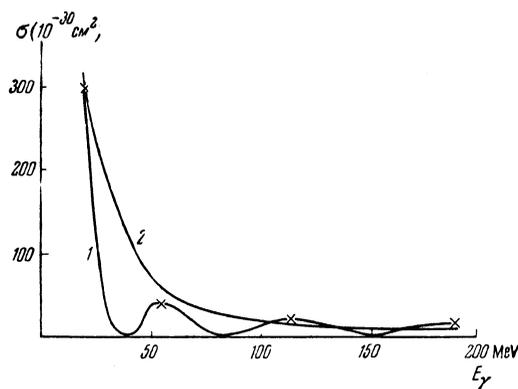


tances should become more and more important in releasing the neutron.



At  $\hbar\omega = 5-8$  Mev Eq. (3) gives  $r_0' \approx 2.6 \times 10^{-13}$  cm. This particular value agrees with the experiment on the collision of the deuteron with the beryllium nucleus, treated in Ref. (2).

If we consider that in the energy range 20–200 Mev, where there is a satisfactory agreement between theory and experiment,  $r_0'$  changes only by a factor 2.5, we can conclude that the assumption of a slow energy dependence of  $r_0'$  will explain the experimental data over a wide range of  $\gamma$ -energies.

In conclusion I wish to thank Prof. V. I. Mamasakhlisov for helpful discussions.

<sup>1</sup>H. Überall, Z. Naturforsch., 8a, 142 (1953).

<sup>2</sup>T. I. Kopaleishvili. Trudy (Transactions), Tbilisi State University, 62, 1957 (in press).

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## Distribution of the Electron-Photon Component on the Periphery of Extensive Air Showers of Cosmic Rays

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**I**NVESTIGATIONS of the lateral distribution function of the electron-photon component of extensive air showers at 3860 m were carried out by means of an experimental set-up<sup>1</sup> consisting of a

large number of hodoscoped counters placed at various distances from each other. Counters intended for the measurement of the flux density of all charged particles in the shower were placed under thin covers made of aluminum and wood ( $\sim 2\text{g}/\text{cm}^2$ ). The particle flux density at a given distance from the axis of the shower was determined according to the formula

$$\rho(r) = \frac{1}{\sigma} \ln \frac{n}{n-m}$$

This formula makes it possible to determine the most probable value of the flux density of charged particles that discharge  $m$  out of the total number of  $n$  counters, each of area  $\sigma$ , situated in the given place of observation.

In the case when  $\rho\sigma n \ll 1$  the determination of the particle flux density in an individual case of shower detection is impossible. In such cases the particle flux density was determined for a group of showers identical with respect to the total number of particles and the position of the shower axes. It was assumed that  $n$  equals the product of the total number of counters by the number of showers of a given group and  $m$  is the total number of counter discharges during the passage of all showers of the group. Test computation showed that the probable value of the particle flux density in a given group of showers is, within the limits of statistical accuracy, the same as the mean weighted value of the probable values of the particle flux density determined for each separate shower of the group.

We considered three groups of extensive air showers with definite axis positions. The first group comprised showers with a total number of particles between  $5 \times 10^4$  and  $1 \times 10^5$ . The energy of primary particles producing these showers can be estimated<sup>2</sup> as  $\bar{E}_0 = 1.6 \times 10^{14}$  ev. The limits of the total number of shower primaries and the corresponding mean energy of primaries for the second and third group were  $1.5 \times 10^5 - 2.6 \times 10^5$  ( $\bar{E}_0 = 5 \times 10^{14}$  ev) and  $5 \times 10^5 - 13 \times 10^5$  ( $\bar{E}_0 = 16 \times 10^{14}$  ev). The lateral distribution functions of all charged particles obtained for the three groups of showers are shown in Fig. 1. The plotted lateral distributions for distances from the axis smaller than 40 m are based on previously published results<sup>3</sup>.

The experimental data given in Fig. 1 characterize the lateral distribution of the electron-photon component of the shower only at distances from the axis smaller than  $\sim 100$  m, where the relative contribution of penetrating particles is not appreciable.

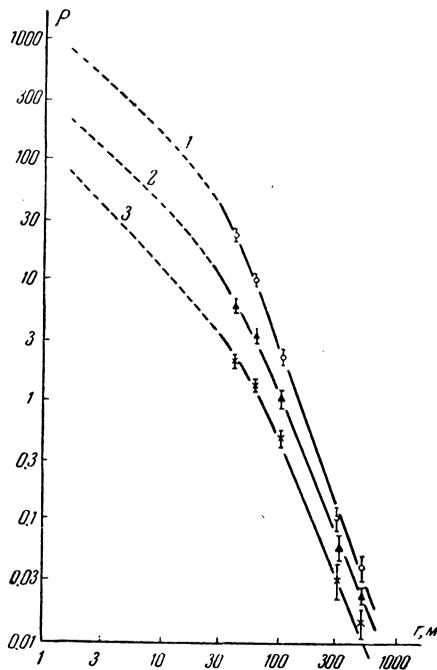


FIG. 1

FIG. 1. Lateral distribution of charged particles in extensive atmospheric showers, caused by particles with different energies. 1 -  $\bar{E}_0 \approx 1.6 \times 10^{15}$ ; 2 -  $\bar{E}_0 \approx 5 \times 10^{14}$ ; 3 -  $\bar{E}_0 \approx 1.6 \times 10^{14}$  ev.

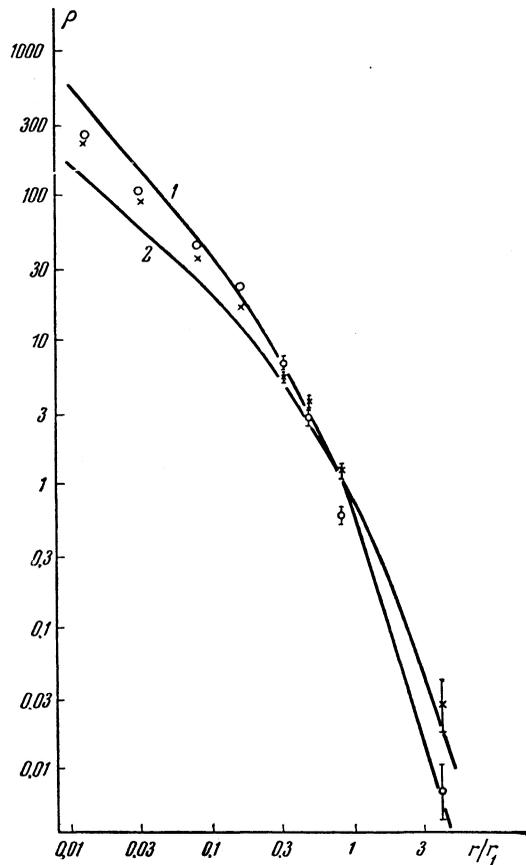


FIG. 2

FIG. 2. Lateral distribution of electron-photon component of extensive atmospheric showers. Number of electrons in showers:  $\circ - 7.4 \times 10^5$ ;  $\times - 7.7 \times 10^4$ ; value of  $s$  is: for 1 - 1, for 2 - 1.4;  $r_1 = 125$  m.

To determine of the lateral distribution of the electron-photon component on the shower periphery it is necessary to account for the share of  $\mu$ -mesons in the total flux of charged particles. The lateral distribution and the energy spectrum at various distances from the axis were obtained in the same series of measurements<sup>1,4</sup>. In consistence with these, the number of  $\mu$ -mesons with energy  $\geq 250$  Mev, multiplied by 1.3, was subtracted from the total flux of charged particles at 100 and 500 meters from the axis, to account for the electron-photon component in equilibrium with the  $\mu$ -mesons. The results for showers with  $7.4 \times 10^5$  and  $7.7 \times 10^6$  electrons not in equilibrium with  $\mu$ -mesons are shown in Fig. 2. The functions of the lateral distribution of electrons calculated according to the electromagnetic cascade theory<sup>5</sup> for two values of the parameter  $s$  are shown

in the same figure. The experimental values of the electron flux density at various distances from the axis and the theoretically calculated functions of lateral distribution are normalized to the same number of electrons at distances between 0 and 1000 m ( $2 \times 10^5$ ).

It can be seen from Fig. 2 that the lateral distribution of electrons on the periphery of extensive air showers varies with the energy of the shower-producing primary. Comparison of the experimental data with the calculations of Nishimura and Kamata<sup>5</sup> indicates that the lateral distribution of electrons in extensive air showers produced by primaries with the energy of  $\sim 2 \times 10^{14}$  ev coincides with the calculated distribution of the electron-photon cascade for the parameter  $s = 1.2$ . Such a value of the parameter can be expected from the altitude develop-

ment of extensive air showers<sup>6,7</sup>.

The lateral distribution of electrons in extensive air showers produced by primaries with the energy of  $(1-2) \times 10^{15}$  ev does not conform with the functions calculated by Nishimura and Kamata for the distribution of electrons in the electron-photon cascade for any value of the parameter  $s$ , including  $s = 1.2$ .

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## On Quasiclassical Single-Electron Wave Functions

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FOR THE PURPOSE OF evaluating the matrix elements for various interactions between matter and a radiation field it is necessary to have good nonrelativistic wave functions of the radiating electron. Hydrogen-like functions or Slater functions are ordinarily used in calculations. However, the results obtained by using these functions are not accurate because they do not take into account the influence of all other electrons of an atom on the radiating electron, or do so insufficiently.

We have constructed approximate wave functions

for an electron in bound states which allow for the electrostatic screening effect of the atomic electrons on the radiating electron. The calculations were based on a combination of the Fermi-Thomas statistical method of obtaining the effective electrostatic field which acts on an individual electron in an atom with a generalized quasiclassical method of calculating wave functions in a given potential field.

The effective electrostatic field acting on a radiating electron in a heavy atom can be described by the Thomas-Fermi potential corrected at small and large distances from the nucleus, as follows:

$$U(r) = \begin{cases} -Z(1 - \alpha r) / r, & r \leq 1/Z; \\ -\frac{1}{r} - \frac{(Z-1)}{r} \varphi_0\left(\frac{r}{\mu}\right); & 1/Z \leq r \leq r_0, \\ -1/r, & r \geq r_0, \end{cases} \quad (1)$$

where  $r_0$  is the boundary radius of a  $(Z-1)$ -fold ionized atom in the Thomas-Fermi statistical model and the constant  $\alpha$  is determined from the continuity of the potential at  $r = 1/Z$  [all quantities in Eq. (1) are measured in atomic units].

Since  $U(r)$  is a centrally symmetrical field the angular dependence of the eigenfunctions  $\psi$  is exactly the same as in the case of the hydrogen atom. The problem is therefore reduced to the calculation of the radial functions  $R_{nl}(r)$  from the equation

$$d^2 x_{nl}(r) / dr^2 + \{2[\epsilon_{nl} - U(r)] - l(l+1)/r^2\} x_{nl}(r) = 0, \\ x_{nl}(r) = r R_{nl}(r), \quad (2)$$

where  $U(r)$  is taken from (1).

The solutions of (2) are obtained quasiclassically in the generalized form first indicated by Fock and by Petrashen'<sup>1</sup> and later by other authors<sup>2</sup>. Using the Fock-Petrashen' method the desired solutions of (2) can be represented approximately as

$$x_{nl}(r) = A(S')^{-1/2} \varphi[S(r)], \quad (3)$$

where  $\varphi(S)$  is the solution of the radial Schroedinger equation with a Coulomb potential:

$$\varphi(S) = S R_{nl}^{\text{hydr}}(S),$$

and  $S = S(r)$ , which we shall call the screening function, is determined from the equation

$$\int_{r_1}^r \sqrt{2[\epsilon_{nl} - U(r)] - \frac{l(l+1)}{r^2}} dr \\ = \int_{S_1}^S \sqrt{-\frac{Z^2}{n^2} + \frac{2Z}{S} - \frac{l(l+1)}{S^2}} dS. \quad (4)$$