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Inelastic Proton-Proton Scattering

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The problem of inelastic proton-proton scattering at 690 Mev is considered. It is assumed that in the intermediate state an isobar is formed which decays into a nucleon and π-meson. Limiting the consideration to isobars in S-states, using the laws of conservation of angular momentum and parity, and taking into account the Pauli principle, it was found possible to obtain the angular distribution of scattered nucleons, with the introduction of only one arbitrary constant.

INTRODUCTION

We base our consideration of proton-proton scattering on the experimental fact[1] of the existence of an excited state of the nucleon (isobar) with ordinary and isotopic spins equal to 3/2. We consider further that in the collision process an isobar with a definite mass $M = 1.31$* is formed in an S-state. Neglect of P- and D-state isobars is possible for energies of the incident nucleon not greatly exceeding the 650 Mev threshold energy for formation on an isobar with mass $M = 1.31$. In connection with this, all calculations were carried out for an incident proton energy of 690 Mev, which is the energy obtained on the accelerator of the Institute for Nuclear Problems, Academy of Sciences. The assumptions made proved to be sufficient to obtain an angular distribution, containing one arbitrary constant, for the scattered protons.

It must be noted, however, that because of the finite lifetime of the isobar ($\sim 10^{-23}$sec) the energy spectrum and angular distribution of the scattered protons at 690 Mev should be rather strongly smeared out, and only upon increasing the energy of the incident protons to 800 Mev and above does the picture become more clear cut.

*An absolute mass unit corresponding to 931 m_e is taken as mass and energy unit.

1. KINEMATICAL CALCULATION

We consider collisions of two particles of mass m_1 and m_2 such that two new particles of mass M_1 and M_2 are formed in place of the initial ones. Let one of the particles (say, m_2) be at rest, and the other in motion, having energy E. Then, making use of the laws of energy and momentum conservation, it is possible to show that the energy of the new particle of mass M_1 emerging at angle φ_1 is equal to

$$E_1 = \frac{Em_2 + A_1 \pm M_1 \frac{E^2 - m_1^2}{E + m_2} \cos^2\varphi_1 \left[1 - \frac{M_1^2 (E + m_2)^2 - (Em_2 + A_1)^2}{M_1^2 (E^2 - m_1^2) \cos^2 \varphi_1}\right]^{1/2}}{E + m - \cos^2 \varphi_1 (E^2 - m_1^2) / (E + m_2)}, \qquad (1.1)$$

$$A_1 = (M_1^2 - M_2^2 + m_1^2 + m_2^2) / 2.$$

a) *Formation of isobars*. The production of an isobar in the collision of two nucleons takes place as a result of one of the six following processes

$$p + p \begin{matrix} \nearrow I^{++} + n \\ \searrow I^{+} + p \end{matrix}, \quad p + n \begin{matrix} \nearrow I^{+} + n \\ \searrow I^{0} + p \end{matrix},$$

$$n + n \begin{matrix} \nearrow I^{0} + n \\ \searrow I^{-} + p \end{matrix}$$

All of these processes occur with different probabilities, but are kinematically equivalent, if the neutron-proton mass difference is neglected. It can be shown that in proton-proton scattering the ratio of probabilities of production of the isobars I^{++} and I^{+} is 3 : 1. In fact, we have

$$\Psi_{1,1} = C_{1,1}^{3/2,\, 1/2;\, 1/2,\, 1/2} \Phi_{3/2,\, 1/2} \chi_{1/2,\, 1/2} + C_{1,1}^{3/2,\, 3/2;\, 1/2,\, -1/2} \Phi_{3/2,\, 3/2} \chi_{1/2,\, -1/2}, \qquad (1.2)$$

where $\Psi_{1,1}$ is the wave function of the system in the intermediate state, $\Phi_{3/2,\, I_z}$ and $\chi_{1/2,\, I_z}$ are the wave functions of the isobar and nucleon, respectively, with isotopic spin projection I_z. Using the expressions for the Clebsch-Gordan coefficients[2] we obtain

$$\Psi_{1,1} = -\frac{1}{2} \Phi_{3/2,\, 1/2} \chi_{1/2,\, 1/2} + \frac{\sqrt{3}}{2} \Phi_{3/2,\, 3/2} \chi_{1/2,\, -1/2}, \qquad (1.3)$$

from which

$$\sigma (I^{+}, p) = 1/3\, \sigma (I^{++}, n). \qquad (1.4)$$

The expression for the energies of the nucleons scattered through angle θ_1 is obtained from the general formula (1.1) by substituting

$$\varphi_1 = \theta_1, \quad m_1 = m_2 = M_1 = 1, \quad A_1 = (3 - M^2) / 2.$$

With the 60 Mev half-width[1] of the isobar level taken into account, the energy E_1 of the scattered nucleons depends on the incident energy E as shown on Fig. 1 for $\theta_1 = 15°$ and $M = 1.26$, 1.31, 1.37. From

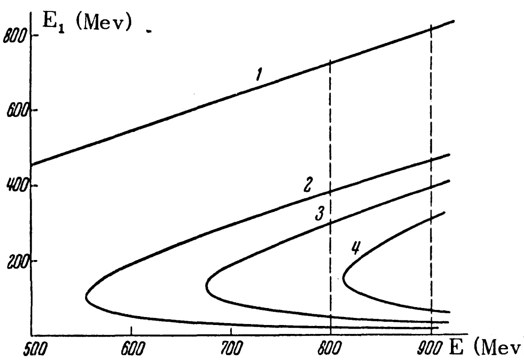

FIG. 1. Dependence of the energies of the scattered nucleons on the energies of incident nucleons for M equal to: *1* – 0; *2* – 1.26; *3* – 1.31; *4* – 1.37.

this figure it can be seen that the presence of the isobar changes the spectrum of the scattered nucleons substantially. Thus, in the case of elastic scattering, the energy of the nucleons scattered through $\theta = 15°$ will be simply 712 Mev for $E = 800$ Mev. If an isobar of mass M lying in the interval $1.26 < M < 1.37$ is produced in the collision of nucleons, then a maximum, which splits into two with increasing incident energy, should appear in the energy spectrum of the scattered nucleons (see Fig. 2).

b) *Decay of isobars*. The isobar decay into nucleon and π-meson proceeds in the following way:

$$I^{++} \to p + \pi^{+}, \quad I^{+} \begin{matrix} \nearrow p + \pi^{0} \\ \searrow n + \pi^{+} \end{matrix}, \quad I^{0} \begin{matrix} \nearrow n + \pi^{0} \\ \searrow p + \pi^{-} \end{matrix},$$

$$I^{-} \to n + \pi^{-}.$$

If only proton-proton scattering is considered, then only the first three processes take place.

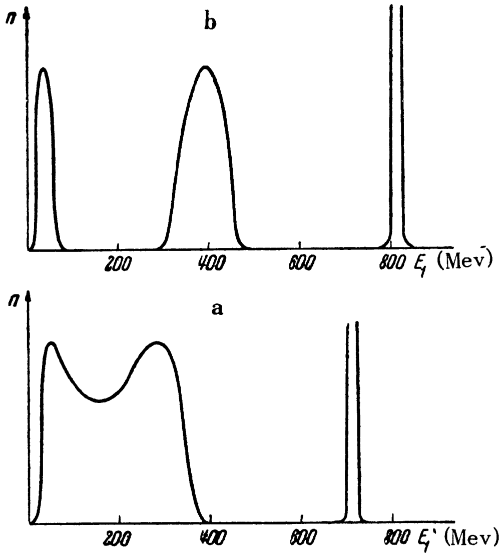

FIG. 2. Setting in of the splitting of the maximum in the energy spectrum of the scattered nucleons for energies E of the incident nucleons equal to: a – 800 Mev and b – 900 Mev.

Using, as in the case of formation of the isobar, the isotopic spin formalism, it is possible to show that the probabilities of the second and third processes are in the ratio 2 : 1, i.e.,

$$^1/_2\, \sigma(p, \pi^0) = \sigma(n, \pi^+). \tag{1.5}$$

Thus, if proton-proton scattering proceeds by way of isobar formation, then from (1.4) and (1.5) it follows that the probabilities of the processes

$$p + p \rightarrow p + n + \pi^+ \text{ and}$$
$$p + p \rightarrow p + p + \pi^0$$

are in the ratio of 5 : 1.

In order to obtain an expression for the energies of nucleons and mesons, coming from the decay of isobars, it is necessary to substitute in the formula (1.1): for nucleons

$$\varphi_1 = \theta, \quad m_1 = M; \quad m_2 = 0, \quad M_1 = 1,$$
$$M_2 = \mu = 0{,}142,$$
$$A = {}^1/_2 (M^2 + 1 - \mu^2);$$

for mesons

$$\varphi_1 = \theta_2, \quad m_1 = M, \quad m_2 = 0, \quad M_2 = 1,$$
$$M_1 = \mu = 0{,}142,$$
$$A = {}^1/_2 (M^2 - 1 + \mu^2).$$

2. FORMATION OF ISOBARS

We determine the probability of formation of isobars with a given spin projection on the direction of motion taking into account only S-scattering in the final state. We designate the scattering matrix which transforms the incident wave into the scattered one by

$$(\mathbf{n}, S_{n_0} \mid R \mid \mathbf{n}_0, S^0_{n_0}),$$

were $\mathbf{n}_0$ and $\mathbf{n}$ are the unit vectors in the direction of motion of the particle before and after collision in the center of mass system; $S^0_{n_0}$ and S_{n_0} are the projections of total spin of the system on direction $\mathbf{n}_0$ before and after collision.

In order to characterize all possible spin states, we introduce the index r, denoting: $r = 1$ – singlet ($S = 0$) state of the two nucleons, $r = 2$ – triplet ($S = 1$) state of the two nucleons, $r = 3$ – triplet ($S = 1$) state of the isobar and nucleon, $r = 4$ – quintet ($S = 2$) state of the isobar and nucleon.

In order to use the conservation of total angular momentum j and its projection m, we expand the scattering matrix in terms of the orthonormal system of eigenfunctions $J^r_{jm}(\mathbf{n}, S_n)$:

$$(\mathbf{n}, S_{n_0} | R \mid \mathbf{n}_0, S^0_{n_0}) = \sum_{j, m, r} \sum_{j_0, m_0, r_0} (j, m, r \mid R \mid j_0, m_0, r_0) \times J^r_{jm}(\mathbf{n}, S_{n_0}) \overset{+}{J}{}^{r_0}_{j_0 m_0}(\mathbf{n}_0, S^0_{n_0}). \tag{2.1}$$

Using the fact that, because of the conservation of j and m and the lack of a preferred direction, $(j, m, r \mid R \mid j_0, m_0, r_0)$ has the form

$$(j, m, r \mid R \mid j_0, m_0, r_0) = (r \mid R_j \mid r_0)\, \delta_{jj_0} \delta_{mm_0}, \tag{2.2}$$

we obtain

$$(\mathbf{n}, S_{n_0} \mid R \mid \mathbf{n}_0, S^0_{n_0}) = \sum_{j, r, r_0} (r \mid R_j \mid r_0)\, W^{r, r_0}_j (\mathbf{n}, S_{n_0}; \mathbf{n}_0, S^0_{n_0}),$$
$$W^{r, r_0}_j (\mathbf{n}, S_{n_0}; \mathbf{n}_0, S^0_{n_0}) = \sum_{m=-j}^{j} J^r_{jm}(\mathbf{n}, S_{n_0}) \overset{+}{J}{}^{r_0}_{j_0 m_0}(\mathbf{n}_0, S^0_{n_0}). \tag{2.3}$$

For $j = 0$ a single state with $l = S$ corresponds to each r. For $j = 1$ the state $r = 1 (S = 0)$ can be obtained in only one way ($l = 1$). The states $r = 2, 3$ ($S = 1$) are possible for the three values $l = 0, 1, 2$. The values

$l=1, 2, 3$ correspond to the state $r=4\,(S=2)$. For $j=2$, one value $l=2$ corresponds to the state $r=1$ $r=1\,(S=0)$. The states $r=2,3\,(S=1)$ are possible for three values $l=1, 2, 3$ and, finally, the state $r=4$ is possible for five values $l=0, 1, 2, 3, 4$*.

Together with the authors of Refs. 3 and 4 we will consider that a pseudoscalar isobar is formed in the nucleon collision process. Then, because of conservation of parity, all matrix elements corresponding to transitions with changes of l of $\pm1, \pm3 \ldots$ should be zero.

Thus, 4 complex or 8 real constants enter into the description of the transition: nucleon + nucleon → isobar + nucleon (in S-states). However, the situation alters materially if one considers the collision of two identical nucleons. In this case it is necessary to take into account the Pauli principle, according to which all matrix elements corresponding to initial states with even S and odd l and with odd S and even l are zero.

Considering that the contribution of terms with $l>0$ is small in the final states, we find that one** real parameter C is sufficient to describe the collision of two identical nucleons.

Thus, in the collision of two identical nucleons, the scattering matrix has the form

$$(\mathbf{n}, S_{n_0} \mid R \mid \mathbf{n}_0, S^0_{n_0}) = C W_2^{4,1}(\mathbf{n}, S_{n_0}; \mathbf{n}_0, S^0_{n_0}). \quad (2.4)$$

Using the well known expression for spherical vectors[5], we obtain

$$J^4_{2m}(\mathbf{n}, S_{n_0}) = Y_{0;0}(\mathbf{n})\,\Phi^{(2)}_m(S_{n_0}),$$

$$J^1_{2m}(\mathbf{n}, S_{n_0}) = Y_{2m}(\mathbf{n}), \quad (2.5)$$

where $\Phi^{(2)}_m(S_{n_0})$ are spin functions corresponding to total spin $S=2$ and projection $S_{n_0}=m$.

Thus, the operator $W_2^{4,1}(\mathbf{n}, S_{n_0}; \mathbf{n}_0, S^0_{n_0})$ can be written as

$$W_2^{4,1}(\mathbf{n}, S_{n_0}; \mathbf{n}_0, S^0_{n_0}) = (4\pi)^{-1/2}\sum_m \Phi^{(2)}_m(S_{n_0})\,Y^*_{2m}(\mathbf{n}_0). \quad (2.6)$$

If the Z-axis is directed along $\mathbf{n}_0$, then

$$W_2^{4,1}(\mathbf{n}, S_{n_0}; \mathbf{n}_0, S^0_{n_0}) = (\sqrt{5}/4\pi)\,\Phi^{(2)}_0(S_{n_0}). \quad (2.7)$$

*The coefficients $(r|R_j|r_0)$ for $j\geqslant3$ are not considered because after collision the state with $l=0$ (S-scattering) is not possible for $j\geqslant3$.

** In general, C is a complex number, but in so far as the absolute value of the phase is not important, only the modulus of C need be considered.

But since we are interested in states with spin projections on the direction $\mathbf{n}$, because different angular distributions correspond to states with different spin projections, it is necessary to transform $\Phi^{(2)}_0(S_{n_0})$ to the new system of coordinates with the Z-axis along the direction $\mathbf{n}$. The matrix corresponding to the irreducible representation of order l of the rotation through Euler angles φ_1, θ, φ_2, has the following form[6]

$$T^l = e^{-im\varphi_1}P^l_{mn}(\cos\theta)\,e^{-in\varphi_2},$$

$$P^l_{mn}(x) = A\,(1-x)^{-(n-m)/2}(1+x)^{-(n+m)/2}\,\frac{d^{l-n}}{dx^{l-n}} \times[(1-x)^{l-m}(1+x)^{l+m}]. \quad (2.8)$$

In our special case $l=2$ and $\varphi_1=\varphi_2=0$. Therefore, we have

$$\Phi'(S_n) = T^2\Phi^{(2)}_0(S_{n_0}) = -\frac{1}{2}\sqrt{\frac{3}{2}}\sin^2\theta\,(\Phi^{(2)}_{-2}(S_n) + \Phi^{(2)}_2(S_n)) + i\sqrt{\frac{3}{2}}\sin\theta\cos\theta\,(\Phi^{(2)}_{-1}(S_n) + \Phi^{(2)}_1 \times(S_n)) + \frac{1}{2}(3\cos^2\theta - 1)\,\Phi^{(2)}_0(S_n).$$

Now (see Ref. 2)

$$\Phi^{(2)}_{\mp2}(S_n) = \Phi_{\mp3/2}(\Sigma_n)\,\chi_{\mp1/2}(\sigma_n),$$

$$\Phi^{(2)}_{\mp1}(S_n) = \frac{1}{2}\Phi_{\mp3/2}(\Sigma_n)\,\chi_{\mp1/2}(\sigma_n) + \frac{\sqrt{3}}{2}\Phi_{\mp1/2}(\Sigma_n)\,\chi_{\mp1/2}(\sigma_n),$$

$$\Phi^{(2)}_0(S_n) = \frac{1}{\sqrt{2}}\Phi_{-1/2}(\Sigma_n)\,\chi_{1/2}(\sigma_n) + \frac{1}{\sqrt{2}}\Phi_{1/2}(\Sigma_n)\,\chi_{-1/2}(\sigma_n).$$

Thus, the probabilities of different states corresponding to different projections of the spin of the isobar Σ_n and spin of the nucleon σ_n on the direction $\mathbf{n}$ are equal to

$$\begin{array}{lll} {}^3/_8\sin^4\theta & \text{for } \Sigma_n = \mp{}^3/_2, & \sigma_n = \mp{}^1/_2; \\ {}^3/_8\sin^2\theta\cos^2\theta & \text{for } \Sigma_n = \mp{}^3/_2, & \sigma_n = \mp{}^1/_2; \\ {}^9/_8\sin^2\theta\cos^2\theta & \text{for } \Sigma_n = \mp{}^1/_2, & \sigma_n = \mp{}^1/_2; \\ {}^1/_8(3\cos^2\theta-1)^2 & \text{for } \Sigma_n = \mp{}^1/_2, & \sigma_n = \mp{}^1/_2 \end{array} \quad (2.9)$$

and, summing over the two possible projections of the nucleon spin, we obtain

$$\begin{array}{ll} {}^3/_8\sin^2\theta & \text{for } \Sigma_n = \pm{}^3/_2, \\ {}^3/_8\cos^2\theta + {}^1/_2 & \text{for } \Sigma_n = \mp{}^1/_2. \end{array} \quad (2.10)$$

Integrating the expression obtained over all solid angle, we find that the total probabilities of states with spin projections of the isobar $\pm\,{}^3\!/_2$ and $\pm\,{}^1\!/_2$ are the same.

3. DECAY OF THE ISOBAR

The determination of the differential cross section for decay of an isobar into a nucleon and π-meson is most simple in the center-of-mass system of the isobar. According to Ref. 7, the components of the wave function of the isobar in the center of mass system are

$$S_z = \frac{3}{2}:\quad B_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\0\end{pmatrix};\quad B_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}i\\0\\0\\0\end{pmatrix};\quad B_3 = B_4 = 0;$$

$$S_z = \frac{1}{2}:\quad B_1 = \frac{1}{\sqrt{6}}\begin{pmatrix}0\\1\\0\\0\end{pmatrix};\quad B_2 = \frac{1}{\sqrt{6}}\begin{pmatrix}0\\i\\0\\0\end{pmatrix};\quad B_3 = \frac{1}{\sqrt{6}}\begin{pmatrix}-2\\0\\0\\0\end{pmatrix};\quad B_4 = 0; \tag{3.1}$$

$$S_z = -\frac{1}{2}:\quad B_1 = \frac{1}{\sqrt{6}}\begin{pmatrix}1\\0\\0\\0\end{pmatrix};\quad B_2 = \frac{1}{\sqrt{6}}\begin{pmatrix}-i\\0\\0\\0\end{pmatrix};\quad B_3 = \frac{1}{\sqrt{6}}\begin{pmatrix}0\\2\\0\\0\end{pmatrix};\quad B_4 = 0;$$

$$S_z = -\frac{3}{2}:\quad B_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\\0\end{pmatrix};\quad B_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\\0\\0\end{pmatrix};\quad B_3 = B_4 = 0. \tag{3.1'}$$

If the 4-momentum of the π-meson in the center of mass system of the isobar is designated by q_μ, then the matrix element of the transition can be written

$$U = g\psi^+ q_\mu B_\mu. \tag{3.2}$$

Using the above expressions for B_μ, we have

$$S_z = \frac{3}{2}:\quad q_\mu B_\mu = -\frac{1}{\sqrt{2}}\, q\sin\theta\, e^{i\varphi}\begin{pmatrix}1\\0\\0\\0\end{pmatrix};$$

$$S_z = \frac{1}{2}:\quad q_\mu B_\mu = \frac{1}{\sqrt{6}}\, q\begin{pmatrix}2\cos\theta\\-\sin\theta e^{i\varphi}\\0\\0\end{pmatrix};$$

$$S_z = -\frac{1}{2}:\quad q_\mu B_\mu = -\frac{1}{\sqrt{6}}\begin{pmatrix}\sin\theta e^{-i\varphi}\\2\cos\theta\\0\\0\end{pmatrix}; \tag{3.3}$$

$$S_z = -\frac{3}{2}:\quad q_\mu B_\mu = -\frac{1}{\sqrt{2}}\, q\sin\theta e^{-i\varphi}\begin{pmatrix}0\\1\\0\\0\end{pmatrix},$$

where $q = |\mathbf{q}|$ and θ, φ are the angles defining the direction of motion of the nucleon in the center of mass system of the isobar.

Substituting in (3.2) definite values of ψ and $q_\mu B_\mu$, corresponding to particular orientations of the spins of nucleon and isobar, we obtain the matrix elements and, consequently, the probabilities of various transitions. Thus, we have

$$\begin{aligned} &W_{3/2,\,1/2} = {}^1\!/_2 \sin^2\theta, && W_{3/2,\,-1/2} = 0,\\ &W_{1/2,\,1/2} = {}^2\!/_3 \cos^2\theta, && W_{1/2,\,-1/2} = {}^1\!/_6 \sin^2\theta\\ &W_{-1/2,\,1/2} = {}^1\!/_6 \sin^2\theta, && W_{-1/2,\,-1/2} = {}^2\!/_3 \cos^2\theta,\\ &W_{-3/2,\,1/2} = 0, && W_{-3/2,\,-1/2} = {}^1\!/_2 \sin^2\theta, \end{aligned} \tag{3.4}$$

where $W_{S_z,\,\sigma_z}$ is the probability of decay of an isobar with spin projection S_z into a π-meson and nucleon with spin projection σ_z.

Summing over different orientations of nucleon spins, we obtain

$$W_{3/2} = \sin^2\theta,\qquad W_{1/2} = {}^4\!/_3\cos^2\theta + {}^1\!/_3\sin^2\theta, \tag{3.5}$$

where $W_{3/2}$, $W_{1/2}$ are the probabilities of decay of an isobar with $|S_z| = {}^3\!/_2,\ {}^1\!/_2$.

4. TRANSITION TO THE LABORATORY SYSTEM

The probability $w^1_{S_z}(\psi)$ of formation of an isobar in the laboratory coordinate system is connected with the probability of formation $W^1_{S_z}(\psi')$ in the system of the center of mass of the colliding nucleons by the following relation

$$W^1_{S_z}(\psi')\sin\psi'\,d\psi' = w^1_{S_z}(\psi)\sin\psi\,d\psi, \tag{4.1}$$

whereas there is the following relation between the angles of emergence of the isobars ψ' and ψ in the different systems:

$$\cos\psi' = (P(\psi)\cos\psi - p/2)/(P^2(\psi) - pP(\psi)\cos\psi + p^2/4)^{1/2}, \quad (4.2)$$

where $P(\psi)$ is the momentum of the isobar, p the momentum of the incident nucleon. But in the laboratory system, to each ψ correspond two values of $P(\psi)$ (P^+ and P^-) and consequently, also two values of ψ'. In this connection, we definite $w^1_{S_z,\lambda}$ to be the probability, in the laboratory system, of formation of an isobar with spin projection S_z and momentum P^+, if $\lambda=1$, and P^-, if $\lambda=2$.

If the total probability of formation of an isobar is normalized to unity, then

$$w^1_{\frac{3}{2},\lambda} = \frac{3}{16\pi}\sin^2\psi \frac{1 - \frac{K_\lambda}{2}\left(\cos\psi + \frac{1}{P_\lambda}\frac{dP_\lambda}{d\psi}\sin\psi\right)}{(1 - K_\lambda\cos\psi + K_\lambda^2/4)^{5/2}},$$

$$w^1_{\frac{1}{2},\lambda} = \frac{\left(\frac{1+3\cos^2\psi}{4} - K_\lambda\cos\psi + \frac{K_\lambda^2}{4}\right)\left(1 - \frac{K_\lambda}{2}\left(\cos\psi + \frac{1}{P_\lambda}\frac{dP_\lambda}{d\psi}\sin\psi\right)\right)}{4\pi(1 - K_\lambda\cos\psi + K_\lambda^2/4)^{5/2}}, \quad (4.3)$$

$$K_\lambda(\psi) = p/P_\lambda(\psi).$$

The transition from the center-of-mass system of the isobar to the system where the Z-axis coincides with the direction of flight of the isobar in the laboratory system is accomplished in an analogous fashion.

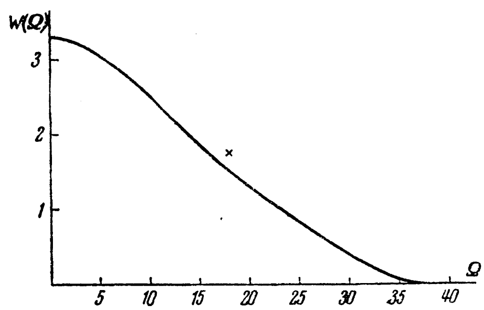

FIG. 3. Angular distribution of the scattered protons in the laboratory system.

Thus, we let $w^1_{S_z,\lambda}(\psi)$ be the probability of formation of an isobar with momentum P_λ in the laboratory system, and $w^2_{S_z,\lambda,\mu}(\theta)$ be the probability of formation of a nucleon with momentum $p_\mu(\theta)$ ($\mu=1$ for p_+ and $\mu=2$ for p_-) as a result of decay of an isobar of type S_z, λ in the system of coordinates connected with the direction of motion of the isobar. Then the probability of emergence of the nucleon at angle Ω in the laboratory system relative to the direction of the incident nucleon is equal to

$$W(\Omega) = \sum_{S_z,\lambda,\mu}\int w^1_{S_z,\mu}(\psi)\, w^2_{S_z,\lambda,\mu}(\theta)\, d\psi\, d\theta, \quad (4.4)$$

$$\cos\theta = \cos\Omega\cos\psi + \sin\Omega\sin\psi\sin\varphi.$$

In order to simplify the calculation of the double integral Eq. (4.4) we replace the probabilities $w^1_{S_z,\lambda}(\psi)$ by δ-functions, corresponding to the most probable angles, taking the exact expressions for $w^2_{S_z,\lambda,\mu}(\theta)$. The results of such calculations are given on Fig. 3. In order to evaluate such an approximation, a numerical integration of the expression Eq. (4.4) was carried out for a single value $\Omega = 18°$. It turned out that the error coming from the replacement mentioned above was 16%.

In conclusion, I should like to use this opportunity to express my deep gratitude to Iu. A. Gol'fand and Acad. I. E. Tamm for valuable advice and critical remarks and constant interest in this work.

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