Investigation of a Model in Quantum Field Theory

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A model is considered in which a spinor field interacts with a pseudoscalar field, the classical pseudoscalar field being independent of the coordinates. Owing to the properties of the model the equations for the Green function are considerably simplified, and this makes it possible to find their exact solution. An investigation of the Green function is carried out.

IN VIEW OF the great difficulties in principle that stand in the way of the solution of the equations of quantum field theory, methodological interest attaches to the study of particular models for which exact solutions of the corresponding equations can be obtained. A large number of papers[1–9] have been devoted to the consideration of various models. The model of a renormalized field theory proposed by T. D. Lee[2] is studied in some papers[3,4], while others[5,6] consider generalizations of this model or use Lee's idea to construct a new model[7,8]. Consideration has been given to a model of a meson pair theory[9] and to a number of other theories.

The purpose of the present paper is the investigation of a new model, in which the equation for the Green function of the fermion can be solved exactly.

1. STATEMENT OF THE MODEL

We consider an interaction of pseudoscalar (for simplicity, neutral) bosons with fermions, which is characterized by the Lagrangian

$$L(x) = g : \overline{\Psi}(x)\gamma_5\Psi(x) : \varphi(x) + M(x), \qquad (1)$$

where $M(x)$ depends on the operator $\varphi(x)$ of the boson field and can include a "classical source" term $J(x)\varphi(x)$ and also counter-terms for renormalization. The model considered is such that the classical boson field does not depend on the coordinates.

Abrikosov and Khalatnikov[10] have examined the point interaction

$$g\overline{\Psi}(x)\Gamma\Psi(x)\varphi(x)$$

as the limit of the "smeared-out" interaction

$$g\int \overline{\Psi}(x)\Gamma\Psi(y)\varphi(z)K(x-y,\, x-z)\,dy\,dz$$

with limiting momenta λ_ψ and λ_ϕ satisfying the condition $\lambda_\psi \gg \lambda_\phi$. In terms of this two-limit technique the model we have studied corresponds to the case in which the limiting momentum λ_ϕ of the boson is equal to zero and the limiting momentum λ_ψ of the fermion has gone to infinity.

To find the Green function of the fermion we employ the formulas obtained by Bogoliubov[11], which express the Green function $G(x, y\,|\,J)$ in terms of the Green function $G_{\text{cl}}(x, y\,|\,\varphi)$ of a single fermion in the classical field φ. In momentum space they have the following form[1]:

$$G(k\,|\,J) = \frac{\int \delta\varphi \exp\left\{-\frac{1}{2}\int dp\,\varphi(p)D^{-1}(p)\varphi(p)\right\} G_{\text{cl}}(k\,|\,\varphi)\times}{\int \delta\varphi \exp\left\{-\frac{1}{2}\int dp\,\varphi(p)D^{-1}(p)\varphi(p)\right\}\times}$$

$$\rightarrow \frac{\times\exp\left\{\overline{g}\int_0^1 d\beta\int dp\,dq\,\mathrm{Sp}\,\gamma_5\, G_{\text{cl}}(p-q, p\,|\,\beta\varphi)\varphi(q) + \int M(p)\,dp\right\}}{\times\exp\left\{\overline{g_0}\int_0^1 d\beta\int dp\,dq\,\mathrm{Sp}\,\gamma_5\, G_{\text{cl}}(p-q, p\,|\,\beta\varphi)\varphi(q) + \int M(p)\,dp\right\}}, \qquad (2)$$

$$(m+\gamma_\mu k_\mu)\,G_{\text{cl}}(k, k'\,|\,\varphi)-\bar{g}\,\gamma_5\int dq\, G_{\text{cl}}(k-q, k\,|\,\varphi)\,\varphi(q)=\delta(k-k'), \tag{3}$$

$$\left[m+\gamma_\mu k_\mu+\int dp\,\gamma_\mu p_\mu\,\varphi(p)\frac{\delta}{\delta\varphi(p)}-\bar{g}\,\gamma_5\int dp\;\varphi(p)\right]G_{\text{cl}}(k\,|\,\varphi)=1. \tag{3'}$$

Here $\bar{g} = g(2\pi)^{-2}$, and the matrices γ and scalar products are defined as follows:

$$\gamma_k=\beta\alpha_k\quad (k=1, 2, 3),\quad \gamma_4=i\beta,$$

$$\gamma_5=\gamma_1\gamma_2\gamma_3\beta,\quad AB=A_4B_4+\mathbf{AB}.$$

In the model under consideration the limiting momentum λ_ϕ of the boson is equal to zero, *i.e.*, the boson is, so to speak, smeared over the entire x-space, and the limiting momentum λ_ψ of the fermion has gone to infinity, *i.e.*, the fermion is a point particle. If we study the behavior of the Green function of the fermion at momenta satisfying the condition $k^2 \gg \lambda_\phi^2$, our present model is the limiting case for $\lambda_\phi \to 0$.

In this model $\varphi(x)$ does not depend on x, so that we get the required formulas if we replace $\varphi(p)$ by $\varphi\delta(p)$, the variational derivative $\delta/\delta\varphi$ by the ordinary $\partial/\partial\varphi$, and the Feynman integral $\int\delta\varphi F(\varphi)$ by the integral over a single variable, $\int_{-\infty}^{\infty} d\varphi F(\varphi)$. Here also the Green function $D(p, p')$ of the free boson field goes over into $d\times\delta(p)\delta(p')$, where d is a constant. The equation for $G_{\text{cl}}(k, x\colon \varphi)$ now takes the simple form

$$(m+\gamma_\mu k_\mu-\bar{g}\,\gamma_5\varphi)\,G_{\text{cl}}(k, x; \varphi)=1. \tag{4}$$

The unrenormalized Green function $G(k; J)$ is then expressed in terms of $G_{\text{cl}}(k; \varphi)$ in the following way:

$$G(k; J)=\frac{\int_{-\infty}^{\infty} d\varphi e^{-\varphi^2/2d}\,G_{\text{cl}}(k;\varphi)\exp\left\{\delta_\lambda\bar{g}'\int_0^1 d\beta\int_\Omega dp\,\text{Sp}\,\gamma_5\,G_{\text{cl}}(p, p; \beta\varphi)\,\varphi+M(\varphi; J)\right\}}{\int_{-\infty}^{\infty} d\varphi e^{-\varphi^2/2d}\exp\left\{\delta_\lambda\bar{g}'\int_0^1 d\beta\int_\Omega dp\,\text{Sp}\,\gamma_5\,G_{\text{cl}}(p, p; \beta\varphi)\,\varphi+M(\varphi; J)\right\}}, \tag{5}$$

where

$$G_{\text{cl}}(k; \varphi)=(m-\gamma_\mu k_\mu+\bar{g}\gamma_5\varphi)\,/\,(m^2+k^2+\bar{g}^2\varphi^2), \tag{6}$$

and here $\lim_{\lambda\to\infty}\delta_\lambda = \delta(0)$, *i.e.*, the δ-function of zero; g' is the priming coupling constant. In Eq. (5) the integral with respect to p is taken over a finite four-dimensional region Ω, prior to carrying out the renormalization.

On the other hand, the expressions obtained in our model for the Green function of the fermion can be regarded as a sort of approximate Green function of the nucleon of quantum field theory, if we assume as a first approximation $\varphi(x)$ = const. It was just this approximation that Feynman used to estimate the role of nucleon-antinucleon pairs.

2. ONE-PARTICLE APPROXIMATION

Let us consider the so-called one-particle approximation, which, in the language of Feynman diagrams, reduces to the neglect of closed fermion loops. In this approximation the Green function $G(k)$ is very simply related to the Green function $G_{\text{cl}}(k; \varphi)$ of a single fermion in the classical field φ, namely:

$$G(k)=(2\pi d)^{-1/2}\int_{-\infty}^{\infty} e^{-\varphi^2/2d}\,d\varphi\,G_{\text{кл}}(k; \varphi). \tag{7}$$

We use expression (6) and get

$$G(k)=2(2\pi d)^{-1/2}\int_0^\infty d\varphi e^{-\varphi^2/2d}\frac{m-\gamma_\mu k_\mu}{m^2+k^2+\bar{g}^2\varphi^2}. \tag{8}$$

The integral in Eq. (8), convergent in the region $k^2 \geq 0$, can be expressed in terms of the probability integral

$$\Phi(x)=\frac{2}{\sqrt{\pi}}\int_0^x dte^{-t^2}.$$

On this basis we write the Green function $G(k)$ in the form

$$G(k)=\frac{m-\gamma_\mu k_\mu}{m^2+k^2}\sqrt{2\pi}\left(\frac{\bar{g}^2 d}{m^2+k^2}\right)^{-1/2}\exp\left\{\frac{m^2+k^2}{2\bar{g}^2 d}\right\}\left[1-\Phi\left(\sqrt{\frac{m^2+k^2}{2d\bar{g}^2}}\right)\right], \tag{9}$$

for $\bar{g}^2 \neq 0$.

Let us find out whether the function $G(k)$ can be represented as a converging series in powers of $\bar{g}^2$. For this purpose we write $G(k)$ in a different way, namely

$$G(k)=\frac{m-\gamma_\mu k_\mu}{\sqrt{m^2+k^2}}\int_0^\infty dxe^{-x}(m^2+k^2+2d\bar{g}^2x)^{-1/2}. \tag{10}$$

Integrating by parts n times we get

$$G(k)=\frac{m-\gamma_\mu k_\mu}{m^2+k^2}\left\{1-\frac{\bar{g}^2 d}{m^2+k^2}+1\cdot 3\cdot\frac{\bar{g}^4 d^2}{(m^2+k^2)^2}+\ldots\right.$$
$$\cdots+(-1)^n(2n-1)!!\frac{d^n\bar{g}^{2n}}{(m^2+k^2)^n}+\bar{g}^{2(n+1)}(2n+1)!!\sqrt{m^2+k^2}\,d^{n+1} \tag{10'}$$
$$\left.\times\int_0^\infty\frac{e^{-x}dx}{(m^2+k^2+2d\bar{g}^2x)^{(2n+3)/2}}\right\}.$$

Let S_n be the sum of the first $(n+1)$ terms of the expansion in power of g^2; then

$$\lim_{\bar{g}^2\to 0}(\bar{g}^2)^{-n}[G(k)-S_n]=\lim_{\bar{g}^2\to 0}\bar{g}^2\frac{(2n+1)!!}{\sqrt{m^2+k^2}}d^{n+1}\int_0^\infty\frac{e^{-x}dx}{(m^2+k^2+2d\bar{g}^2x)^{(2n+3)/2}}=0.$$

From this it follows that $G(k)$ can be represented by the following asymptotic series

$$G(k)\sim\frac{m-\gamma_\mu k_\mu}{m^2+k^2}\sum_{n=0}^\infty(-1)^n(2n-1)!!\,\bar{g}^{2n}\left(\frac{d}{m^2+k^2}\right)^n. \tag{11}$$

This series is of the type summable by Borel's method[13,14]. This means that from the coefficients of the asymptotic expansion one can recover the original function by using the generalized Borel method. To show this, we represent unity in each term of the series (11) as

$$1 = \frac{2}{\sqrt{2\pi d}} \frac{1}{d^n} \frac{1}{(2n-1)!!} \int_0^\infty e^{-\varphi^2/2d} \varphi^{2n}\, d\varphi .$$

We interchange the order of summation and integration and obtain

$$G(k) \sim \frac{2}{\sqrt{2\pi d}} \frac{m - \gamma_\mu k_\mu}{m^2 + k^2} \int_0^\infty e^{-\varphi^2/2d}\, d\varphi \sum_{n=0}^\infty \bar{g}^{2n} \left(- \frac{\varphi^2}{m^2 + k^2} \right)^n . \tag{12}$$

We then write unity in each term of the series (12) in the form

$$1 = \frac{1}{n!} \int_0^\infty e^{-x} x^n\, dx,$$

interchange once more the order of summation and integration, and get the convergent series

$$G(k) = \frac{2}{\sqrt{2\pi d}} \frac{m - \gamma_\mu k_\mu}{m^2 + k^2} \int_0^\infty d\varphi e^{-\varphi^2/2d} \int_0^\infty e^{-x}\, dx \sum_{n=0}^\infty \frac{(-1)^n}{n!} x^n \bar{g}^{2n} \left(\frac{\varphi^2}{m^2 + k^2} \right)^n .$$

We thus recover the original function (8) from the asymptotic series (11) by using the generalized Borel method.

It must be remarked that in the case of the symmetric (not the neutral) theory the Green function of the fermion must be written as follows:

$$G(k) = \int_0^\infty dx\, (m - \gamma_\mu k_\mu)\, e^{-(m^2+k^2)x} (1 + 2d\bar{g}^2 x)^{-3/2}. \tag{13}$$

3. THE EXACT GREEN FUNCTION OF THE FERMION

For our model it is possible to find the exact Green function of the fermion, neglecting none of the Feynman diagrams.

First let us consider the nonrenormalized Green function $G(k)$. We substitute into Eq. (5) the function $G_{cl}(k; \varphi)$ in the form (6) and get

$$G(k) = \frac{\displaystyle\int_{-\infty}^{\infty} \frac{m - \gamma_\mu k_\mu}{m^2 + k^2 + \bar{g}^2\varphi^2} \exp\left\{ -4\bar{g}'^2 \delta_\lambda \int_0^1 \beta d\beta \int_\Omega \frac{\varphi^2 dp}{m^2 + p^2 + \bar{g}^2\varphi^2\beta^2} + M(\varphi) - \frac{\varphi^2}{2d} \right\} d\varphi}{\displaystyle\int_{-\infty}^{\infty} \exp\left\{ -4\bar{g}'^2 \delta_\lambda \int_0^1 \beta d\beta \int_\Omega \frac{\varphi^2 dp}{m^2 + p^2 + \bar{g}^2\varphi^2\beta^2} + M(\varphi) - \frac{\varphi^2}{2d} \right\} d\varphi} . \tag{14}$$

Since the model under consideration corresponds to a certain degree to the behavior of the nucleon Green function of pseudoscalar meson theory for large k^2, *i.e.*, very far from the pole $k^2 = m^2$, the mass of the fermion should be set equal to zero. But we do not do this, in order to avoid divergences of the type of the infrared catastrophe.

If we carry out the renormalization

$$\bar{g}^2 = \bar{g}'^2 \delta_\lambda m^4,$$

the Green function $G(k)$ degenerates into

$$G(k) = (m - \gamma_\mu k_\mu)/(m^2 + k^2),$$

i.e., into the Green function of the free fermion field. This occurs because of incorrect performance of the renormalization. In the carrying out of the renormalization there are certain peculiar features, owing to the nature of the model, *i.e.*, to the fact that all the virtual mesons transfer only zero momentum. Associated with this are divergences of the form $\delta(0)$. There are also divergences in those Feynman diagrams that involve fermion pairs, *i.e.*, in the boson self-energy diagrams, the diagrams of boson-boson scattering, and also the corresponding overlapping diagrams.

Owing to the presence of these two types of divergences we carry out the renormalization in two stages: first we renormalize the fermion Green function

$$G''(k) = (\delta_\lambda m^4)\, G(k)$$

and corresponding to this we renormalize the charge, and then, in the second stage, we remove the ordinary divergences associated with the fermion pairs.

To carry out the renormalization

$$G''(k) = (\delta_\lambda m^4)\, G(k)$$

one must either add to the Lagrangian a counter-term having the operator structure of the free fermion field, *i.e.*, a term of the form

$$L'(x) = -{}^1/_2\,[\delta_\lambda m^4 - 1] : \overline{\Psi}(x)$$
$$i\,(m - i\gamma_\mu \partial/\partial x_\mu)\,\Psi(x) : + \text{ compl. conj.}$$

or predetermine the pairing of two fermion operators in the following way:

$$\langle T\{\Psi(x)\,\overline{\Psi}(x')\}\rangle_0 = (\delta_\lambda m^4)^{-1}\, S_F(x - x').$$

In both cases this leads to the charge renormalization $g'' = (\delta_\lambda m^4)^{-1} g'$.

To carry out the second stage of the renormalization we use the counter-terms $B\varphi^4$ and $A\varphi^2$, where A and B are certain constants that go to infinity as the region of integration Ω is extended to the entire four-dimensional space. The term $B\varphi^4$ corresponds to the direct interaction of bosons; a counter-term of this form is necessary in carrying out the renormalization in pseudoscalar meson theory. In our case it is contained in $M'(\varphi)$. The counter-term $A\varphi^2$ is obtained by the predetermination of the T-product of two boson functions (*cf.* Ref. 15) and occurs in the expression for the fermion Green function in the form

$$\exp\{-\varphi^2(d^{-1} + A)/2\}.$$

The presence of these two counter-terms makes it possible to cancel from the integral

$$\int_\Omega dp \frac{\bar{g}^2 \varphi^2}{m^2 + p^2 + \bar{g}^2 \varphi^2 \beta^2}$$

the terms proportional to φ^2 and φ^4, which diverge for $\Omega \to \infty$, with the result having the appearance

$$\lim_{\Omega\to\infty} \int_\Omega dp \left\{ \frac{\bar{g}^2 \varphi^2}{m^2 + p^2 + \bar{g}^2 \varphi^2 \beta^2} - \frac{\bar{g}^2 \varphi^2}{m^2 + p^2} + \frac{\bar{g}^4 \varphi^4 \beta^2}{(m^2 + p^2)^2} \right\} = \int dp \frac{\bar{g}^6 \beta^4 \varphi^6}{(m^2 + p^2)^2 (m^2 + p^2 + \bar{g}^2 \varphi^2 \beta^2)}. \tag{15}$$

The divergent part of the coefficients A and B is uniquely determined, but in the determination of the finite parts there is a certain arbitrariness. As is well known, for the term $B\varphi^4$ this leads to the appearance of an additional coupling constant. The arbitrariness in the determination of the finite part of the coefficient A exists only in our model and is due to the absence of the condition

$$\lim_{\gamma_\mu k_\mu \to -m} G(k)\,(m + \gamma_\mu k_\mu) = 1.$$

imposed in field theory. In view of the presence of this arbitrariness we can subtract from the expression (15) some finite function $N(\varphi) = a\varphi^2 + b\varphi^4$.

As the result of the renormalization, the Green function $G'(k)$ takes the following form

$$G'(k) = \frac{\int\limits_{-\infty}^{\infty} \frac{m - \gamma_\mu k_\mu}{m^2 + k^2 + \bar{g}^2\varphi^2} \exp\left\{-\frac{\bar{g}^6}{m^4}\int\limits_0^1 \beta^5 d\beta \int \frac{\varphi^6 dp}{(m^2+p^2)^2(m^2+p^2+\bar{g}^2\varphi^2\beta^2)} - N(\varphi) - \frac{\varphi^2}{2d}\right\} d\varphi}{\int\limits_{-\infty}^{\infty} \exp\left\{-\frac{\bar{g}^6}{m^4}\int\limits_0^1 \beta^5 d\beta \int \frac{\varphi^6 dp}{(m^2+p^2)^2(m^2+p^2+\bar{g}^2\varphi^2\beta^2)} - N(\varphi) - \frac{\varphi^2}{2d}\right\} d\varphi}. \tag{16}$$

We evaluate the integral in the exponential function and obtain the final expression

$$G'(k) = \frac{\int\limits_0^{\infty} e^{-\varphi^2/2d} d\varphi \frac{m - \gamma_\mu k_\mu}{m^2 + k^2 + \bar{g}^2\varphi^2} \exp\left\{f_1\left(\frac{\varphi}{m}\right)^4 + f_2\left(\frac{\varphi}{m}\right)^2\right\}\left(1 + \bar{g}^2\frac{\varphi^2}{m^2}\right)^{-\pi^2(1+\bar{g}^2\varphi^2/m^2)^2}}{\int\limits_0^{\infty} e^{-\varphi^2/2d} d\varphi \exp\left\{f_1\left(\frac{\varphi}{m}\right)^4 + f_2\left(\frac{\varphi}{m}\right)^2\right\}\left(1 + \bar{g}^2\frac{\varphi^2}{m^2}\right)^{-\pi^2(1+\bar{g}^2\varphi^2/m^2)^2}}, \tag{17}$$

where f_1 and f_2 are arbitrary constants. The integrals over φ converge for $k^2 \geq 0$, and the values of the constants f_1 and f_2 cannot affect the convergence of these integrals.

From these considerations it can be seen that the renormalization procedure can be carried out consistently without the use of perturbation theory.

The properties of the propagation functions of fermions and bosons and of the renormalization constants have been studied by Lehmann[16], Gell-Mann and Low[17], and Källen[18].

Lehmann considered a neutral pseudoscalar field $\varphi(x)$ interacting with a spinor field $\Psi(x)$. Without making any special assumptions about the form of the interaction, but assuming that the theory is relativistically invariant and that the energy operator possesses a smallest eigenvalue, which is normalized to zero, he obtained the conditions that must be satisfied by the elements of the suitably written propagation functions and renormalization constants.

It is of interest to write our fermion Green function $G'(k)$ in the form given by Lehmann and examine whether these conditions are satisfied. If we take into account our notations and the properties of the model considered (absence of a discrete level at $k^2 = m^2$), Lehmann's formulas can be written in the following way: the fermion Green function has the form

$$G'(k) = \int\limits_{m^2}^{\infty} d(\varkappa^2) \frac{(\varkappa - \gamma_\mu k_\mu)\rho_1(\varkappa^2) - \rho_2(\varkappa^2)}{k^2 + \varkappa^2}, \tag{18}$$

where $\rho_1(\varkappa^2)$ and $\rho_2(\varkappa^2)$, defined for positive values of the argument, must satisfy the inequalities

$$\rho_1(\varkappa^2) \geqslant 0, \quad 2\varkappa\rho_1(\varkappa^2) \geqslant \rho_2(\varkappa^2) \geqslant 0. \tag{19}$$

The renormalization constants have the form

$$Z_2^{-1} = \int\limits_{m^2}^{\infty} \rho_1(\varkappa^2)\, d(\varkappa^2),$$

$$\delta m = Z_2 \int\limits_{m^2}^{\infty} [(m - \varkappa)\rho_1(\varkappa^2) + \rho_2(\varkappa^2)]\, d(\varkappa^2), \tag{20}$$

where Z_2 must satisfy the inequality

$$Z_2 \geqslant 0. \tag{21}$$

In order to get $G'(k)$ in the form (18), we make in Eq. (17) the change of variable $\varkappa^2 = m^2 + \bar{g}^2\varphi^2$, valid for $\bar{g}^2 \neq 0$; then

$$G'(k)=\frac{\int\limits_{m^2}^{\infty}\frac{d(\varkappa^2)}{\sqrt{\varkappa^2-m^2}}\frac{(\varkappa-\gamma_\mu k_\mu)-(\varkappa-m)}{\varkappa^2+k^2}\left(\frac{\varkappa^2}{m^2}\right)^{-\pi^2(\varkappa^2/m^2)^2}e^{-\varkappa^2/2\bar{g}^2d}}{\int\limits_{m^2}^{\infty}\frac{d(\varkappa^2)}{\sqrt{\varkappa^2-m^2}}\left(\frac{\varkappa^2}{m^2}\right)^{-\pi^2(\varkappa^2/m^2)^2}e^{-\varkappa^2/2\bar{g}^2d}}, \tag{22}$$

here $\varkappa = (\varkappa^2)^{1/2}$ and, for simplicity, $f_1 = f_2 = 0$. Comparing Eqs. (22) and (18), we get

$$\rho_1(\varkappa^2)=\frac{\frac{1}{\sqrt{\varkappa^2-m^2}}\left(\frac{\varkappa^2}{m^2}\right)^{-\pi^2(\varkappa^2/m^2)^2}e^{-\varkappa^2/2\bar{g}^2d}}{\int\limits_{m^2}^{\infty}\frac{d(\varkappa^2)}{\sqrt{\varkappa^2-m^2}}\left(\frac{\varkappa^2}{m^2}\right)^{-\pi^2(\varkappa^2/m^2)^2}e^{-\varkappa^2/2\bar{g}^2d}},\quad \rho_2(\varkappa^2)=(\varkappa-m)\,\rho_1(\varkappa^2). \tag{23}$$

It can easily be seen that $\rho_1(\varkappa^2)$ and $\rho_2(\varkappa^2)$ satisfy the conditions (19). Substituting the values (23) for ρ_1 and ρ_2 into Eq. (20), we get

$$Z_2=1,\quad \delta m=0. \tag{24}$$

The fulfillment of the conditions (19) and (21) shows that our model is free from internal contradictions.

It has been shown by Lehmann that if

$$\int\limits_{m^2}^{\infty}\rho_1(\varkappa^2)\,d(\varkappa^2)<\infty,$$

then at large momenta the exact Green functions exhibit the same behavior as the free Green functions. In our model this integral is equal to unity, so that the fermion Green function $G'(k)$ behaves like $1/\gamma k$ at very large momenta.

From the consideration of the fermion Green function $G'(k)$, taken in the form (17), it can be seen that $G'(k)$ cannot be represented as a convergent series in powers of g^2. Moreover, neither in the numerator nor in the denominator of (17) can the integrand be expanded in convergent power series in g^2. It turns out to be impossible to introduce a finite number of supplementary integrations over the parameters in such a way that the integrand could be represented in the form of a convergent series in powers of the interaction constant g^2. This suggests that the series of perturbation theory cannot serve as the basis for carrying out any investigations in pseudoscalar meson theory.

We proceed further to write $G'(k)$ in the form

$$G'(k)=\frac{\int\limits_{m^2}^{\infty}\frac{d(\varkappa^2)}{\sqrt{\varkappa^2-m^2}}\frac{m-\gamma_\mu k_\mu}{\varkappa^2+k^2}\left(\frac{\varkappa^2}{m^2}\right)^{-\pi^2(\varkappa^2/m^2)^2}e^{-\varkappa^2/2\bar{g}^2d}}{\int\limits_{m^2}^{\infty}\frac{d(\varkappa^2)}{\sqrt{\varkappa^2-m^2}}\left(\frac{\varkappa^2}{m^2}\right)^{-\pi^2(\varkappa^2/m^2)^2}e^{-\varkappa^2/2\bar{g}^2d}}, \tag{25}$$

valid for $\bar{g}^2 \neq 0$. From this expression it can be seen that both in the numerator and in the denominator the integrands can be expanded in series of inverse powers of the coupling constant. These series under the sign of integration over $\varkappa^2$ are convergent, and even if one carries out the integration term by term each term of the series obtained will be finite.

It must be remarked that the contribution from the polarization of the vacuum is included in the first term of the expansion in inverse powers of the coupling constant; the vacuum polarization does not occur in the perturbing term.

If in the case of the one-particle approximation one expands the integrand in a series of inverse powers of the coupling constant, then after integration each term of the series, except the first, will be infinite. This leads to the conclusion that the inclusion of the vacuum polarization decidedly strengthens the coupling. Moreover, it follows from a consideration of Eqs. (17) and (25) that the strong-coupling approximation is in better correspondence with the nature of the interaction of the type γ_5 than the weak-coupling approximation.

The model we have considered gives an idea of the behavior of the Green function of the nucleon in pseudoscalar meson theory in the region $k^2 \gg m^2$, *i.e.*, far from the pole $k^2 = m^2$. Feynman[12] calculated the polarization of the vacuum in the approximation $\varphi(x)$ = const; on the basis of the study of our model it can be said that his conclusion about the large part played by the polarization of the vacuum relates only to the region $k^2 \gg m^2$.

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Extension of the Spin-Wave Model to the Case of Several Electrons Surrounding Each Site

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The energy of a weakly excited state of a ferromagnetic or antiferromagnetic crystal in which each site is surrounded by several electrons is calculated by the method of approximate second quantization, applied to a system consisting of two types of interacting Fermi particles. It is found that besides the usual excitations of the ferromagnon-antiferromagnon type, some additional excitations, which depend weakly on the quasi-momentum, appear in these systems. A physical interpretation of these excitations is proposed.

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1. THE PICTURE OF a weakly excited state of a ferromagnetic or antiferromagnetic crystal, when there is only one magnetically active electron at each lattice site, is now fairly well understood. In the approximations of the spin-wave model it is possible to approximate the energy of a weakly-excited state of these crystals by the energy of an ideal gas of separate Bose-type quasi-particles—ferromagnons[1] antiferromagnons[2, 3, 4] obeying the dispersion laws