the photoproduction amplitude at infinity. Since the degree of reaction amplitude growth is a dynamic factor, it cannot be obtained from any kinematic considerations. We shall assume henceforth that the degree of growth of the photoproduction amplitude is zero.

Based on the above assumption, one can use the Cauchy theorem to write for the combination (5.1) the following dispersion relations:

$$S_e R_{\nu\rho}(E,\lambda \mathbf{e}) = \frac{1}{\pi i}\,\mathrm{Vp}\int_{-\infty}^{+\infty} \frac{S_e R_{\nu\rho}(E',\lambda' \mathbf{e})}{E'-E}\, dE' + C, \qquad (5.2a)$$

$$\mathfrak{A}_{\mathbf{e}} R_{\nu\rho}(E, \lambda\mathbf{e}) = \frac{1}{\pi i} \mathrm{Vp} \int_{-\infty}^{+\infty} \frac{\mathfrak{A}_{\mathbf{e}} R_{\nu\rho}(E', \lambda'\mathbf{e})}{E' - E} dE', \tag{5.2b}$$

or, separating the Hermitian parts of (5.2a) and (5.2b), we obtain a dispersion relation that connects the Hermitian part of the amplitude $X_{\nu\rho}$ with the anti-Hermitian part $Y_{\nu\rho}$:

$$S_{\mathbf{e}} X_{\nu\rho}(E, \lambda\mathbf{e}) = \frac{1}{\pi} \mathrm{Vp} \int_{-\infty}^{+\infty} \frac{S_{\mathbf{e}} Y_{\nu\rho}(E', \lambda'\mathbf{e})}{E' - E} dE' + \mathrm{Re}\,\hat{C}, \tag{5.3a}$$

$$\mathfrak{A}_{\mathbf{e}} X_{\nu\rho}(E, \lambda\mathbf{e}) = \frac{1}{\pi} \mathrm{Vp} \int_{-\infty}^{+\infty} \frac{\mathfrak{A}_{\mathbf{e}} Y_{\nu\rho}(E', \lambda'\mathbf{e})}{E' - E} dE'. \tag{5.3b}$$

Using (2.16) in the dispersion relations (5.3a) and (5.3b), one can eliminate the region of negative energies and rewrite these expressions as

$$S_{\mathbf{e}} X_{\nu\rho}^{ps;\,p's'}(E, \lambda\mathbf{e}) = \frac{1}{\pi} \mathrm{Vp} \int_{0}^{+\infty} dE' \left[\frac{S_{\mathbf{e}} Y_{\nu\rho}^{ps;\,p's'}(E', \lambda'\mathbf{e})}{E' - E} + \frac{P_{ss'} S_{\mathbf{e}} Y_{\nu\rho}^{*ps;\,p's'}(E', \lambda'\mathbf{e})}{E' + E}\right] + \mathrm{Re}\,\hat{C}, \tag{5.4a}$$

$$\mathfrak{A}_{\mathbf{e}} X_{\nu\rho}^{ps;\,p's'}(E, \lambda\mathbf{e}) = \frac{1}{\pi} \mathrm{Vp} \int_{0}^{+\infty} dE' \left[\frac{\mathfrak{A}_{\mathbf{e}} Y_{\nu\rho}^{ps;\,p's'}(E', \lambda'\mathbf{e})}{E' - E} + \frac{P_{ss'} \mathfrak{A}_{\mathbf{e}} Y_{\nu\rho}^{*ps;\,p's'}(E', \lambda'\mathbf{e})}{E' + E}\right]. \tag{5.4b}$$

According to (4.5), the most general expression for energy-independent constant can be written as

$$\mathrm{Re}\,\hat{C} = \hat{C}_1(p^2)\, \boldsymbol{\sigma}\boldsymbol{\pi}^{\nu} + \hat{C}_2(p^2)\, (\boldsymbol{\sigma}\mathbf{p})\, (\mathbf{p}\boldsymbol{\pi}^{\nu}), \tag{5.5}$$

where $\hat{C}_1$ and $\hat{C}_2$ have an isotopic structure analogous to (4.3). Using (3.3) and (5.5), the dispersion relation (5.4a) can be represented as

$$S_e X_{\nu\rho}^{ps;\,p's'}(E, \lambda e) = -\frac{M^2 - \mu^2/4}{M^2 + \mathbf{p}^2} \sum_{\mu s''} \pi_{\mu}^{\nu} \left\{ S_e \frac{\langle \Psi^{*}_{p's'} I_{\mu}(0) \Psi_{p''s''}\rangle \langle \Psi_{p''s''} J_{\rho}(0) \Psi_{ps}\rangle}{E_2 - E} + S_e \frac{\langle \Psi^{*}_{ps'} J_{\rho}(0) \Psi_{p''s''}\rangle \langle \Psi^{*}_{p''s''} I_{\mu}(0) \Psi_{p's}\rangle}{E_2 + E} \right\} \tag{5.6}$$

$$+ \frac{1}{\pi} \mathrm{Vp} \int_{\frac{M\mu + \mu^2/4 - \mathbf{p}^2}{\sqrt{M^2 + \mathbf{p}^2}}}^{\infty} dE' \left[\frac{S_e Y_{\nu\rho}^{ps;\,p's'}(E', \lambda' e)}{E' - E} + \frac{P_{ss'} S_e \bar{Y}_{\nu\rho}^{*ps;\,p's'}(E', \lambda' e)}{E' + E}\right] + \hat{C}_1(p^2)\,(\boldsymbol{\sigma}\boldsymbol{\pi}^{\nu}) + \hat{C}_2(p^2)\,(\boldsymbol{\sigma}\mathbf{p})\,(\mathbf{p}\boldsymbol{\pi}^{\nu}).$$

A complete analysis of the dispersion relations is given in Ref. 6.

In conclusion I wish to express deep gratitude to Academician N. N. Bogoliubov, under whose guidance the work was performed, and also to B. M. Stepanov for evaluating this work.

Note added in proof (May 27, 1957). The results obtained in this article show that the dispersion relations given by B. L. Ioffe for the photoproduction processes [J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 583 (1956); Soviet Phys. JETP 4, 534 (1957)] are in error.

[1] M. L. Goldberger, Phys. Rev. 99, 979 (1955).

[2] Bogoliubov, Medvedev, and Polivanov, *Dispersion Relations*, Gostekhizdat, 1957 (in press).

[3] A. A. Logunov and B. M. Stepanov, Dokl. Akad. Nauk SSSR 110, 368 (1957); Soviet Phys. "Doklady" 1, 552 (1956). Logunov, Stepanov, and Tavkhelidze, Dokl. Akad. Nauk SSSR 112, 45 (1957); Soviet Phys. "Doklady" 2, 12 (1957).

[4] Koba, Katani, and Nakai, Progr. Theoret. Phys. 6, 849 (1951).
[5] G. F. Chew and F. E. Low, Phys. Rev. 101, 1579 (1956).
[6] A. A. Logunov and A. N. Tavkhelidze, Nuclear Physics (in press).

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Application of the Dirac-Fock-Podol'skii Method to a Mechanical Many-Body Problem

I. G. FIKHTENGOL'TS

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The general form of all the classical integrals of the motion and the generalized expression for the inertial mass of a body, in the relativistic many-body problem, are established with the aid of the many-time formalism. The results are applied to a system of interacting electric charges and to a system of bodies interacting gravitationally.

THE TREATMENT of the relativistic many-body problem is physically more coherent if the motion of the bodies is described by four-dimensional rather than by three-dimensional vectors. In view of this, we shall describe each of the n bodies of the system not merely by its three spatial coordinates x_i, y_i, z_i $(i = 1, 2, \ldots, n)$ but also by its time coordinate t_i. Correspondingly, we shall also apply the term "four-dimensional" to all functions and relations in which the motion of the bodies is described by four-dimensional vectors. Such a method was applied first in quantum electrodynamics in the work of Dirac, Fock, and Podol'skii,[1] and later in classical electrodynamics in the work of Markov.[2] In the present article this method is applied to the mechanical n-body problem.

The system of bodies is supposed isolated, and only the translational motion of the bodies is considered; no account is taken of the dependence of this motion on their shape and other parameters (the bodies are supposed spherically symmetric, and the distances between them are supposed much greater than their linear dimensions).

Treatment of the many-body problem from a purely mechanical point of view is naturally approximate, and permissible only when radiation may be neglected. Therefore we assume that the speeds of the mechanical motion are small in comparison with the speed of light, and we retain only quantities of order $\dot{\mathbf{r}}_i^2/c^2$ in the case of electrical interaction, and only quantities of order $\dot{\mathbf{r}}_i^4/c^4$ in the case of gravitational interaction between the bodies ($\dot{\mathbf{r}}_i^2$ is the square of the velocity of translational motion of the ith body).

1. THE FOUR-DIMENSIONAL EQUATIONS OF MOTION OF A SYSTEM OF BODIES, AND THEIR INTEGRALS

The equations of motion in which each body is described by means of its own time can be written in the form

$$d\mathcal{P}_{\nu i}/dt = \mathcal{F}_{\nu i}\,(\nu = 0, 1, 2, 3;\ i = 1, 2, \ldots, n). \quad (1)$$

Here

$$\mathcal{P}_{0i} = -c^{-1}\partial\mathcal{L}/\partial\dot{x}_{0i},\ \mathcal{P}_{ji} = \partial\mathcal{L}/\partial\dot{x}_{ji}, \quad (2)$$

$$\mathcal{F}_{0i} = -c^{-1}\partial\mathcal{L}/\partial x_{0i},\quad \mathcal{F}_{ji} = \partial\mathcal{L}/\partial x_{ji}\,(j = 1, 2, 3), \quad (3)$$

where $\mathcal{L}$ is the four-dimensional Lagrangian function (to be determined later), dependent on the variables t, $x_{\nu i}$, and $\dot{x}_{\nu i}$; t is the independent variable, for which we use the proper time of the coordinate system; $x_{0i} = t_i$ is the time coordinate, and $x_{1i} = x_i$, $x_{2i} = y_i$, $x_{3i} = z_i$ are the spatial coordinates, of the ith body in the chosen coordinate system. A superior dot indicates differentiation with respect to the variable t.

To establish relations between the general integrals of the four-dimensional equations (1) and the groups of transformations with respect to which these equations are invariant, we change from the variables t and $x_{\nu i}$ in Eqs. (1) to new variables τ