On Low-Frequency Oscillations in the Plasmas of Electronegative Gases

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The oscillations of ions in a plasma whose charged components are electrons, and positive and negative ions are considered. Two branches of low-frequency oscillations have been established whose frequencies depend differently on the concentration of the charged particles.

ONE of the charged components in the gas-discharge plasmas of electronegative gases (or mixtures of such gases) is composed of negative ions which arise as a result of electrons becoming attached to neutral molecules. This situation causes several peculiarities in gas discharges in these cases.* So far, however, effects due to negative ions have been taken into account only in determining steady-state distributions of charged particles[3-5], and the study of small pertubations has been restricted to plasmas containing only electrons and positive ions. It is therefore of interest to determine those changes in the behavior of small perturbations of the charged-particle densities which are due to ence of negative ions in the plasma.

We shall examine the influence of negative ions on the behavior of small density perturbations in the hydrodynamic approximation. The gas-discharge plasma in this case is considered a mixture of three ideal gases (the electron gas, the gas of positive ions, and the gas of negative ions)[6] interacting by means of a self-consistent field which leads to the appearance of additional forces on the electrons and ions. The present treatment shall differ from our previous one[6] in that it shall not take account of interactions with a neutral gas or of effects due to creation and annihilation of charged particles, so that the applicability of the results is restricted to low-frequency oscillations at pressures which are not too high. With these assumptions, the hydrodynamic equations describing the behavior of the gases can be written

$$\frac{\partial N_e}{\partial t} + \frac{\partial}{\partial x_\alpha}(N_e v_\alpha^e) = 0, \quad P_e = \Theta_e N_e,$$

$$\frac{\partial}{\partial t}(N_e v_\alpha^e) + \frac{\partial}{\partial x_\beta}(N_e v_\alpha^e v_\beta^e) = -\frac{1}{M_e}\frac{\partial P_e^\alpha}{\partial x_\alpha} + \frac{eN_e}{M_e}E_\alpha \tag{1}$$

for the electrons, with similar equations for the positive (index p) and negative (index n) ions (in the equations for N_p the opposite sign must be taken for the charge e). Here N_e, v_α^e, P_e, M_e, and Θ_e are the concentration, α-component of the velocity, partial pressure, mass of the individual particle, and temperature in ergs of the electron gas, respectively. The field is determined from Maxwell's equations for the static case, namely

$$\partial E_\alpha / \partial x_\alpha = 4\pi e (N_e + N_n - N_p),$$
$$\partial E_\alpha / \partial x_\beta - \partial E_\beta / \partial x_\alpha = 0. \tag{2}$$

Let us consider the behavior of small perturbations in the linear approximation. For the zeroth approximation we shall take the stationary case in which the gases are considered at rest and to have constant density distributions in space. The densities N_e^0, N_n^0, and N_p^0 satisfy the quasi-neutrality conditions[4]

$$N_e^0 + N_n^0 = N_p^0. \tag{3}$$

We shall look for the charged-particle densities in the first approximation in the form

$$N_e = N_e^0 + n_e, \; N_p = N_p^0 + n_p, \; N_n = N_n^0 + n_n, \tag{4}$$

where $n_e, \ldots$ are small perturbations in the electron and ion densities. Eliminating the momentum and electric field from Eqs. (1), and restricting ourselves

*It has been established experimentally that mixing electro-negative gasses makes stratified discharge more likely, influences the length of the stratifications, *etc.*[1,2]

to the linear terms in the deviations of the densities, velocities, and self-consistent field, we obtain a set of equations determining the behavior of small simultaneous perturbations of the charged-particle densities in the isothermal case:

$$\frac{\partial^2 n_e}{\partial t^2} - \frac{\Theta_e}{M_e}\frac{\partial^2 n_e}{\partial x^2} + \omega_e^2 n_e - \omega_e^2 n_p + \omega_e^2 n_n = 0,$$

$$\frac{\partial^2 n_p}{\partial t^2} - \frac{\Theta_p}{M_p}\frac{\partial^2 n_p}{\partial x^2} + \omega_p^2 n_p - \omega_p^2 n_e - \omega_p^2 n_n = 0,$$

$$\frac{\partial^2 n_n}{\partial t^2} - \frac{\Theta_n}{M_n}\frac{\partial^2 n_n}{\partial x^2} + \omega_n^2 n_n + \omega_n^2 n_e - \omega_n^2 n_p = 0,$$

$$\omega_e^2 = 4\pi e^2 N_e^0/M_e, \ldots \qquad (5)$$

We attempt to find solutions of Eqs. (5) proportional to exp $(i\omega t - ikx)$. We then obtain a dispersion which, on the assumption of highly nonisothermal plasmas and low frequencies, breaks up into two equations; the first of these is

$$\omega^2 - (k^2\Theta_e/M_e) - \omega_e^2 = 0 \qquad (6)$$

which describes high-frequency oscillations of the electrons while the ions remain at rest*, and the second is

$$\left[\omega^2 - \left(\frac{k^2\Theta_p}{M_p} + \omega_p^2\right)\right]\left[\omega^2 - \left(\frac{k_2\Theta_n}{M_n} + \omega_n^2\right)\right] = \omega_p^2\omega_n^2 \qquad (7)$$

which corresponds to low-frequency oscillations of the ions about a background of uniformly distributed electrons. From Eqs. (7) we have

$$\omega_{1,2}^2 = \frac{a+b}{2} \pm \frac{a-b}{2}\sqrt{1 + \frac{4c}{(a-b)^2}}, \qquad (8)$$

$$a = k^2\Theta_p/M_p + \omega_p^2, \; b = k^2\Theta_n/M_n + \omega_n^2, \; c = \omega_p^2\omega_n^2.$$

Since c is determined by the product of the positive and negative ion densities, and $(a-b)^2$ by the square of the positive ion density, for low negative ion concentrations $c \ll (a-b)^2$. Restricting ourselves to terms only of first order in $c/(a-b)^2$, we obtain from Eq. (8)

$$\omega_1^2 = \frac{k^2\Theta_p}{M_p} + \omega_p^2 + \frac{\omega_p^2\omega_n^2}{(k^2\Theta_p/M_p) + \omega_p^2 - (k^2\Theta_n/M_n) - \omega_n^2}, \qquad (9)$$

$$\omega_2^2 = \frac{k^2\Theta_n}{M_n} + \omega_n^2 - \frac{\omega_p^2\omega_n^2}{(k^2\Theta_p/M_p) + \omega_p^2 - (k^2\Theta_n/M_n) - \omega_n^2}. \qquad (10)$$

If we take account of the fact that[4] $\Theta_p/M_p \approx \Theta_n/M_n$, operations on (9) and (10) lead to

$$\omega_1^2 = (k^2\Theta_p/M_p) + \omega_p^4/(\omega_p^2 - \omega_n^2), \qquad (11)$$

$$\omega_2^2 = (k^2\Theta_n/M_n) - \omega_n^4/(\omega_p^2 - \omega_n^2). \qquad (12)$$

Thus the result when negative ions are taken into account differs from the previous ones in that there appear two branches of low-frequency oscillations whose frequencies depend in different ways on the charged-particle concentration.

Let us consider in more detail the oscillations described by Eqs. (11) and (12). When $N_p^0 \gg N_n^0$, the Langmuir frequency of free oscillations of the negative ions will be significantly lower than that of the positive ions. Then (11) reduces to

$$\omega_1^2 = (k^2\Theta_p/M_p) + \omega_p^2, \qquad (13)$$

which is a dispersion equation for the positive ion

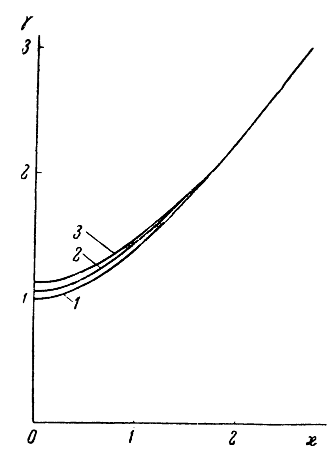

FIG. 1. The dependence of the frequency on the wave vector as given by Eq. (11) for the first branch of oscillations, in dimensionless units. The parameter taken for the negative ion concentration is N_n^0/N_p^0, and for the various curves it is equal to 1-0, 2-0.1, 3-0.2; $\gamma = \omega/\omega_p$, $\varkappa = k/k_D$ is the reciprocal of the Debye distance for positive ions.

*A similar result is obtained elsewhere.[6,8] The differences, however, should be noticed: in the previous works ω_e^2 was determined by the total concentration of negatively charged particles, whereas in our case it is determined only by the electron concentration.

oscillations about a background of negative charges.* The frequencies given by Equation (11) will be larger than the corresponding ones of Eq. (13). This is due to the fact that Eq. (13) describes only the oscillations of positive ions, whereas Eq. (11) determines simultaneous oscillations of positive and negative ions in opposite phase. This last situation leads to an increase of the electric field as well as of the quasielastic force and frequency determined by the field. These statements can be verified by calculating the amplitude. The relation between the amplitudes of the oscillations of the negative and positive ion concentrations corresponding to the approximation of Eq. (10) can be written in the form

$$n_n^0 = [(k^2\Theta_p / M_p) + \omega_p^2 - \omega^2]\, n_p^0 / \omega_p^2. \qquad (14)$$

If the value of ω^2 from Eq. (13) is inserted into (14), the oscillation amplitude of the negative ion concentration vanishes; if, on the other hand, the value of ω^2 from Eq. (11) is inserted, the relation between the amplitudes becomes

$$n_n^0 = -\left(\frac{\omega_n^2}{\omega_p^2 - \omega_p^2} - 1\right) n_p^0. \qquad (15)$$

Analysis of this expression shows that the simultaneous positive and negative ion waves have opposite phases.

Equation (12), as opposed to the case already considered, describes in-phase ion oscillations. If ω^2 as given by Eq. (12) is inserted into (14), we are led to the following relation between the amplitudes:

$$n_n^0 = \left(1 + \frac{\omega_n^4}{\omega_p^2(\omega_p^2 - \omega_n^2)}\right) n_p^0, \qquad (16)$$

which shows immediately that the positive and negative ion wave have the same phase.

We should note also another property of the oscillations described by Eq. (12). As the charged-particle density increases, the oscillation frequency decreases monotonically and finally vanishes. The wavelength at this point takes on a certain limiting value

*In previously studied cases, an equation such as (13) described oscillations about the background of uniformly distributed electrons, whereas here the negative charge background contains both electrons and negative ions.

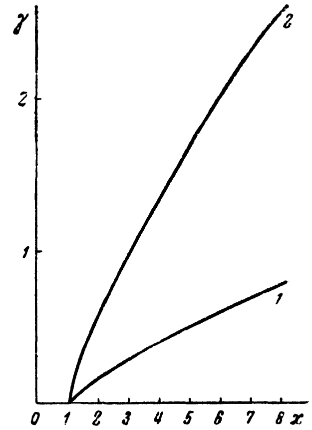

FIG. 2. The dependency of the frequency the wave vector as given by Equation (12) for the second branch of oscillations, in dimensionless units. The negative ion concentration parameter, N_n^0/N_p^0: 1 — 0,01, 2 — 0,1. For a concentration of 0.01, a temperature of 300° K and a positive ion density of 10^9 cm^{-3}, we have $k_D^2 = 4\pi e^2 N_n^0/\Theta_n = 10^2$ cm^{-2}, $\lambda_{\lim} \approx 1$ cm; for a concentration of 0.1, the other conditions remaining the same, $k_D \approx 30$ cm^{-1}, $\lambda_{\lim} \approx 0{,}1$ cm. Here $\gamma = \omega/\omega_n$, $\varkappa = k/k_D$ in units of $\sqrt{N_n^0/(N_p^0 - N_n^0)}$

$$\lambda_{\lim} = 2\pi\,[\Theta_n\,(\omega_p^2 - \omega_n^2) / M_n\omega_n^4]^{1/2}. \qquad (17)$$

Long wavelengths (for a given charged-particle density) correspond to negative frequencies which means that they cannot occur.* As the charged-particle density is increased, this limiting wavelength decreases so that the spectrum of possible wavelengths moves into the region of lower values.

So far, the oscillations and waves only of plasmas containing electrons and positive ions have been experimentally studied, and therefore it is impossible to compare the results obtained here with experiment.

In conclusion I consider it my duty to express my gratitude to Professor Ia. P. Terletskii and A. A. Zaitsev for discussions and valuable advice during the performance of this work, as well as to Professor G. V. Spivak and Professor N. A. Kaptsov for discussing the results of the work.

[1] R. Neubert, Phys. Z. **15**, 430 (1914).

*This is due to the fact that the pressure gradients which cause the propagation of the wave are compensated by the action of electric forces which arise because of the displacements.

[2] K. Darrow, *Electrical Phenomena in Gases*, DNTVU, 1937, p 303 (Russian Translation)

[3] R. Seelinger and K. Kruschke, Phys. Z. **34**, 883 (1933).

[4] B. Bohm et al., *The Characteristics of Electrical Discharges in Magnetic field*, 173 (1949).

[5] V. L. Granvskii, *Electric Discharge in Gases*, GITTL, 1952, p 311.

[6] M. V. Koniukov and Ia. P. Terletskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **27**, 542 (1954)

[7] J. J. Thomson and G. P. Thompson, *Conduction of Electricity Through Gases*, Cambridge **2**, 353 (1933).

[8] A. A. Vlasov, *Many-Particle Theory*, GTTI, Moscow, 1950, p 285.

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Contribution to the Theory of Collective Motion of Particles in Quantum Mechanical Systems

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A method for separating the collective motions in a system of interacting particles is presented. The connection between this method and Refs. 1-4 is established. An attempt is made to give a basis to the generalized heavy nucleus model.

1. MOST quantum mechanical systems that are dealt with in the various fields of physics consist of a large number of particles. The problems of finding the energy spectrum of such a system presents great mathematical difficulties. From among the quantum mechanical systems with large number of particles, we can separate out that class of systems in which collective motion, reminiscent of the motion of a continuous system, takes place. Among such systems are, for instance, heavy atomic nuclei, the electrons and ions of a metal, and others.

The problem of the approximate methods for describing such systems reduces to finding those terms in the Hamiltonian which correspond to collective motion. In the case of central forces, the problem of separating out the collective motions was solved by Zubarev[1], and for the case of Coulomb forces by Bohm and Pines[2] using a different method. An interesting method for describing collective motion, well set forth and of great generality, has been developed by Tomonaga[3,4]. The present article describes a method for separating out collective motions is systems, which is very simple in the sense that it does not require the complex apparatus of second quantization and unitary transformations. At the same time this method is quite general.

2. At the basis of the method lies the transformation of the Hamiltonian with the aid of auxiliary variables introduced into the wave function of the system, as has been described by Zubarev[1].

We consider the wave function $\Psi(r_1 \ldots r_N; t)$ describing a system whose Hamiltonian is

$$H = \sum_j \mathbf{p}_j^2/2M + {}^1/_2 \sum_{ij} G(\mathbf{r}_i, \mathbf{r}_j), \tag{1}$$

and introduce the auxiliary ("superfluous") variable functions $\varphi_j(r_j)$ (where $j=1, 2, \ldots, N$), which for the time being are arbitrary, and instead of Ψ we shall consider the new wave function (a functional of the φ_j)

$$\Phi(r_1 \ldots r_N;\ \varphi_1(r_1) \ldots \varphi_N(r_N); t). \tag{2}$$

since Φ depends on r_j explicitly as well as through the φ_j, the operator $\mathbf{p}_j$ must be replaced by

$$-i\hbar\,[\nabla_j + (\nabla_j\varphi_j)\partial/\partial\varphi_j]. \tag{3}$$

As will be shown below, part of the potential energy can also be described in terms of the $\varphi_j(r_j)$.

Let us now make some comments about the $\varphi_j(r_j)$. We assume that $\varphi_j(r_j)$ is the wave function of the stationary state of the j-th particle in the zeroth