$$P \approx (e^2 c/2l^2)\sqrt{\varepsilon'/\mu'}, \quad P_l \approx (2\rho^2 c/l)\sqrt{\varepsilon'/\mu'}. \tag{6}$$

These expressions can be interpreted in an analogous fashion as the previous ones.

The force acting on the particle in a direction normal to the surface of the medium is given by

$$F = -\frac{e^2}{\pi\theta l^2}\left\{\frac{\mu'\varepsilon'-1}{\mu'-\varepsilon'} L(\varepsilon') - \left(\frac{\mu'\varepsilon'-1}{\mu'-\varepsilon'} + \mu'\theta^2\right) L(\mu') + \frac{\pi}{4}\theta^2\right\}, \tag{7}$$

$$L(\lambda) = \frac{\lambda}{\sqrt{(\lambda^2-1)(\lambda^2+\Gamma^2)}} \tan^{-1}\left(\sqrt{\frac{\lambda^2-1}{\lambda^2+\Gamma^2}}\,\Gamma\right) + \int_0^{\tan^{-1}\Gamma} \frac{d\alpha}{\lambda+\sqrt{1-(\Gamma^2+1)\sin^2\alpha}}. \tag{8}$$

The equivalent force per unit length acting on a charged line is

$$F_l = -(\rho^2/l)(\varepsilon'^2-\Gamma^2)/(\varepsilon'^2+\Gamma^2). \tag{9}$$

One sees from (8) that in the case of a magnetic medium ($\mu' \gg 1$, $\varepsilon' = 1$) the force is attractive up to $\Gamma \sim 1$ and becomes attractive again at $\Gamma \approx \mu'$. For $\mu' \gg \Gamma \gg 1$ the repulsive force equals $e^2/(4l^2)$. On the other hand, in the case of a dieletric, the force is always attractive and has a value close to $-e^2/(4l^2)$.

For comparison we note that for a charged line the force is repulsive beginning at the inversion velocity (*i.e.*, at the velocity where $\Gamma = \varepsilon'$) and remains repulsive from there on for all velocities.

In conclusion, the author wishes to express his gratitude to Prof. A. A. Sokolov for his interest in this work, to I. Kvasnitsa for the discussion of the calculations, and to V. Pafomov for acquainting the author with his work[2].

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Photon Green Function Accurate to e^4

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THE SUM OF DIAGRAMS for the self energy of the photon, inclusive of fourth-order diagrams, was calculated. After carrying out renormalization in the usual way and evaluating the integrals over the momenta, the following expression was obtained

$$\begin{aligned} iG_{\mu\nu} = \frac{k^2\delta_{\mu\nu} - k_\mu k_\nu}{k^4} \Bigg\{ 1 &+ \frac{e^2}{2\pi^2}\int_0^1 dx\, X \ln\left(1+\frac{k^2}{m^2}X\right) + \left[\frac{e^2}{2\pi^2}\int_0^1 dx\, X \ln\left(1+\frac{k^2}{m^2}X\right)\right]^2 \\ &+ \frac{e^4}{16\pi^4}\Bigg[\left[\frac{1}{2}\int_0^1 dx \ln\left(1+\frac{k^2}{m^2}X\right)\right]^2 + \int_0^1 dx\,(2-3x+2x^2)\ln\left(1+\frac{k^2}{m^2}X\right) \\ &+ \int_0^1 dx \int_0^1 dy \left(y - 2y^2 - \frac{y}{x}\right)\ln\left(1+\frac{k^2 Y x}{m^2(1-y+yx)}\right) + \int_0^1 dx\int_0^1 dy\int_0^1 dz\int_0^1 dt \\ &\times \frac{k^2[1-x(1-t)-zt][x(1-t)+zt] + m^2\{1+2[x(1-t)+zt][yx(1-t)+yzt+x(1-y)]\}}{(1-t)(k^2X+m^2) + y[t(k^2Z+m^2)+k^2T(z-x)^2]} \\ &- \int_0^1 dx\int_0^1 dy\int_0^1 dz \frac{2m^2(3-x)\,yz^2(1-yz)}{k^2x(Yz+Zy^2)+m^2(1-z+zx)} + \frac{49}{54} + \frac{5}{12} - \frac{11}{6}\zeta(2)\Bigg] + \cdots \Bigg\}, \end{aligned} \tag{1}$$

where

$$X = x(1-x),\quad Y = y(1-y),$$
$$Z = z(1-z),\quad T = t(1-t).$$

We will consider that the sum

$$A = \alpha + \beta \ln \frac{k^2}{m^2} + \gamma\left(\ln \frac{k^2}{m^2}\right)^2 \tag{2}$$

is the asymptotic form of the function f if

$$\lim\left[f(k^2/m^2) - A(k^2/m^2)\right] = 0 \text{ as } k^2/m^2 \to \infty. \tag{3}$$

Then the asymptotic form of the Green function of the photon is, in the approximation considered

$$iG_{\mu\nu} \sim \frac{k^2\delta_{\mu\nu} - k_\mu k_\nu}{k^4}\left\{1 + \frac{e^2}{12\pi^2}\left(\ln \frac{k^2}{m^2} - \frac{5}{3}\right) + \left[\frac{e^2}{12\pi^2}\left(\ln \frac{k^2}{m^2} - \frac{5}{3}\right)\right]^2 + \frac{e^4}{64\pi^4}\left(\ln \frac{k^2}{m^2} + \frac{139}{54} - \frac{22}{3}\zeta(2) + 4\zeta(3)\right)\right\}, \tag{4}$$

where $\zeta(2)$ and $\zeta(3)$ are the Riemann Zeta functions [see Eq. (5.10) of Ref. 1].

The coefficient $e^4/64\pi^4$ coincides with the coefficient obtained earlier by Jost and Luttinger[2] by a different procedure.

We give the numerical value of the constant contained in the asymptotic form:

$$C = \frac{139}{54} - \frac{22}{3}\zeta(2) + 4\zeta(3) = -4{,}680\,548\ldots \tag{5}$$

Taking the constant C into account does not change the structure of Eq. (30) of Ref. 3 for the asymptotic form of the Green function of the photon, but the charge e which comes into this formula is given now by the expression

$$e^2 = e_0^2 \Bigg/ \left[1 + \frac{5}{3}\frac{e_0^2}{3\pi} + \frac{e_0^4}{4\pi^2}4{,}68\right]. \tag{6}$$

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Reduction of the Two-Nucleon Problem to a Single-Nucleon Problem in the Nonrelativistic Range

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WE SHALL CONSIDER the interaction between two nucleons at the fixed points $\mathbf{r}_1$ and $\mathbf{r}_2$ and shall attempt to express the renormalized two-nucleon matrix elements in terms of renormalized single-nucleon matrix elements. We shall use as a basis the papers of Chew and Low[1] and Wick[2], in which single-nucleon problems are treated.

The energy operator is

$$H = H_0 + U_1 + U_2, \tag{1}$$

$$U_A = \sum_{\mathbf{k}} V_{\mathbf{k}}^0(A)\, e^{i\mathbf{k}\mathbf{r}_A} a_{\mathbf{k}} + V_{\mathbf{k}}^0(A)\, e^{-i\mathbf{k}\mathbf{r}_A} a_{\mathbf{k}}^+;\quad A = 1, 2. \tag{2}$$

Here $V_{\mathbf{k}}^0(A)$ contains the operators σ_A and τ^A, which apply to nucleon A; the rest of the notation is taken from Ref. 1.

The state Ψ_σ with two interacting physical nucleons is an eigenfunction of the Hamiltonian (1):

$$H\Psi_\sigma(1, 2, \bar{a}) = [2E_0 + E_\sigma(\rho)]\,\Psi_\sigma(1, 2, \bar{a}),\quad (\rho = \mathbf{r}_1 - \mathbf{r}_2), \tag{3}$$

where E_0 is the nucleon self-energy and $E_\sigma(\rho)$ is the static interaction energy of the nucleons. The symbol $\sigma \equiv (I', S', I_3', S_3')$ denotes the eigenvalues of the total spin, the total isotopic spin, and their three projections. In the representation where the creation operator $a_{\mathbf{k}}^+$ is equivalent to multiplication by $\bar{a}_{\mathbf{k}}$ *i.e.*, $a_{\mathbf{k}}^+\Psi = \bar{a}_{\mathbf{k}}\Psi$, the state vector Ψ_σ will be a function of $\bar{a}_{\mathbf{k}}$.

As the basic set of functions we shall use the products of single-nucleon state vectors $F_{\alpha\beta}(1, 2, \bar{a}) = F_\alpha(1, \bar{a})F_\beta(2, \bar{a})$, where α and β are spin and isotopic spin indices. $F_\alpha(1, \bar{a})$, which describes a nucleon in a meson cloud, is the solution of the Schroedinger equation