where

$$X = x(1-x), \quad Y = y(1-y),$$
$$Z = z(1-z), \quad T = t(1-t).$$

We will consider that the sum

$$A = \alpha + \beta \ln \frac{k^2}{m^2} + \gamma \left(\ln \frac{k^2}{m^2}\right)^2 \quad (2)$$

is the asymptotic form of the function f if

$$\lim [f(k^2/m^2) - A(k^2/m^2)] = 0 \text{ as } k^2/m^2 \to \infty. \quad (3)$$

Then the asymptotic form of the Green function of the photon is, in the approximation considered

$$iG_{\mu\nu} \sim \frac{k^2\delta_{\mu\nu} - k_\mu k_\nu}{k^4} \left\{1 + \frac{e^2}{12\pi^2}\left(\ln \frac{k^2}{m^2} - \frac{5}{3}\right) + \left[\frac{e^2}{12\pi^2}\left(\ln \frac{k^2}{m^2} - \frac{5}{3}\right)\right]^2 + \frac{e^4}{64\pi^4}\left(\ln \frac{k^2}{m^2} + \frac{139}{54} - \frac{22}{3}\zeta(2) + 4\zeta(3)\right)\right\}, \quad (4)$$

where $\zeta(2)$ and $\zeta(3)$ are the Riemann Zeta functions [see Eq. (5.10) of Ref. 1].

The coefficient $e^4/64\pi^4$ coincides with the coefficient obtained earlier by Jost and Luttinger[2] by a different procedure.

We give the numerical value of the constant contained in the asymptotic form:

$$C = \frac{139}{54} - \frac{22}{3}\zeta(2) + 4\zeta(3) = -4{,}680\,548\ldots \quad (5)$$

Taking the constant C into account does not change the structure of Eq. (30) of Ref. 3 for the asymptotic form of the Green function of the photon, but the charge e which comes into this formula is given now by the expression

$$e^2 = e_0^2 \Big/ \left[1 + \frac{5}{3}\frac{e_0^2}{3\pi} + \frac{e_0^4}{4\pi^2}4{,}68\right]. \quad (6)$$

The author is very grateful to N. P. Klepikov for help in this work, and also to V. G. Solov'ev for valuable advice.

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Translated by G. E. Brown
262

Reduction of the Two-Nucleon Problem to a Single-Nucleon Problem in the Nonrelativistic Range

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(Submitted to JETP editor February 25, 1957)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1262-1264 (May, 1957)

WE SHALL CONSIDER the interaction between two nucleons at the fixed points $\mathbf{r}_1$ and $\mathbf{r}_2$ and shall attempt to express the renormalized two-nucleon matrix elements in terms of renormalized single-nucleon matrix elements. We shall use as a basis the papers of Chew and Low[1] and Wick[2], in which single-nucleon problems are treated.

The energy operator is

$$H = H_0 + U_1 + U_2, \quad (1)$$

$$U_A = \sum_{\mathbf{k}} V_{\mathbf{k}}^0(A)\, e^{i\mathbf{k}\mathbf{r}_A} a_{\mathbf{k}} + V_{\mathbf{k}}^0(A)\, e^{-i\mathbf{k}\mathbf{r}_A} a_{\mathbf{k}}^+; \quad A = 1, 2. \quad (2)$$

Here $V_{\mathbf{k}}^0(A)$ contains the operators σ_A and τ^A, which apply to nucleon A; the rest of the notation is taken from Ref. 1.

The state Ψ_σ with two interacting physical nucleons is an eigenfunction of the Hamiltonian (1):

$$H\Psi_\sigma(1, 2, \bar{a}) = [2E_0 + E_\sigma(\rho)]\,\Psi_\sigma(1, 2, \bar{a}), \quad (\rho = \mathbf{r}_1 - \mathbf{r}_2), \quad (3)$$

where E_0 is the nucleon self-energy and $E_\sigma(\rho)$ is the static interaction energy of the nucleons. The symbol $\sigma \equiv (I', S', I_3', S_3')$ denotes the eigenvalues of the total spin, the total isotopic spin, and their three projections. In the representation where the creation operator $a_{\mathbf{k}}^+$ is equivalent to multiplication by $\bar{a}_{\mathbf{k}}$ *i.e.*, $a_{\mathbf{k}}^+\Psi = \bar{a}_{\mathbf{k}}\Psi$, the state vector Ψ_σ will be a function of $\bar{a}_{\mathbf{k}}$.

As the basic set of functions we shall use the products of single-nucleon state vectors $F_{\alpha\beta}(1, 2, \bar{a}) = F_\alpha(1, \bar{a})F_\beta(2, \bar{a})$, where α and β are spin and isotopic spin indices. $F_\alpha(1, \bar{a})$, which describes a nucleon in a meson cloud, is the solution of the Schroedinger equation

$$(H_0 + U_1) F(1, \bar{a}) = E_0 F(1, \bar{a}). \tag{4}$$

It can be shown[3] that for $\rho \to \infty$ the products $F_{\alpha\beta}(1, 2, \bar{a}) = F_\alpha(1, \bar{a}) F_\beta(2, \bar{a})$ are solutions of (3) and are subject to the orthogonality condition

$$(F_{\alpha\beta}(1, 2, \bar{a}), \quad F_{\alpha'\beta'}(1, 2, \bar{a})) = \delta_{\alpha\alpha'}\delta_{\beta\beta'}.$$

However for finite ρ these products are nonorthogonal functions of ρ.

We shall obtain Ψ_σ in the form

$$\Psi_\sigma = \Phi_\sigma + \chi_\sigma,$$

where $\Phi_\sigma = \sum c^\sigma_{\alpha\beta} F_{\alpha\beta}$ coincides with Ψ_σ for $\rho \to \infty$. When χ_σ is expanded in eigenfunctions of the total Hamiltonian H we shall restrict ourselves to the states Ψ_μ (without real mesons) and Ψ^q_μ (with one real meson) so that

$$\Psi_\sigma = \frac{1}{(\Psi_\sigma, \Phi_\sigma)} \Big[\Phi_\sigma - \sum_{\mu \neq \sigma} (\Psi_\mu, \Phi_\sigma) \Psi_\mu - \sum_{\mu, q} \frac{1}{q_0} (\Psi^q_\mu, [H - 2E_0 - E_\sigma] \Phi_\sigma) \Psi^q_\mu \Big]. \tag{5}$$

In the nonrelativistic approximation where small distances are unimportant Ψ_σ in the right-hand side can be replaced by Φ_σ. The principal difficulty here lies in the calculation of the matrix elements

$$(\alpha\beta \mid L \mid \alpha'\beta') = (F_\alpha(1, \bar{a}) F_\beta(2, \bar{a}), L(a, a^+) F_{\alpha'}(1, \bar{a}) F_{\beta'}(2, \bar{a})) \tag{6}$$

without being able to use the explicit single-nucleon states $F(1, \bar{a})$ and $F(2, \bar{a})$.

We introduce a different notation for the meson field variables in $F_{\alpha'}(1, \bar{a})$ and $F_{\beta'}(2, \bar{a})$, as follows:

$$F_{\alpha'}(1, \bar{a}) = F_{\alpha'}(1, \bar{a}_1), \quad F_{\beta'}(2, \bar{a}) = F_{\beta'}(2, \bar{a}_2)$$

(without any special assumptions). Then, for example, the matrix element (6) with $L = 1$ will be written as

$$F^*_\alpha\left(1, \frac{\partial}{\partial \bar{a}_1} + \frac{\partial}{\partial \bar{a}_2}\right) F^*_\beta\left(2, \frac{\partial}{\partial \bar{a}_1} + \frac{\partial}{\partial \bar{a}_2}\right) \times F_{\alpha'}(1, \bar{a}_1) F_{\beta'}(2, \bar{a}_2)\big|_{\bar{a}_1 = \bar{a}_2 = 0}. \tag{7}$$

Assume now that a meson cloud interacts much more strongly with its "own" nucleon than with another nucleon. Then in $F^*_\alpha(1, \partial/\partial\bar{a}_1 + \partial/\partial\bar{a}_2)$ the operator $\partial/\partial\bar{a}_2$ will be small compared with $\partial/\partial\bar{a}_1$ and in $F^*_\beta(2, \partial/\partial\bar{a}_1 + \partial/\partial\bar{a}_2)$ the operator $\partial/\partial\bar{a}_1$ will be small compared with $\partial/\partial\bar{a}_2$. Since for small $\bar{a}_2$

$$F(1, \bar{a}_1 + \bar{a}_2) \approx F(1, \bar{a}_1) + \sum_{\mathbf{k}} a^+_{2\mathbf{k}} a_{1\mathbf{k}} F(1, \bar{a}_1) + \dots, \tag{8}$$

we obtain when we limit ourselves to the linear term in (8)

$$(\alpha\beta \mid \alpha'\beta') = (F_\alpha(1, \bar{a}_1) F_\beta(2, \bar{a}_2), (1 + \hat{N}) F_{\alpha'}(1, \bar{a}_1) F_{\beta'}(2, \bar{a}_2)), \tag{9}$$

$$N = \sum_{\mathbf{q}} [a^+_{1\mathbf{q}} a_{2\mathbf{q}} + a^+_{2\mathbf{q}} a_{1\mathbf{q}}], \tag{10}$$

with $[a_{1\mathbf{q}}, a^+_{2\mathbf{q}}] = 0$, $[a_{1\mathbf{q}}, a^+_{1\mathbf{q}'}] = \delta_{\mathbf{q}\mathbf{q}'}$ etc.

In general, for the calculation of (6) all $a_{\mathbf{k}}$ and $a^+_{\mathbf{k}}$ must first operate on the functions $F(1, \bar{a})$ and $F(2, \bar{a})$, following which (7) and (8) will be used. For example,

$$(\alpha\beta \mid H - 2E_0 \mid \alpha'\beta') = (F_\alpha(1, \bar{a}_1) F_\beta(2, \bar{a}_2), (1 + \hat{N}) [U^+_1(a_2) + U^+_2(a_1)] F_{\alpha'}(1, \bar{a}_1) F_{\beta'}(2, \bar{a}_2)), \tag{11}$$

where $U^+_1(a_2)$ is the annihilation component of the operator U_1 with annihilation operators $a_{2\mathbf{k}}$. The right-hand sides of (9) and (11) can be expressed in terms of the single-nucleon matrix elements $(F_\alpha, V^0_{\mathbf{k}} F_{\alpha'})$ and $(F^q_\alpha, V^0_{\mathbf{k}} F_{\alpha'})$, where F^q_α is the state with a nucleon and one (real) meson q. The first of these matrix elements is known to be $(u_\alpha, V_{\mathbf{k}} u_{\alpha'})$, where u is the spin-isotopic spin function of the bare nucleon and $V_{\mathbf{k}}$ contains the renormalized charge f. The second matrix element is associated with the meson-nucleon elastic scattering amplitude. The rule for calculating expressions such as (6) can be expressed as follows. In the coordinate representation the annihilation component $\varphi^{(+)}(\mathbf{r}_1)$ of the meson operator $\varphi(\mathbf{r}_1)$ is

$$\varphi^{(+)}(\mathbf{r}_1) = \int \Delta^{(+)}(\mathbf{r}_1 - \mathbf{r}) \frac{\delta}{\delta\varphi^{(-)}_{(\mathbf{r})}} d^3\mathbf{r}, \tag{12}$$

$$\text{where} \quad \Delta^{(+)}(\mathbf{r}_1 - \mathbf{r}) = [\varphi^{(+)}(\mathbf{r}_1), \varphi^{(-)}(\mathbf{r})],$$

where $\varphi^{(-)}(\mathbf{r})$ is the creation component of $\varphi(\mathbf{r})$. If τ_1 and τ_2 are the regions occupied by the meson clouds of nucleons 1 and 2, $F(1, \bar{a})$ will depend on $\varphi^{(-)}(\mathbf{r})$, where $\mathbf{r}$ lies in the region τ_1, and $F(2, \bar{a})$

will depend on $\varphi^{(-)}(\mathbf{r})$ in the volume τ_2. Then division of the operator $\partial/\partial\bar{a}_1 + \partial/\partial\bar{a}_2$ in $F_\alpha^*(1)$ [Eq. (7)] into a larger part $\partial/\partial\bar{a}_1$ and a smaller part $\partial/\partial\bar{a}_2$ corresponds to division of $\varphi^{(+)}(\mathbf{r}_1)$ into two terms – an integral over τ_1 (the larger part) and an integral over τ_2 (the smaller part).

Equation (8) is not an expansion with respect to renormalized charge because the single-nucleon matrix element $(F_\alpha^q, V_\mathbf{k}^0 F_\alpha')$ cannot be calculated by ordinary perturbation theory. Its value must either be calculated exactly or obtained from pion-nucleon scattering experiments.

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Translated by I. Emin
263