

### Bremsstrahlung of Polarized Electrons

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(Submitted to JETP editor May 3, 1956)

J. Exptl. Theoret. Phys. 32, 1078-1082 (May, 1957)

The polarization properties of the bremsstrahlung arising when a polarized electron beam strikes a target are considered. It is shown that in this case the bremsstrahlung contains a circularly polarized component. For highly relativistic electrons which are completely polarized in the direction of motion, the circular polarization near the upper limit of the spectrum amounts to as much as 25%; multiple scattering of electrons in the target has practically no effect on this value. Equations have been derived for the polarization of bremsstrahlung photons produced by electrons of arbitrary energy.

**T**HE POLARIZATION PROPERTIES of bremsstrahlung have been studied in detail in a series of papers<sup>1, 2, 3, 4</sup> on the assumption that the electron beam striking the target is not polarized. In this case the bremsstrahlung contains a linearly polarized component in addition to the unpolarized part. If an integration is carried out over the direction of the final momentum of the electron, the bremsstrahlung turns out to be partly polarized along the normal to the plane of emission. At high energies the effective polarization falls off strongly as a result of multiple scattering of electrons in the target.

In connection with the possibility mentioned by one of the authors<sup>5</sup> of obtaining an intense beam of polarized electrons, the following question arose: what properties would the bremsstrahlung have if the electron beam striking the target had arbitrary polarization? It has already been pointed out by Zel'dovich<sup>6</sup> that polarization of the incident electrons leads to circular polarization of the bremsstrahlung photons. The present paper is devoted to a quantitative investigation of this question.

In order to describe the polarization properties of the beam of photons we will make use of density matrices<sup>7, 8</sup>

$$\rho_{ph} = \frac{1}{2} (1 + \xi \Omega), \tag{1}$$

where  $\Omega$  is a "matrix vector" with components

$$\Omega_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \Omega_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \Omega_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{2}$$

$\xi_1, \xi_2, \xi_3$  are Stokes parameters. We recall that the set of values  $\xi_1 = 1$  ( $\xi_1 = -1$ ),  $\xi_2 = \xi_3 = 0$  designates linear polarization along the OX (OY) axis; the set  $\xi_2 = 1$  ( $\xi_2 = -1$ ),  $\xi_1 = \xi_3 = 0$  designates lin-

ear polarization at an angle of 45° with respect to the OX(OY) axis; and the set  $\xi_3 = 1$  ( $\xi_3 = -1$ ),  $\xi_1 = \xi_2 = 0$  designates clockwise (counterclockwise) circular polarization. The quantities  $\xi_i$  are not components of a vector and under a rotation of the coordinate axes they are transformed according to a different law. The vector symbol  $\xi$  has to be understood in a strictly formal sense; in particular,  $\xi \Omega$  is an abbreviation for the sum  $\xi_1 \Omega_1 + \xi_2 \Omega_2 + \xi_3 \Omega_3$ .

If the electron beam striking the target is not polarized, the bremsstrahlung parameters  $\xi_i$  are determined by the expression

$$\xi \Omega = {}_{\lambda \lambda'} \text{Sp} (S_\lambda S_{\lambda'}^+) / \text{Sp} (S_\lambda S_\lambda^+), \tag{3}$$

where  $S_\lambda$  is the element of the scattering matrix corresponding to the emission of a bremsstrahlung photon with polarization vector  $e_\lambda$  ( $\lambda = 1, 2$ ;  $e_\lambda e_{\lambda'} = \delta_{\lambda \lambda'}$ ,  $e_\lambda k = 0$ ); we assume that the corresponding operators of the projection are already included in  $S_\lambda$ , and it is possible to write down the "spurs" at once.

We define  $Q_{\lambda \lambda'} = \text{Sp}(S_\lambda S_{\lambda'}^+)$ . Calculations yield

$$q^2 Q_{\lambda \lambda'} \sim 4 \left( \frac{\varepsilon_0}{x_0} \mathbf{p} + \frac{\varepsilon}{x} \mathbf{p}_0, \mathbf{e}_\lambda \right) \left( \frac{\varepsilon_0}{x_0} \mathbf{p} + \frac{\varepsilon}{x} \mathbf{p}_0, \mathbf{e}_{\lambda'} \right) - q^2 \left( \frac{\mathbf{p}}{x_0} + \frac{\mathbf{p}_0}{x}, \mathbf{e}_\lambda \right) \left( \frac{\mathbf{p}}{x_0} + \frac{\mathbf{p}_0}{x}, \mathbf{e}_{\lambda'} \right) - \frac{\delta_{\lambda \lambda'}}{x_0 x} [\mathbf{p} - \mathbf{p}_0, \mathbf{k}]^2 \tag{4}$$

[we have ignored factors which play no role when  $Q_{\lambda \lambda'}$  is substituted into equation (3)]; here

$$\begin{aligned}x_0 &= -\frac{2\varepsilon\omega}{m^2}(1 - v \cos \vartheta), \\x &= \frac{2\varepsilon_0\omega}{m^2}(1 - v_0 \cos \vartheta_0),\end{aligned}\quad (5)$$

$\mathbf{p}_0$ ,  $\mathbf{p}$  are the initial and final momenta of the electron,  $\mathbf{k}$  the momentum of the photon,  $\varepsilon_0$ ,  $\varepsilon$  and  $\omega$  the corresponding energies,  $\theta_0$ ,  $\theta$  the angles between  $\mathbf{k}$  and  $\mathbf{p}_0$ ,  $\mathbf{q}$  the momentum transferred to the nucleus ( $\mathbf{q} = \mathbf{p} + \mathbf{k} - \mathbf{p}_0$ ).

Except for a factor, the denominator of the right hand side of equation (3) agrees with the bremsstrahlung cross-section<sup>9</sup>:

$$\begin{aligned}q^4(Q_{11} + Q_{22}) &\sim q^4 Q(\mathbf{p}_0, \mathbf{p}, \mathbf{k}) \\&= \frac{4}{\omega^2} \left[ \mathbf{k}, \frac{\varepsilon_0}{x_0} \mathbf{p} + \frac{\varepsilon}{x} \mathbf{p}_0 \right]^2 - \frac{q^2}{\omega^2} \left[ \mathbf{k}, \frac{\mathbf{p}}{x_0} + \frac{\mathbf{p}_0}{x} \right]^2 \\&\quad - \frac{2}{x_0 x} [\mathbf{k}, \mathbf{p} - \mathbf{p}_0]^2.\end{aligned}\quad (6)$$

As coordinate axes we select mutually perpendicular vectors  $\mathbf{e}_1$ ,  $\mathbf{e}_2$ , and  $\mathbf{k}/\omega$ . The choice of  $\mathbf{e}_1$  and  $\mathbf{e}_2$  is arbitrary; for definiteness in what follows we set  $\mathbf{e}_1 = \mathbf{n} \equiv [\mathbf{p}_0 \mathbf{k}] / |[\mathbf{p}_0 \mathbf{k}]|$ ,  $\mathbf{e}_2 = \mathbf{l} \equiv [\mathbf{k} \mathbf{n}] / \omega$ . Since  $Q_{12} = Q_{21}$ , the third Stokes parameter is equal to zero; i.e., the bremsstrahlung is linearly polarized:

$$\begin{aligned}\xi_1 &= \frac{Q_{11} - Q_{22}}{Q_{11} + Q_{22}} \\&= \frac{2(4\varepsilon_0^2 - q^2)(\mathbf{p} \mathbf{n})^2 - 2\frac{x_0}{x} [\mathbf{p} - \mathbf{p}_0, \mathbf{k}]^2}{x_0^2 q^4 Q(\mathbf{p}_0, \mathbf{p}, \mathbf{k})} - 1,\end{aligned}\quad (7)$$

$$\begin{aligned}\xi_2 &= \frac{2Q_{12}}{Q_{11} + Q_{22}} \\&= \frac{2(\mathbf{p} \mathbf{n})}{x_0 q^4 Q} \left( \frac{4\varepsilon_0^2 - q^2}{x_0} \mathbf{p} + \frac{4\varepsilon_0 \varepsilon - q^2}{x} \mathbf{p}_0, \mathbf{l} \right),\end{aligned}\quad (8)$$

$$\xi_3 = i(Q_{12} - Q_{21}) / (Q_{11} + Q_{22}) = 0. \quad (9)$$

In Eqs. (7)–(9) the direction of  $\mathbf{p}$  is fixed. Since it does not interest us, we have to multiply  $\xi$  by the probability of a given direction  $\mathbf{p}$ , which is equal to

$$\omega(\mathbf{p}) d\omega_p = Q(\mathbf{p}_0, \mathbf{p}, \mathbf{k}) d\omega_p / \int Q(\mathbf{p}_0, \mathbf{p}', \mathbf{k}) d\omega_{p'},$$

and integrate over all directions  $\mathbf{p}$ :

$$\bar{\xi} = \int \Omega_{\lambda\lambda'} Q_{\lambda\lambda'} d\omega_p / \int Q d\omega_{p'}. \quad (10)$$

The integral of equation (8) gives zero (as was to be expected from symmetry considerations), and for the single non-vanishing parameter  $\bar{\xi}_1$  we obtain a result well-known from previous work<sup>2, 4</sup>.

The state of polarization of electrons with momentum  $\mathbf{p}_0$  is described by a four-row density matrix<sup>7, 10</sup>

$$\rho_e = \eta^{(+)}(\mathbf{p}_0) \frac{1}{2} (1 + \zeta \Sigma \gamma_4) \eta^{(+)}(\mathbf{p}_0), \quad \Sigma = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix}, \quad (11)$$

where  $\eta^{(+)}(\mathbf{p}_0)$  is an operator of the projection onto a state with positive energy  $\varepsilon_0 = \sqrt{m^2 + \mathbf{p}_0^2} > 0$

$$\eta^{(+)}(\mathbf{p}_0) = \frac{1}{2\varepsilon_0} (m - i\hat{p}_0) \gamma_4, \quad \hat{p}_0 = p_{0\mu} \gamma_\mu, \quad (12)$$

and the vector  $\zeta$ , characterizing the polarization, is equal to

$$\zeta = \zeta^0 + \frac{\varepsilon_0}{m} \zeta^i; \quad (13)$$

$\zeta^0$  is the polarization vector in the coordinate system in which the electrons are at rest,  $\zeta^i$  ( $\zeta^{0i}$ ) is its component perpendicular (parallel) to the vector  $\mathbf{p}_0$ .

When the electron beam is polarized [matrix density of Eq. (11)] the Stokes parameters of the bremsstrahlung are equal to

$$\bar{\xi} = \frac{\Omega_{\lambda\lambda'} \text{Sp} \{ S_\lambda [1 + (\zeta \Sigma) \gamma_4] S_\lambda^+ \}}{\text{Sp} [S_\lambda (1 + \zeta \Sigma \gamma_4) S_\lambda^+]}; \quad (14)$$

where for brevity, as in (3), we do not write out the factors  $\eta^{(+)}(\mathbf{p}_0)$ ,  $\eta^{(+)}(\mathbf{p})$ , taking them to be included in  $S_\lambda$ , i.e.,

$$S_\lambda = \eta^{(+)}(\mathbf{p}) S_\lambda^{(0)} \eta^{(+)}(\mathbf{p}_0),$$

where  $S_\lambda^{(0)}$  is the scattering matrix in the usual sense.

We define  $R_{\lambda\lambda'} = \text{Sp} (S_\lambda \zeta \Sigma \gamma_4 S_\lambda^+)$ . It is easily shown that  $R_{11} + R_{22} = 0$ , and the denominator  $\text{Sp} [S_\lambda (1 + \zeta \Sigma \gamma_4) S_\lambda^+] = Q$  in (14) is the same as in (3). Consequently the polarization of the electrons has no effect on the bremsstrahlung cross-section, as calculated from the Born approximation. Analogously,  $R_{11} - R_{22} = R_{12} + R_{21} = 0$ . i.e., the second component in  $\rho_e$  makes no contribution to  $\bar{\xi}_1$  and  $\bar{\xi}_2$ , which as before are determined

by Eqs. (7) and (8). But the parameter  $\xi_3$ , which was equal to zero for unpolarized electrons, now does not vanish:

$$\begin{aligned} \xi_3 &= i(R_{12} - R_{21}) / (Q_{11} + Q_{22}) \\ &= \frac{m}{2\varepsilon_0\omega^2q^4Q} \left\{ \left( \frac{p_0}{x} + \frac{p}{x_0}, [k [k, p_0 - p]] \right) \left( \zeta, \frac{p_0 + \varepsilon_0 k}{x_0} + \frac{\omega p_0 - \varepsilon_0 k}{x} \right) \right. \\ &\quad \left. + \frac{1}{2} m^2 \omega \left[ \frac{x_0}{x} - \frac{x}{x_0} + \frac{4\omega}{m^2} \left( \frac{\varepsilon_0}{x_0} + \frac{\varepsilon}{x} \right) \right] \left( \zeta, \frac{\varepsilon}{x} (\omega p_0 - \varepsilon_0 k) + \frac{\varepsilon_0}{x_0} (\omega p - \varepsilon k) \right) \right\}. \end{aligned} \tag{15}$$

We introduce the unit vectors  $a = p_0/p_0$  and  $b = [an]$ . Clearly

$$\bar{\xi}_3 = i \int (R_{12} - R_{21}) d\omega_p / \int Q d\omega_p = \Lambda_a (\zeta^0 a) + \Lambda_b (\zeta^0 b), \tag{16}$$

that is, the projection of the polarization vector of the electrons in the direction of the normal  $n$  vanishes when the integration is performed.

In the limiting case of non-relativistic energies Eqs. (7), (8) and (15) simplify greatly:

$$\xi_1 = 2\omega^2 (pn)^2 [k, p - p_0]^{-2} - 1, \quad \xi_2 = 2\omega^2 (pn) (p - p_0, l) [k, p - p_0]^{-2}, \quad \xi_3 = (\zeta k) / 4m. \tag{17}$$

In the opposite limiting case of extreme relativistic energies it is necessary to take account of the screening effect. The approximate expression for the parameter  $\bar{\xi}_1$  in this case has been given in Ref. 2. If in the expressions  $R_{12} - R_{21}$  and  $Q$  under the integral in Eq. (16) we replace  $q^{-4}$  by  $(q^2 + g^2)^{-2}$ , where  $g = mZ^{1/2}/137$  and carry out an integration over the directions of the vector  $p$ , with the approximation

$$m^2 x_0 \equiv \varepsilon_0^2 \sin^2 \vartheta_0, \quad m^2 x \equiv \varepsilon^2 \sin^2 \vartheta \ll \varepsilon_0^2, \quad \varepsilon^2, \quad \omega^2, \tag{18}$$

then we obtain corresponding expressions for  $\Lambda_a$  and  $\Lambda_b$ :

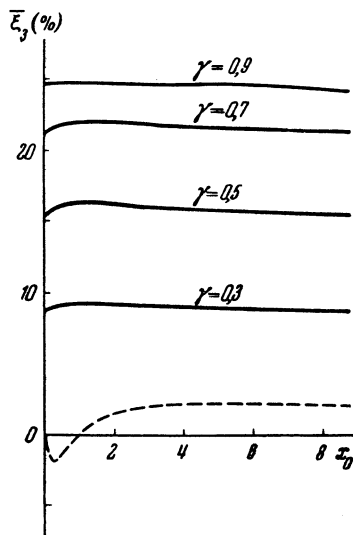
$$\begin{aligned} \Lambda_a &= \gamma \Phi^{-1} \left\{ \frac{1}{4} [(1 + x_0)^2 + (1 - \gamma)(1 - x_0)^2] \left( \ln \frac{1+x_0}{f} - 1 \right) + \frac{1}{8} \gamma (1 + x_0)^2 + x_0 (1 - \gamma) \right\}, \\ \Lambda_b &= -\gamma \Phi^{-1} (1 - \gamma) (1 - x_0) \sqrt{x_0} \left( \frac{1}{2} \ln \frac{1+x_0}{f} - 1 \right), \end{aligned} \tag{19}$$

$$\begin{aligned} \Phi &= 2 \left[ \left( 1 - \gamma + \frac{1}{2} \gamma^2 \right) (1 + x_0)^2 - 2(1 - \gamma)x_0 \right] \ln \frac{1+x_0}{f} - \frac{1}{2} \gamma^2 (1 + x_0)^2 - 2(1 - \gamma)(1 - x_0)^2; \\ \gamma &= \omega / \varepsilon_0, \quad f^2 = q_{\min}^2 + g^2 \approx (m^2 \omega / 2\varepsilon_0 \varepsilon)^2 + g^2. \end{aligned} \tag{20}$$

The dependence of  $\bar{\xi}_3$  on  $x_0$  and  $\gamma$  for a beam of completely polarized, highly relativistic electrons is shown in the figure. For  $\zeta^0$  parallel to  $a$ , the quantity  $\bar{\xi}_3$  depends weakly on  $x_0$ ; for photons with

energy\*  $\omega \sim 0.9 \epsilon_0$  with  $|\zeta^0| = 1$  we have  $\bar{\xi}_3 \sim 25\%$ .

\* We note that Eq. (19) loses its applicability in the immediate vicinity of the limit of the spectrum, where the condition (18) is not satisfied.



Circular polarization of the bremsstrahlung of highly relativistic electrons, averaged over the directions of the final moments of the electrons, shown as a function of  $x_0 = (\epsilon_0/m)^2 \sin^2 \theta_0$  for fixed values of  $\gamma = \omega/\epsilon_0$ : solid curve—electron beam totally polarized along  $p_0$ ; dotted curve—electron beam totally polarized in the plane of emission perpendicular to the vector  $p_0$ ,  $\gamma = 0.5$ ,  $Z = 13$ .

Because of the weak dependence of  $\Lambda_a$  on  $\theta_0$ , multiple scattering of electrons in the target of the beam has little effect on that part of  $\xi_3$  which is determined by the projection of the vector  $\zeta^0$  onto  $\mathbf{a}$ . The other part of  $\xi_3$ , corresponding to the projection of  $\zeta^0$  onto  $\mathbf{b}$ , is small, and for targets of ordinary thickness ( $t \sim 10^2 t_0$ , where  $t_0$  is the radiation

unit of length), practically disappears because of the multiple scattering of electrons preceding the emission. Only for very thin targets ( $t \sim 10^4 t_0$ ) can the effect connected with  $\Lambda_b$  become noticeable\*.

In the general case of arbitrary energy we obtain rather cumbersome expressions for  $\Lambda_a$  and  $\Lambda_b$ :

$$2\Gamma\Lambda_a = -\gamma v_0^2 [v_0(1 + \gamma^2 + v_0^2) - (1 + \gamma)(\gamma + 2v_0^2) \cos \vartheta_0 + 3\gamma v_0 \cos^2 \vartheta_0] L_1 + (\gamma/2v_0) \Delta_0^{-2} \{v_0[1 - 3\gamma + v_0^2(1 + 2\gamma)] + [4\gamma - v_0^2(2 + 5\gamma) + 2\gamma v_0^4] \cos \vartheta_0 + v_0[6\gamma - v_0^2(3 + 5\gamma) + v_0^4] \cos^2 \vartheta_0 - [6\gamma - 2v_0^2(1 + 3\gamma) + \gamma v_0^4] \cos^3 \vartheta_0\} L_2 + v(1 - \gamma) \Delta_0^{-2} \{v_0(1 - 3\gamma + 2\gamma v_0^2) + 2[2\gamma - v_0^2(1 + 2\gamma)] \cos \vartheta_0 + v_0[6\gamma + v_0^2(1 - 3\gamma)] \cos^2 \vartheta_0 - 2\gamma(3 - 2v_0^2) \cos^3 \vartheta_0 - v_0^2 v(1 - \gamma)(v_0 - \gamma \cos \vartheta_0) T^{-2}, \quad (21)$$

$$-2(\gamma \sqrt{1 - v_0^2 \sin^2 \vartheta_0})^{-1} \Gamma \Lambda_b = v_0^2 [3\gamma \Delta_0 + v^2(1 - \gamma)^2] L_1 + (1/2v_0) \Delta_0^{-2} \{\gamma + v_0^2(1 - 2\gamma) + 2v_0[3\gamma - v_0^2(2 + \gamma)] \cos \vartheta_0 + [-6\gamma + v_0^2(2 + 3\gamma) + v_0^4] \cos^2 \vartheta_0\} L_2 + v(1 - \gamma) \Delta_0^{-2} [1 - 2v_0^2 + 6v_0 \cos \vartheta_0 - (6 - v_0^2) \cos^2 \vartheta_0] + v_0^2 v(1 - \gamma) T^{-2}, \quad (22)$$

$$\Gamma = v_0^2 [(2 - 3\gamma \Delta_0 - 2v_0^2) T^2 - \gamma(v_0^2 - \gamma^2) \Delta_0] L_1 + v_0^{-1} \{2(1 - v_0^2) \Delta_0^{-2} [3\gamma - v_0^2(1 + 2\gamma)] \sin^2 \vartheta_0 + \gamma^2 - 5\gamma + v_0^2(3 + \gamma + \gamma^2) + v_0^4 + \gamma \Delta_0(1 - \gamma + v_0^2)\} L_2 - 2v_0^2 \Delta_0 L_3 + v(1 - \gamma) \{4(1 - v_0^2)(3 - v_0^2) \Delta_0^{-2} \sin^2 \vartheta_0 - 10 + 2\gamma + 3v_0^2 + 2(1 - \gamma) \Delta_0 + v_0^2(\gamma^2 - v_0^2) T^{-2}\}, \quad (23)$$

\*See the analogous considerations of May<sup>2</sup> concerning the influence of multiple scattering on the effective value of the parameter  $\xi_1$ .

where

$$L_1 = T^{-3} \ln \frac{T + v(1 - \gamma)}{T - v(1 - \gamma)}, \quad L_2 = \ln \frac{v_0^2 + v_0 v(1 - \gamma) - \gamma}{v_0^2 - v_0 v(1 - \gamma) - \gamma}, \quad L_3 = \ln \frac{1 + v}{1 - v}; \quad (24)$$

$$T^2 = \gamma^2 + v_0^2 - 2\gamma v_0 \cos \vartheta_0, \quad \Delta_0 = 1 - v_0 \cos \vartheta_0. \quad (25)$$

Equations (21)–(23) are valid for values of  $\epsilon_0$  which are not too large ( $\epsilon_0 \ll 137 mZ^{1/3}$ ) and for values of  $\gamma$  which are not too small, for which the screening effect is unimportant.

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Translated by W. M. Whitney  
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SOVIET PHYSICS JETP

VOLUME 5, NUMBER 5

DECEMBER, 1957

### Theory of Kinetic Phenomena in Liquid He<sup>3</sup>

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(Submitted to JETP editor May 9, 1956)

J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 1083-1091 (May, 1957)

The kinetic coefficients of liquid He<sup>3</sup> have been computed on the basis of the theory of a Fermi liquid by Landau. The temperature dependences of the coefficients and their numerical order of magnitude have also been computed.

IN THE PRESENT WORK, which is based on the theory of a Fermi liquid developed by Landau<sup>1</sup>, we shall consider the problem of the viscosity and thermal conductivity of He<sup>3</sup>. In accord with the Landau theory, the excitation energy in a Fermi liquid is a functional of the distribution function  $n$ . At temperatures close to  $T = 0$ , where the diffuse region of the Fermi function is not large, we can, according to the Landau theory, represent this functional dependence in the form of a decomposition in the deviation of the distribution function from its equilibrium value at  $T = 0$ . Limiting ourselves to terms up to first order of smallness, we have

$$\begin{aligned} \varepsilon &= \varepsilon(p) + \int f(p, p') v d\tau', \\ d\tau &= 2dp_x dp_y dp_z / (2\pi\hbar)^3, \end{aligned} \quad (1)$$

where  $v$  is the difference between the actual distribution function and its value at  $T = 0$ .

It is most natural to consider that the distribution is the Fermi sphere at  $T = 0$ . Then at not too high temperatures the excitation energy will be described by the expression

$$\varepsilon(p) = a + p_0(p - p_0) / m, \quad (2)$$

where  $p_0$  is the limiting momentum and  $a$  and  $m$  are constants (in the ideal gas case, this expression becomes  $\varepsilon = p^2/2m$ ). By Ref. 2, it follows from the measurement of the density and entropy of He<sup>3</sup> that  $p_0/\hbar = 0.76 \times 10^9 \text{ cm}^{-1}$ ,  $m = 1.43 m_{\text{He}^3}$ . In view of the fact that the energy  $\varepsilon$  enters into the Fermi distribution in the combination  $\varepsilon - \mu$ , the constant  $a$