Disintegration of Light Nuclei in a Coulomb Field

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General formulae for the effective cross section for disintegration of light nuclei in a Coulomb field are derived. The formulae obtained are applied to the cases of the Li^7 and O^{17} nuclei, where it is assumed that the former splits into an α-particle and a triton, and the latter into O^{16} and a neutron.

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1. IN CONNECTION WITH the development of techniques of acceleration it has proved possible to accelerate a beam of light nuclei to high energies. In going through the Coulomb field of other nuclei, these nuclei can split, analogously to what happens in the case of the deuteron nucleus moving in a Coulomb field. The analogy with deuterons is complete for those nuclei in which the odd neutron is weakly bound with the remaining nucleus. Such nuclei are, for example, Be^9 (the binding energy of the last neutron is $\epsilon = 1.67$ Mev), C^{13} ($\epsilon = 4.9$ Mev), O^{17}($\epsilon = 4.1$ Mev) and others. Sawicki[1] considered the splitting of Be^9 into the residual Be^8 and a neutron in a Coulomb field. He showed that for energies of the Be^9 nucleus of 170 Mev the effective disintegration cross section of this nucleus was equal to $4.10^{-29} Z^2$ cm^2, where Z is the charge of the nucleus through whose Coulomb field the beryllium nucleus passes. It is clear that analogous calculations can be carried out for the nuclei C^{13}, O^{17} and others. The difference will consist in the fact that the odd neutron in these nuclei can be in states different from the states it is in for the deuteron or beryllium. For example, according to the shell model the unpaired neutron in the O^{17} nucleus is in a $d_{5/2}$ state. Formulae will be given below for the calculation of the disintegration of a nucleus in a Coulomb field for the general case where the odd neutron in these nuclei occurs in an arbitrary state of angular momentum l.

We consider more interesting, however, the consideration of those cases in which the light nucleus splits in the Coulomb field into two charged particles. In Ref. 2, as a result of the study of stars in photographic emulsions irradiated by 340 Mev protons, it is proposed that light nuclei have a clear cut structure. Such nuclei, according to these authors, can be considered as formed from groups of nucleons (for example, α-particles, tritons and deuterons).

A number of authors[3,4] have obtained satisfactory results by considering the Li^6 nucleus as a system consisting of an α-particle and a deuteron. There are grounds to believe that the Li^7 nucleus can be viewed as a system consisting of an α-particle and a triton, in some approximation. We note, first of all, that the binding energy of the triton in the Li^7 nucleus is equal to only 2.52 Mev, whereas the internal binding energies of the α-particle and triton are 28.2 and 8.48 Mev, respectively.

The binding energy in the Li^7 nucleus is unequally distributed. The nucleons in the triton and α-particle are bound considerably more tightly than the triton and α-particle are bound with each other. In this context one can speak of the triton and α-particle as constituting the nucleus.

We indicate, further, that such a model of the Li^7 nucleus leads to satisfactory agreement with experiment in relation to the magnetic moment of this nucleus. If the g-factor of triton is defined by the formulae

$$\mu_l = (e\hbar / 2M_t c)\, g_l\, l, \quad \mu_s = (e\hbar / 2Mc)\, g_s\, s,$$

where M_t denotes the reduced mass of the triton, then the magnetic moment (in nuclear magnetons)

$$\mu = {}^7/_{12}\, g_l + g_s / 2 = 3{,}56,$$

is obtained, assuming that the Li^7 nucleus is in a $p_{3/2}$ state, and the experimental value of the magnetic moment of the triton is taken for μ_s. The value of the magnetic moment of Li^7 obtained agrees well with the experimental value of 3.25.

It is interesting to note that for several other light nuclei, also, values of the magnetic moments which are in satisfactory agreement with experimental data are obtained if it is assumed that in these nuclei of the type X^{4k+3}_{2k+1} the magnetic moment is determined not by the odd proton, but by the triton moving in the field of the remaining part X^{4k}_{2k}. For example, the magnetic moment of the Al^{27}_{13} nucleus which, as is well known, is in a $d_{5/2}$ state, is equal to 3.72 under this assumption. This agrees well with the experimental value of 3.64. More or less acceptable values of the magnetic moments of B^{10}_{5} (2.11) and F^{19}_{9}(2.98) are obtained on the basis of this assumption.

It is natural to assume that those nuclei which, with relation to asymmetry in distribution of mass and charge, can be represented as consisting of two particles with relatively weak binding between them, split into two particles in going through the Coulomb field of a heavy nucleus if the frequency of variation of the field is such that the corresponding energy quanta are larger than the energy of binding of these particles with each other.

The present work is devoted to a study of the probability of splitting of light nuclei in a Coulomb field.

2. We denote the masses and charges of the constituents of the nucleus by M_1 and M_2 and Z_1' and Z_1'', respectively. In the case where, in addition to the residual X^{4k}_{2k}, there is a weakly bound neutron, $Z_1'' = 0$ and $M_2 = M$, where M is the mass of the neutron. We assume that a light nucleus of mass $M_0 = M_1 + M_2$ and charge $Z_1 = Z_1' + Z_1''$ goes through the Coulomb field of a heavy nucleus of charge Z. The Coulomb interaction, in the general case, will have the form

$$V = Z_1 Ze^2 / r_1 + Z_1'' Z^2 e / r_2, \tag{1}$$

where r_1 and r_2 are the distances from the center of mass of both particles of the light nucleus to the center of mass of the heavy nucleus. We can consider the radius of the light nucleus to be significantly less than the distance between centers of the nuclei considered. Therefore, in Eq. (1), we can set approximately $r_1 = r_2 \approx r$, where r is the distance between centers of the given nuclei. In the system of reference in which the light nucleus, as a whole, is at rest but the heavy nucleus is moving, the electrostatic energy of the light nucleus, viewed as a perturbation, is equal to

$$V = Z_1 Z e^2 / [b^2 + (z - vt)^2]^{1/2}, \tag{2}$$

where $Z_1 = Z_1' + Z_2''$, v is the velocity of the heavy nucleus moving along the z-axis, z and b are the projections of the center of mass of the light nucleus—where, in the present approximation its charge Ze is concentrated—on the z-axis and on the perpendicular to it.

The effective differential cross section of the process considered can, as is well known, be written in the form[5]

$$d\sigma = |A|^2 L^2 d\varepsilon_1 d\Omega_{\mathbf{k}} (L / 2\pi)^3 d\,\mathbf{k}_1, \tag{3}$$

where ϵ_1 is the energy of relative motion of the constituents of the light nucleus after disintegration, $\mathbf{k}_1$ is the wave vector of the center of mass of the light nucleus after disintegration, $\mathbf{k}$ is the wave vector of relative motion of the products of the light nucleus after disintegration, L is the edge of the cube,

$$A = \frac{1}{\hbar} \int V_{if} e^{i\omega t}\, dt, \tag{4}$$

$$V_{if} = \int \psi_i^* V \psi_f^* \, d\,\mathbf{r}\, d\,\mathbf{r}_c. \tag{5}$$

Here $\mathbf{r}_c$ is the radius vector of the center of mass of the moving nucleus and $\mathbf{r}$ is the radius vector of the relative motion of the products of the light nucleus after splitting. The wave functions of the initial and final states, which enter into Eq. (5), have the form

$$\Psi_i = L^{-3/2} e^{i\mathbf{k}_0\mathbf{r}} \psi_l(\mathbf{r})\, e^{iE_0 t/\hbar},$$
$$\Psi_f = L^{-3/2} e^{i\,\mathbf{k}_1\mathbf{r}} \psi(\mathbf{r})\, e^{iE_1 t/\hbar}, \tag{6}$$

where $\mathbf{k}_0$ is the wave vector of the center of mass of the light nucleus before disintegration, $E_0 = -\epsilon$ is the binding energy of the constituents of the light nucleus before disintegration, $E_1 = \epsilon_1 + \hbar^2 k_1^2 / 2(M_1 + M_2)$ is the total energy of the light nucleus after splitting, $\psi_l(\mathbf{r})$ is the wave function of the ground state of the light nucleus before disintegration, normalized with respect to volume, $\psi(\mathbf{r})$ is the wave function of relative motion of the products of the light nucleus after disintegration. The wave function of the final state can be taken as a plane wave, normalized with respect to energy, *i.e.*,

$$\psi(\mathbf{r}) = (p\mu / (2\pi\hbar)^3)^{1/2} e^{i\mathbf{kr}}, \tag{7}$$

where p is the momentum of relative motion of the products of disintegration, and $\mu = M_1 M_2 / (M_1 + M_2)$. According to the law of conservation of energy we have

$$\hbar\omega = E_1 - E_0 = \varepsilon_1 + \varepsilon + \hbar^2 k_1^2 / (M_1 + M_2). \quad (8)$$

As Dancoff[5] has shown, the matrix element A can be represented in the form

$$A = 2L^{-3} (Z_1 Ze^2 / \hbar v) J_k J_r, \quad (9)$$

$$J_k = \frac{4\pi}{q^2} \frac{\sin [q_z - \omega / v) L / 2]}{(q_z - \omega / v)},$$

$$J_r = \int d\mathbf{r} \exp \left[-i \frac{M_1}{M_1 + M_2} (\mathbf{k}_1 \mathbf{r}) \right] \psi_l(\mathbf{r}) \psi^*(\mathbf{r}), \quad (10)$$

where q_z is the projection of the vector $\mathbf{q} = \mathbf{p}_1/\hbar$ on the z-axis. It is possible to show that for sufficiently high energies of the light nucleus ~ 100 Mev the quantity $k_1 r \ll 1$. Therefore, it is possible to set

$$\exp \left\{ -i \frac{M_1}{M_1 + M_2} (\mathbf{k}_1 \mathbf{r}) \right\} = 1 - i \frac{M_1}{M_1 + M_2} (\mathbf{k}_1 \ \mathbf{r}). \quad (11)$$

Taking Eq. (11) and the orthogonality condition of the Ψ-functions, J_r can be written

$$J_r = -i \frac{M}{M_1 + M_2} \int (\mathbf{k}_1, \mathbf{r}) \psi^* (\mathbf{r}) \psi_l (\mathbf{r}) r^2 \sin \theta d\theta \, d\varphi. \quad (12)$$

Here M_1 denotes the mass of the lighter of the products, freed in the breaking up of the light nucleus, *i.e.*, the mass of the neutron, deuteron or triton.

Denoting the angle between $\mathbf{k}_1$ and $\mathbf{r}$ by ϑ we have

$$\cos \vartheta = \cos \gamma \cos \theta + \sin \gamma \sin \theta \cos (\varphi - \varphi'), \quad (13)$$

where γ is the angle between the vectors $\mathbf{k}_1$ and $\mathbf{k}$. The last term in Eq. (13) does not contribute to the integral in Eq. (12). Therefore, we can write J_r as

$$J_r = -i \frac{M_1}{M_1 + M_2} k_1 \cos \gamma \int \psi_l (\mathbf{r}) \psi^* (\mathbf{r}) r^3 dr. \quad (14)$$

The wave function $\Psi(\mathbf{r})$ is given by Eq. (7). In order to determine the wave function of the bound state $\Psi_l(\mathbf{r})$, the form of the interaction between the decay products must be chosen. We assume that this interaction can be represented as a potential well of depth V_0 and radius R. In this case we can take

$$\psi_l (\mathbf{r}) = R_l (r) Y_{lm} (\theta, \varphi),$$

$$R_l (r) = C_l f_l (\beta r) \quad \text{for} \quad r < R, \quad (15)$$

$$R_l (r) = C_l \frac{f_l (\beta R)}{k_l (\alpha R)} k_l (\alpha r) \quad \text{for} \quad r > R,$$

where

$$f_l (x) = \sqrt{\frac{\pi}{2x}} J_{l+1/2} (x),$$

$$k_l (x) = \sqrt{\frac{2}{\pi x}} K_{l+1/2} (x) = \sqrt{\frac{2}{\pi x}} \frac{i\pi}{2} e^{il\pi/2} H^{(1)}_{l+1/2} (ix),$$

$$\beta^2 = 2\mu (V_0 - \varepsilon)/\hbar^2, \quad \alpha^2 = 2\mu\varepsilon/\hbar^2. \quad (16)$$

Here $J_{l+1/2}$ and $H^{(1)}_{l+1/2}$ are the Bessel and Hankel functions.

The conditions of continuity at $r = R$ and normalization give

$$-\alpha k_{l-1} (\alpha R)/k_l (\alpha R) = \beta f_{l-1} (\beta R)/f_l (\beta R), \quad (17)$$

$$C_l^2 = -2\alpha^2/(\alpha^2 + \beta^2) R^3 f_{l-1} (\beta R) f_{l+1} (\beta R). \quad (18)$$

It is easy to show that, after averaging over initial states, we finally obtain for $|J_r|^2$

$$|J_r|^2 = \left(\frac{M_1}{M_1 + M_2} \right)^2 \frac{p_1 \cos^2 \gamma}{\hbar^2} \frac{4\pi\mu p}{(2\pi\hbar)^3} \left\{ \frac{(l+1)^2}{(2l+1)^2} D^2_{l,\, l+1} + \frac{l^2}{(2l+1)^2} D^2_{l,\, l-1} \right\},$$

$$D_{l,\, l\pm 1} = C_l \int_0^R f_l (\beta r) f_{l\pm 1} (kr) r^3 dr + C_l \frac{f_l (\beta R)}{k_l (\alpha R)} \int_R^\infty k_l (\alpha r) f_{l\pm 1} (kr) r^3 dr. \quad (19)$$

3. We consider several special cases. We assume that $l = 1$ in the initial state. This is the case, for example, in the Li^7 nucleus. We will view the latter as consisting of an α-particle and triton. Here the orbital moment of relative motion of the triton is $l = 1$ ($p_{3/2}$ state).

As a result of calculation, we obtain for the differential cross section

$$d\sigma = \frac{1080}{2989} \left(\frac{Z_1 Ze^2}{\hbar v} \right)^2 \Phi_1 (\varepsilon_1) \frac{d\varepsilon_1}{\varepsilon_1^{1/2}} \ln \frac{\hbar v}{(\varepsilon + \varepsilon_1) R}, \quad (20)$$

where

$$\Phi_1 (\varepsilon_1) = \left\{ \left(a_1 + \frac{a_2}{\sqrt{\varepsilon_1}} \right) \sin \sqrt{\rho \varepsilon_1} - \left(\frac{a_3}{\varepsilon_1} + a_4 - \frac{a_5}{\sqrt{\varepsilon_1}} \right) \cos \sqrt{\rho \varepsilon_1} \right\}^2;$$

$$a_1 = C_1/\beta^2, \quad a_2 = \hbar (2\mu)^{-1/2} [\beta R^2 f_0 (\beta R) - 1/\beta] C_1,$$

$$a_3 = (\hbar^2/2\mu) 2\beta R f_0 (\beta R) C_1,$$

$$a_4 = [(1 + \alpha R)/\alpha^2] \beta R f_0 (\beta R) C_1,$$

$$a_5 = \hbar R C_1/\sqrt{2\mu}, \quad \rho = 2\mu R^2/\hbar^2.$$

The coefficient C_1 is defined by Eq. (18); the quantity ϵ, which enters into Eq. (20), denotes the binding energy of the triton in the Li^7 nucleus and is equal to 2.52 Mev.

4. We consider further the case in which the light nucleus has orbital momentum $l = 2$ in the initial state. This case is encountered, for example, in the O^{17} nucleus, where the odd neutron is in a $d_{5/2}$ state.

For the effective differential cross section of the process we obtain

$$d\sigma = \frac{1}{61}\left(\frac{Z_1 Ze^2}{\hbar v}\right)^2 \frac{\hbar}{\sqrt{8\mu}} \frac{\sin^2\sqrt{\rho\varepsilon_1}}{\varepsilon_1^{3/2}} \Phi_1(\varepsilon_1)\, d\varepsilon_1 \ln\frac{\hbar v}{(\varepsilon+\varepsilon_1)R}, \tag{21}$$

where

$$\Phi_1(\varepsilon_1) = \left\{\frac{B}{\varepsilon_1} - \frac{D\,(1+\cot^2\sqrt{\rho\varepsilon_1})}{\sqrt{\varepsilon_1}} + A - D_1\cot\sqrt{\rho\varepsilon_1}\right\}^2,$$

$$A = \frac{6\alpha^2}{(\alpha^2+\beta^2)\, f_1(\beta R)\, f_3(\beta R)\, R^3}\left[\frac{1-f_0(\beta R)}{\beta^2} + \frac{\beta^2(1+\alpha R)}{\alpha^4} f_0(\beta R)\right],$$

$$B = 6\left(\frac{\hbar^2}{2\mu}\right)\frac{\beta^2\alpha}{(\alpha^2+\beta^2)R^2}\,\frac{f_0(\beta R)}{f_3(\beta R)\, f_1(\beta R)},$$

$$D = \frac{\hbar^2}{(2\mu)^{3/2}}\,\frac{2\beta^2}{(\alpha^2+\beta^2)R^2}\,\frac{f_0(\beta R)}{f_3(\beta R)\, f_1(\beta R)}, \qquad D_1 = \frac{\hbar^2}{(2\mu)^{3/2}} D.$$

All formulae for the effective differential cross section which we introduced above contain the quantity (βR), which is the solution of the transcendental equation (17). By giving the nuclear radius R and requiring the depth of the well representing the interaction of particles of the light nucleus to lie in an acceptable interval of values between 5 and 30 Mev, β can be unambiguously determined by solving Eq. (17) for a given value of l. The formulae obtained by us hold for the system in which the light nucleus is at rest. In order to obtain expressions for the effective cross section in the laboratory system, in which the heavy nucleus is at rest, it is necessary to carry out a transformation analogous to that which is made in the theory of disintegration of the deuteron in a Coulomb field[5].

We note, in conclusion, that the order of magnitude of the effective cross section for the disintegration of light nuclei in a Coulomb field, at energies in the range ~100 Mev turns out to be equal to $10^{-29}(ZZ_1)^2$ cm^2.

[1] J. Sawicki, Acta Phys. Polon. **14**, 135 (1955).

[2] P. Cüer and G. Combe, Compt. rend. (Paris) **239**, 351 (1954).

[3] I. Sh. Vashakidze and G. A. Ghilashvili, J. Exptl. Theoret. Phys. (U.S.S.R.) **26**, 254 (1954).

[4] J. Dabrowski and J. Sawicki, Phys. Rev. **97**, 1002 (1955).

[5] S. Dancoff, Phys. Rev. **72**, 1017 (1947).

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