tribution of decay electrons:

$$d^2W_\mu/d\Omega d\varepsilon = (2\pi)^{-4}\varepsilon_m^5\varepsilon^2\{(a_1+a_5)(1-\varepsilon)+{}^8/_3a_3(3-\varepsilon) + {}^2/_3(a_2+a_4)(3-2\varepsilon)+[(b_1+b_5)(1-\varepsilon) - {}^8/_3b_3(1+\varepsilon)+{}^2/_3(b_2+b_4)(1-2\varepsilon)](\mathbf{sl})\}, \quad (4)$$

where **l** denotes a unit vector in the direction of the momentum of the electron, and

$$a_i = g_i^2(1+|\alpha_i|^2), \quad b_i = g_i^2(\alpha_i+\alpha_i^*).$$

Integrating the probability (4) over the energy of the electron yields

$$\frac{dW_\mu}{d\Omega} = \frac{\varepsilon_m^5}{(2\pi)^4}\left\{\left(\frac{a_1+a_5}{12}+2a_3+\frac{a_4+a_2}{3}\right) + \left(\frac{b_1+b_5}{12}-\frac{14}{9}b_3-\frac{b_2+b_4}{9}\right)(\mathbf{sl})\right\} = v_1+v_2(\mathbf{sl}). \quad (5)$$

Note that the coefficient a which characterizes the degree of parity violation may be either real (for invariance under time reversal) or purely imaginary (for invariance under charge conjugation). In the latter case, the coefficients u_2 and v_2 are easily seen to be identically equal to zero, and there is no correlation. If a is real, then the correlation between the direction of the momentum of the μ-meson and that of the electron is given by

$$W(\mathbf{l},\mathbf{n}) = \sum_{s_1s_2} W_\pi(\mathbf{ns}_1)\,W_d(\mathbf{s}_1,\mathbf{s}_2)\,W_\mu(\mathbf{s}_2\mathbf{l}). \quad (6)$$

W_π and W_μ are obtained from Eqs. (2) and (5), and $W_d(\mathbf{s}_1, \mathbf{s}_2)$ is the probability for depolarization of the μ-meson as it is slowed down. If the depolarization is small, *i.e.*, if $W_d \sim \delta(\mathbf{s}_1-\mathbf{s}_2)$, Eq. (6) is easily solved:

$$W(\mathbf{ln}) = u_1v_1 + u_2v_2(\mathbf{ln}). \quad (7)$$

Note that the effect of parity violation shows up in (7) through the appearance of a scalar quantity (**ln**), and not a pseudoscalar as is usually the case. This is linked to the fact that parity is violated twice in the process under consideration, during the decay of the π-meson, and during that of the μ-meson.

In conclusion, we remark that experimental observation of this correlation is very difficult due to the fact that strong depolarization takes place during the lifetime of the μ-meson ($\sim 10^{-6}$ sec).

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Polarization in Reverse Reactions

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In this note, we shall demonstrate a relation which holds between the polarization of final particles in forward and reverse reactions. Consider an arbitrary reaction of the form $a + X \to b + Y$, where a, b, X, Y are arbitrary particles of spin ½; we shall assume that the product of the intrinsic parities of the particles before the reaction is the same as the product of the intrinsic parities of the particles formed after the reaction. We shall denote the spin states of the initial and final systems by the components ζ_{sm} of a column vector, where s is the total spin and m its z-component. It may be shown that the amplitude of the final particles has the form

$$\frac{e^{ik_1r}}{r}\frac{\sqrt{\pi}}{ikk_1}\cdot\begin{vmatrix} a-f & c & -d_2 \\ b & g-b & 0 \\ c & f & a-d_2 \\ d_1 & 0 & d_1 & e\end{vmatrix}\cdot\begin{vmatrix}\zeta_{11}\\ \zeta_{10}\\ \zeta_{1-1}\\ \zeta_{00}\end{vmatrix} \equiv \frac{e^{ik_1r}}{r}M\cdot\begin{vmatrix}\zeta_{11}\\ \zeta_{10}\\ \zeta_{1-1}\\ \zeta_{00}\end{vmatrix}, \quad (1)$$

where k and k_1 are the wave vectors before and after the reaction, ζ_{sm} is the spin function for the initial state (particles a and X), and the coefficients $a, b, c, \ldots$ are expressed by means of the elements of the reaction matrix $M_{ls,\,l's'}$ and the scattering angle θ. If we know the operator M, we can solve for the average values of the spin operators of particles b and Y and the reaction cross section $d\sigma$ $d\sigma/do$.

$$\overline{\sigma_i^{(b)}} = \langle\sigma_i^{(b)}\rangle/(d\sigma/do);\ \overline{\sigma_i^{(Y)}} = \langle\sigma_i^{(Y)}\rangle/d\sigma/do$$
$$\langle\sigma_i^{(b,Y)}\rangle = (k_1/k)\ \mathrm{Sp}\,(M\rho M^+\sigma_i^{(b,Y)});$$
$$d\sigma/do = (k_1/k)\,\mathrm{Sp}\,(M\rho M^+). \quad (2)$$

ρ is here the spin density matrix for the initial state, and $i = x, y, z$.

In the center-of-mass system of coordinates (x, y, z)*, we have:

$$d\sigma/do = (d\sigma/do)_0 + P_y (d\sigma/do)_y;$$
$$\langle\sigma_y\rangle = \langle\sigma_y\rangle_0 + P_y \langle\sigma_y\rangle_y,\ \langle\sigma_x\rangle = P_x \langle\sigma_x\rangle_x + P_z \langle\sigma_x\rangle_z,$$
$$\langle\sigma_z\rangle = P_x \langle\sigma_z\rangle_x + P_z \langle\sigma_z\rangle_z, \quad (3)$$

where P_i is the polarization component of particles a (particles X are assumed unpolarized), and $(d\sigma/do)_{0,y}$ and $\langle\sigma_i\rangle_{x,y,z}$ are expressible in terms of the coefficients $a, b, c, \ldots$ appearing in formula (1). The polarization of particles b and Y is conveniently expressed in the coordinate system (x', y', z'), wherein the z'-axis is in the direction of motion of particle b, and the y'-axis coincides with the y-axis; The components of the vectors $\langle\sigma^{(b)}\rangle$ and $\langle\sigma^{(Y)}\rangle$ in this system are obtained from the components of these vectors in the system (x, y, z) by applying the usual rules of vector algebra. Their dependence upon the polarization of particle a has the same form as in the (x, y, z) system [see Eq. (3)]. For example,

$$\langle\sigma_{x'}^{(b)}\rangle = P_x [\cos\vartheta \langle\sigma_x^{(b)}\rangle_x - \sin\vartheta \langle\sigma_z^{(b)}\rangle_x] + P_z [\cos\vartheta \langle\sigma_x^{(b)}\rangle_z - \sin\vartheta \langle\sigma_z^{(b)}\rangle_z] \equiv P_x \langle\sigma_{x'}^{(b)}\rangle_x + P_z \langle\sigma_{x'}^{(b)}\rangle_z.$$

Consider now the reverse reaction $b+Y \to a+X$. The cross section for this reaction $(d\sigma/do)'$ and the average values of the spin operators for particles a and X may be obtained in exactly the same way as in the reaction $a+X \to b+Y$. One need only interchange a and X with b and Y in the preceding analysis, and correspondingly change the directions of the coordinate axis (*i.e.*, the z-axis must now be directed along the direction of motion of particle b, the z'-axis along the direction of motion of particle a, etc. ...). Then in all the formulas , $\sigma^{(a)}$ will replace $\sigma^{(b)}$, $\sigma^{(Y)}$ will replace $\sigma^{(X)}$, k_1 will replace k, and some new coefficients a', b', c', d'_1 ... will replace a, b, c, d_1, ...; furthermore the quantities P_i must then be taken as describing the polarization of particle b before the reaction. The coefficients a', b', c',... which describe the reaction $b + Y \to a + X$ depend upon the elements of the matrix for this reaction, $\mathcal{M}^I_{ls,\, l's'}$, in exactly the same way as the coefficients a, b, c, ... depend upon $M^I_{ls,\, l's'}$. Symmetry under time reversal implies[1] that $\mathcal{M}^I_{ls,\, l's'} = M^I_{ls,\, l's'}$. Making use of this relation, and of the explicit form of the elements $a, b, c, \ldots$, it may easily be shown that

$$a - a' = \mathfrak{M};\ g - g' = -2\mathfrak{M};\ c - c' = -\mathfrak{M};\ b - b' = -\sqrt{2}\operatorname{ctg}\vartheta\cdot\mathfrak{M},$$
$$f - f' = \sqrt{2}\operatorname{ctg}\vartheta\cdot\mathfrak{M};\ d_2 = d_1';\ d_1 = d_2';\ e' = e, \quad (4)$$
$$\mathfrak{M} = {}^1/_2 \sin^2\vartheta\,[(a - g - c) - \sqrt{2}\operatorname{ctg}\vartheta\,(b - f)].$$

Substituting the expressions obtained from (4) for a', b', ... into the formulas for the cross section and the polarization of particles in the reaction $b + Y \to a + X$, one finds that the cross section and polarization for this reaction are related to the cross section and polarization for the reverse reaction in the following way:

$$(d\sigma/do)_0' = (k/k_1)^2 (d\sigma/do)_0;\ \langle\sigma_{y'}^{(a)}\rangle = (k/k_1)^2 (d\sigma/do)_y;$$
$$\langle\sigma_{y'}^{(X)}\rangle_0 = (k/k_1)^2 \langle\sigma_{y'}^{(b)}\sigma_{y'}^{(Y)}\rangle_y;$$
$$\langle\sigma_{y'}^{(a)}\rangle_y = (k/k_1)^2 \langle\sigma_{y'}^{(b)}\rangle_y;\ \langle\sigma_{y'}^{(X)}\rangle_y = (k/k_1)^2 \langle\sigma_{y'}^{(Y)}\rangle_y; \quad (5)$$
$$\langle\sigma_{x'}^{(a)}\rangle_x = (k/k_1)^2 \langle\sigma_{x'}^{(b)}\rangle_x;\ \langle\sigma_{x'}^{(a)}\rangle_z = -(k/k_1)^2 \langle\sigma_{z'}^{(b)}\rangle_x;\ \langle\sigma_{z'}^{(a)}\rangle_z = (k/k_1)^2 \langle\sigma_{z'}^{(b)}\rangle_z;$$
$$\langle\sigma_{x'}^{(X)}\rangle_x = (k/k_1)^2 \langle\sigma_{x'}^{(Y)}\rangle_x^*;\ \langle\sigma_{x'}^{(X)}\rangle_z = -(k/k_1)^2 \langle\sigma_{z'}^{(Y)}\rangle_x^*;\ \langle\sigma_{z'}^{(X)}\rangle_z = (k/k_1)^2 \langle\sigma_{z'}^{(Y)}\rangle_z^*.$$

The first formula in (5) gives the well known relation between cross sections in the forward and reverse reactions. The second formula in (5) relates the asymmetry of angular distribution in the forward reaction to the polarization in the reverse reaction. Indeed the right-and-left symmetry of the angular distribution in the reaction $a + X \to b + Y$, $e = P_y (d\sigma/do)_y/(d\sigma/do)_0$ (P_y is the polarization component of particle a, perpendicular to the reaction plane), may be found from the second formula in (5) to be equal to

* The z-axis is directed along the trajectory of particle a, the x-axis lies in the reaction plane (so that the direction of motion of the particle b corresponds to an azimuthal angle $\varphi = 0$), and the y-axis is perpendicular to the reaction plane.

$$e = P_y^{(a)} \langle \sigma_{y'}^{(a)} \rangle_0 / (d\sigma/do)_0' = P_y \overline{\sigma_{y'}^{(a)}}, \quad (6)$$

where $\overline{\sigma_y^{(a)}}$ is the polarization of particle a in the reaction $b + Y \to a + X$, when b and Y are unpolarized. In the case of elastic scattering $(a + X \to a + X)$ equation (6) becomes the well known formula of Wolfenstein.[2] Note that Wolfenstein only proved his theorem for the case where inelastic scattering is absent (he assumed the scattering operator M was unitary). Our demonstration is free of this limitation. The third formula in (5) relates the polarization of particle Y in the reaction $a + X \to b + Y$ to the correlated polarization in the reverse reaction. The remaining formulas in (5) need no special elucidation except for the formula of line four. The asterisk attached to the brackets < >, such as in $\langle \sigma_x^{(b)} \rangle_x^*$, denotes the fact that it describes the polarization of particle b for the reaction $a + X \to b + Y$ wherein particle X was polarized in the initial state while particle a was completely unpolarized. The index x attached to the bracket < >, refers here to the x-component of polarization of particle X. The rest of the formulas of line four may be interpreted in this way.

In conclusion the author wishes to thank Ia. A. Smorodinskii for a discussion of the results.

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Polarization of Cerenkov Radiation

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In order to analyze the dependence of Cerenkov radiation upon the spin of the charged particles, we have utilized the method developed in Ref. 1 (see also Ref. 2)., which allows one to solve for the intensities of both linearly and circularly polarized radiation.

When we consider linear polarization, we must resolve the amplitude of the vector potential of the quantized photon field into two mutually perpendicular components in the following fashion:

$$\mathbf{a} = \mathbf{a}_2 + \mathbf{a}_3 = \beta_2 q_2 + \beta_3 q_3, \quad \beta_2 = [\varkappa^0 \mathbf{k}^0] / \sqrt{1 - (\varkappa^0 \mathbf{k}^0)}, \quad \beta_3 = [\varkappa^0 \beta_2]. \quad (1)$$

$\varkappa^0 = \varkappa/\varkappa$ is a unit vector which characterizes the motion of the photon and the unit vector $\mathbf{k}^0$ must be assigned some definite direction (in our problem, we shall assume that the vector $\mathbf{k}^0$ is in the direction of the electron motion, *i.e.*, along the z-axis).

In the case of circular polarization the vector potential is resolved into two different components:

$$\mathbf{a} = \mathbf{a}_1 + \mathbf{a}_{-1} = \beta_1 q_1 + \beta_{-1} q_{-1}, \quad \sqrt{2}\beta_\lambda = \beta_2 + i\lambda\beta_3, \quad \lambda = 1, -1. \quad (2)$$

The quantized part of the vector potential appearing in Eq. (1) and (2) must satisfy the relations

$$q_{j'}^+ q_j = 0, \quad q_j q_{j'}^+ = \delta_{jj'}, \quad j, j' = 2, 3, 1, -1.$$

In contructing the quantized transverse electromagnetic field in a medium of refractive index $n (n = \sqrt{\varepsilon}, \mu = 1)$ (*cf.* Refs. 3 and 4, where the quantum theory of the Cerenkov effect is developed), we find that the vector potential $\mathbf{A}$ is related to the quantized amplitudes $\mathbf{a}$ (*cf.* Ref. 1 or 2) through the following expression

$$\mathbf{A} = L^{-3/2} \sum_\varkappa \sqrt{\frac{2\pi c\hbar}{n\varkappa}} (\mathbf{a} \exp\{-ic(\varkappa t/n) + i\varkappa \mathbf{r}\} + \mathbf{a}^+ \exp\{ic(\varkappa t/n - i\varkappa \mathbf{r}\}). \quad (3)$$

We shall choose to write the wave function for a free electron in the form

$$\psi = L^{-3/2} \sum_{\mathbf{k}'} C' b' \exp\{-icK't + i\mathbf{k}'\mathbf{r}\}, \quad (4)$$

where $\hbar\mathbf{k}$ is the momentum of the electron, $c\hbar K = c\hbar\sqrt{k^2 + k_0^2}$ the electron energy and $\hbar k_0/c$ its mass. We shall denote the initial state of the electron by an unprimed symbol and its final state by a primed symbol

Introducing the perturbation energy $U = (e/c)(\alpha\mathbf{A})$, we must consider the coefficients C' as time-dependent, satisfying the initial condition: $C' = C(k', 0) = \delta_{\mathbf{k}'\mathbf{k}}$.

When solving the Dirac equation (*i.e.*, including