Letters to the Editor

Energy Dependence of the Effective Cross Section of Inelastic Scattering of Particles near the Threshold

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A METHOD is presented in the present paper for the investigation of the energy dependence of inelastic scattering near the threshold. As an illustration, let us consider a system A consisting of a fixed force center and a particle a bound to it. Let particle b fall on this system. We shall consider that a and b are different, in order to avoid difficulties connected with the Pauli principle (in the case of identical particles a and b, the results are not changed in principle, but the calculations are more complicated).

We are interested in the inelastic scattering of b, in which a undergoes a transition from a state with energy E_0 to another state with energy E_1. Expanding the wave function of the system $A + b\,\Psi(r_1r_2)$ in the eigenfunctions ψ_α of system A

$$\Psi(\mathbf{r}_1\mathbf{r}_2) = \sum_\alpha \psi_\alpha(\mathbf{r}_1)\, F_\alpha(\mathbf{r}_2), \tag{1}$$

we get for F_α the system of equations[1]

$$\Delta F_\alpha + k_\alpha^2 F_\alpha = \sum_\beta V_{\alpha\beta} F_\beta \tag{2}$$

with asymptotic behavior

$$F_\alpha \sim \exp(ik_\alpha r_\alpha)\,\delta_{\alpha 0} + q_\alpha r^{-1}\exp(ik_\alpha r). \tag{3}$$

We introduce the notation $\hbar^2 k_0^2/2m_b$ for the energy of the particle before collision:

$$\hbar^2 k_\alpha^2/2m_b = \hbar^2 k_0^2/2m_b + (E_0 - E_\alpha), \tag{4}$$

$$k_0^2 = k_\alpha^2 + k_{\alpha 0}^2,\quad \varkappa_\alpha = k_0 - k_{\alpha 0} = (k_\alpha^2 + k_{\alpha 0}^2)^{1/2} - k_{\alpha 0}. \tag{5}$$

The effective cross section of the transition $0 \to \alpha$ is related to the amplitude q_α by the equation:

$$\sigma_{0\alpha} = (k_\alpha/k_0)\int |q_\alpha|^2 d\Omega. \tag{6}$$

We shall consider the system (2) in an approximation in which we can separate the two equations for F_0 and F_1 and neglect the coupling of these equations with the other equations (regarding this approximation, see Refs. 1, 2).

We shall then have

$$(\Delta + k_0^2)\,F_0 = V_{00}F_0 + V_{01}F_1,\quad (\Delta + k_1^2)\,F_1 = V_{11}F_1 + V_{10}F_0. \tag{7}$$

We limit ourselves to the simplest case of a transition between s states; moreover, we shall take into account only the s state of particle b which undergoes scattering. Then F_1 and F_2 will be spherically symmetric. Setting $F_\alpha = f_\alpha/r$, we get

$$d^2 f_\alpha/dr^2 + k_\alpha^2 f_\alpha = \sum_{\beta=0}^{1} V_{\alpha\beta} f_\beta \quad (\alpha = 0;\ 1). \tag{8}$$

For large r,

$$f_\alpha \sim k_\alpha^{-1}\sin k_\alpha r\,\delta_{\alpha 0} + q_\alpha \exp ik_\alpha r. \tag{9}$$

We can convert the system (8) with asymptotic conditions (9) to the integral equations

$$f_\alpha = c_\alpha \varphi_\alpha + \sum_{\beta=0}^{1}\int_0^r G_{\alpha\beta}(r,s)\, f_\beta(s)\,ds \quad (\alpha = 0;\ 1), \tag{10}$$

$$G_{\alpha\beta}(r,s) = \frac{1}{k_\alpha}\sin k_\alpha (r-s)\, V_{\alpha\beta}(s);\quad \varphi_\alpha = \frac{\sin k_\alpha r}{k_\alpha},$$

$$c_\alpha = \delta_{\alpha 0} - \sum_{\beta=0}^{1}\int_0^\infty e^{ik_\alpha s}\, V_{\alpha\beta}(s)\, f_\beta(s)\,ds.$$

For q_α we have

$$q_\alpha = -\frac{1}{k_\alpha}\sum_{\beta=0}^{1}\int_0^\infty \varphi_\alpha(s)\, V_{\alpha\beta}(s)\, f_\beta(s)\,ds. \tag{11}$$

We set

$$f_\alpha = \sum_{\beta=0}^{1} c_\beta x_{\alpha\beta} \quad (\alpha = 0;\ 1) \tag{12}$$

and introduce the notation

$$M_{\alpha\gamma} = \sum_{\beta=0}^{1}\int_0^\infty e^{ik_\alpha s}\, V_{\alpha\beta} x_{\beta\gamma}(s)\,ds. \tag{13}$$

We then obtain

$$x_{\alpha\beta} = \varphi_\alpha \delta_{\alpha\beta} + \sum_{\gamma=0}^{1}\int_0^r G_{\alpha\gamma}(r,s)\, x_{\gamma\beta}(s)\,ds, \tag{14}$$

$$c_\alpha = \delta_{\alpha 0} - \sum_{\gamma=0}^{1} M_{\alpha\gamma} c_\gamma. \tag{15}$$

Solving the system (15), and taking Eqs. (11)-(13) into account, we get for the amplitude of the excitation

$$q_1 = \frac{M_{10}k_1^{-1}\,\mathrm{Im}\,M_{11} - (1+M_{11})\,k_1^{-1}\,\mathrm{Im}\,M_{10}}{(1+M_{00})(1+M_{11}) - M_{01}M_{10}}\,. \quad (16)$$

Taking it into consideration that $\sin k_\alpha r/k_\alpha$ is an analytic function of k_α^2, we can represent $x_{\alpha\beta}$ in the form of a series in k_1^2. Substituting this series in $M_{1\gamma}$, and expanding $\exp ik_1 s$ in the radicand in powers of k_1, we get

$$M_{1\gamma} = M_{1\gamma}^{(0)} + ik_1 M_{1\gamma}^{(1)} + k_1^2 M_{1\gamma}^{(2)} + ik_1^3 M_{1\gamma}^{(3)} + \ldots \quad (\gamma = 0; 1). \quad (17)$$

The series (17) converges, generally speaking, only over some region of variation of k_1. The radius of convergence is determined by the form of $V_{\alpha\beta}(r)$. So far as $M_{0\gamma}$ is concerned, we can, by making use of (5), transform Eq. (13) to the form

$$M_{0\gamma} = \sum_\beta \int_0^\infty e^{i\varkappa_1 s + ik_{01}s}\, V_{0\beta}(s) x_{\beta\gamma}(s)\, ds. \quad (18)$$

Expanding $\exp i\varkappa_1 s$ in powers of $\varkappa_1$, we get

$$M_{0\gamma} = M_{0\gamma}^{(0)} + i\varkappa_1 M_{0\gamma}^{(1)} + \varkappa_1^2 M_{0\gamma}^{(2)} + k_1^2 M_{0\gamma}^{(3)} + \ldots \quad (19)$$

Substituting (17) and (19) in Eq. (16), and then in Eq. (6), and limiting ourselves to terms of no higher order in k_1 and $\varkappa_1$ than the second, we obtain

$$\sigma = 4\pi\frac{k_1}{k_0}\, a_1 \frac{1 + a_2 k_1^2}{1 + a_3 k_1 + a_4\varkappa_1 + a_5\varkappa_1^2 + a_6\varkappa_1 k_1 + a_7 k_1^2}\,. \quad (20)$$

The coefficients $a_1 \ldots a_7$ are expressed in terms of $M_{\alpha\gamma}^{(i)}$. In the case in which the threshold of excitation is sufficiently high, we can expand $\varkappa_1$ near the threshold in powers of k_1^2. We then have, with accuracy up to terms in k_1^2,

$$\sigma = 4\pi\,(k_1/k_0)a_1\,(1 + a_2 k_1^2)\,/\,(1 + a_3 k_1 + a_4 k_1^2). \quad (21)$$

In a number of cases, we can neglect the term $V_{10}F_0$ in (7). This is the so-called method of distorted waves.[1,2] In such an approximation, $M_{01} = 0$, whence, as is easy to show, $a_3 = 0$. Thus, in the approximate method of distorted waves, the linear term of Eqs. (20) and (21) is absent.

[1] N. Mott and G. Massey, *Theory of atomic collisions*, IIL, Ch. VIII (Russian translation).

[2] H. S. W. Massey, Rev. Mod. Phys. **28**, 3 (1956).

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133

Statistical Retardation in Dielectric Breakdown of Solid Dielectrics

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SEITZ[1] made the assumption that, in computing the dielectric strength of crystals on the basis of Hippel's criterion, it is necessary to account for the fluctuations in the behaviour of the electrons in the energy exchange with the lattice. The possibility of such fluctuations is described by a certain probability that the electron will be accelerated to the ionization potential without collisions with the lattice. This assumption leads to the possibility of statistical retardation in the breakdown of thin layers of solid dielectrics, and also to the dependence of the dielectric strength of crystals on thickness (for the thickness range 10^{-3} to 10^{-4} cm).[2,3] In 1952-1954, Japanese investigators[2,5] obtained data on the dependence of dielectric strength on the time of application of the voltage and on the thickness for samples of mica 3 to 5×10^{-4} cm thick (dotted lines in Figs. 1, 2). These authors concluded that Seitz' assumptions were confirmed experimentally. The size of electron avalanches and also the number of initial electrons formed per second were computed. The quantities computed in this manner were in accordance with the approximate computations of Seitz.

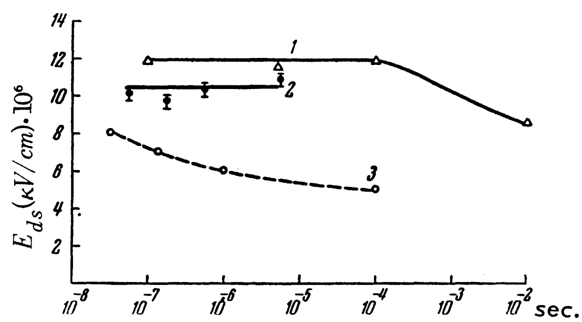

FIG. 1. Dependence of the dielectric strength of glass and mica on the duration of voltage application. 1 – glass, 2 – mica, 3 – mica according to the data, Ref. 5