# Elastic Scattering of 360 Mev Positive $\pi$-Mesons by Protons

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Photographic plates were used to study the angular distribution of 360 ±10 Mev $\pi^+$ mesons elastically scattered by protons. The differential cross section derived from 218 scattering events has the form (1) for *SP* analysis and the form (2) for *SPD* analysis. The phase shifts which correspond to these distributions are (*A*) and (*B*).

EXPERIMENTS involving a measurement of differential cross sections for the elastic scattering of $\pi$ mesons by protons make it possible to determine which states take part in the scattering. It is also possible to calculate the phase shifts which correspond to those states. A phase analysis of the data for the scattering of $\pi^+$mesons was carried out recently for energies up to 310 Mev[1,2].

In the present work the angular distribution of $\pi^+$mesons with energies of 360 ±10 Mev was measured using photographic plates. The mesons were scattered by hydrogen nuclei contained in the photographic emulsions. The experiments were carried out on the synchrocyclotron of the Joint Institute for Nuclear Research.

Electron-sensitive photographic plates with emulsions 400 $\mu$ thick were exposed in a beam of $\pi^+$ mesons emerging from a magnetic meson spectrometer[3]. $\pi$ mesons were produced in a carbon target 5 cm thick by a beam of 657 Mev protons brought out from the synchrocyclotron chamber. Mesons emitted from the target at an angle of 8.5° were deflected by the magnetic field of the spectrometer after preliminary collimation. The narrow converging beam of measons then passed through a 4 meter collimator at whose exit the photographic plates were placed. A copper shield 3 cm thick was used to protect the plates from protons. The energy of the mesons passing through this filter was measured to be 360 ±10 Mev. The intensity of the meson beam was about 4 mesons/cm$^2$ sec. The search for events was carried out by examination of photographic plates in sections by using immersion lenses, with about half the area examined at a magnification 630 times and the remainder at a magnification of 450 times. A significant fraction of the area of photographic plates was subjected to a careful second examination by various observers. This second examination disclosed a total of 8 previously-overlooked scattering events, evidencing a very high degree of efficiency in the examination of photographic plates.

The criteria used in the identification of the elastic scattering events were the angular correlation between the scattered mesons and the proton recoil, and co-planarity. Those events in which the deviation from co-planarity and the calculated angular correlation exceeded 1° were discarded.

A total of 218 scattering events were found in the angular interval of 10° to 170° in the center of mass coordinate system. The summation interval was 20°. A distribution of the number of scattering events with the angular intervals is given in the table.

| Angular Intervals in center of mass system, in degrees | Corresponding angular intervals in the laboratory system of coordinates | Number of events in given interval | $\frac{d\sigma}{d\Omega}\cdot 10^{27}$ cm$^2$/sterad |
|---|---|---|---|
| 10—30 | 7—21 | 42 | 11,2±1,7 |
| 30—50 | 21—36 | 64 | 9,2±1,2 |
| 50—70 | 36—52 | 49 | 5,2±0,7 |
| 70—90 | 52—70 | 24 | 2,2±0,5 |
| 90—110 | 70—89 | 10 | 0,9±0,3 |
| 110—130 | 89—111 | 10 | 1,1±0,3 |
| 130—150 | 111—138 | 13 | 1,9±0,5 |
| 150—170 | 138—166 | 6 | 1,6±0,6 |
| | Total | 218 | |

The angular distribution was normalized to satisfy the following equality

$$\sum_i \frac{d\sigma}{d\Omega}(\vartheta_i)\,\Delta\Omega_i = \sigma_t.$$

The total cross section of the elastic scattering $\sigma_t$

was chosen to be equal to $43.4 \times 10^{-27}$ cm$^{2}$ [4].

The experimental data is shown in Figure 1. Substitution of the experimental results into the series of Legendre polynomials

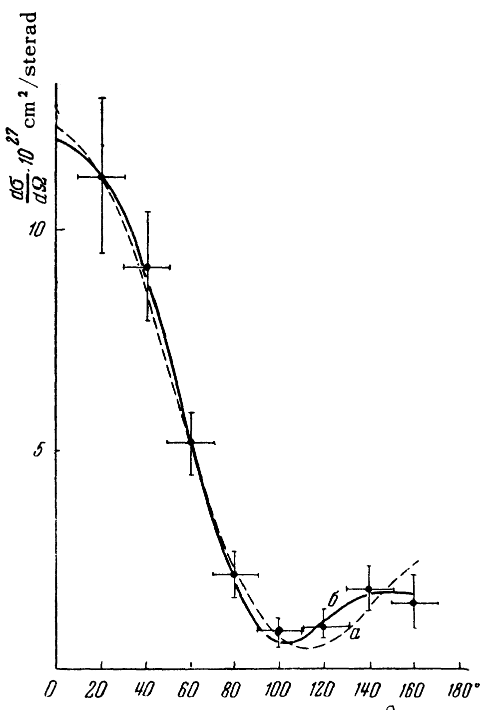


FIG. 1. Differential cross section for elastic scattering of mesons. Curves $a$ and $b$ are calculated from Eq. (1) and Eq. (2) respectively.

$$\frac{d\sigma}{d\Omega} = \sum_n A_n P_n(\cos\vartheta),$$

where $A_n$ is computed from the equation

$$A_n = \frac{2n+1}{2}\sum_i \frac{d\sigma}{d\Omega}(\vartheta_i)\, P_n(\cos\vartheta_i)\,\Delta\cos\vartheta_i$$

leads to the following expression for the differential cross section:

$$d\sigma/d\Omega = [3{,}43P_0 + 4{,}63P_1 + 4{,}20P_2 + 0{,}57P_3 - 0{,}81P_4]\cdot 10^{-27}\,\text{cm}^2/\text{sterad}$$

By writing out his expression it is possible to express the differential cross section in a power series in $\cos\vartheta$. This power series has the form:

$$d\sigma/d\Omega = [(1{,}33 \pm 0{,}20) + (4{,}63 \pm 0{,}62)\cos\vartheta + (6{,}30 \pm 1{,}42)\cos^2\vartheta]\cdot 10^{-27}\ \text{cm}^2/\text{sterad} \quad (1)$$

if only the first three terms are retained, and

$$d\sigma/d\Omega = [(1{,}02 \pm 0{,}16) + (3{,}78 \pm 0{,}48)\cos\vartheta + (9{,}35 \pm 1{,}31)\cos^2\vartheta + (1{,}42 \pm 0{,}86)\cos^3\vartheta - (3{,}56 \pm 1{,}64)\cos^4\vartheta]\cdot 10^{-27}\ \text{cm}^2/\text{sterad} \quad (2)$$

if the terms up to $n=4$ are included (see Fig. 1).

Owing to the low statistical accuracy of the obtained results it is difficult at first sight to find preference for one or the other expression for the differential cross section, even though it is evident from the figure that in the region of large angles curve $b$ fits the experimental results better than curve a.

Scattering of $\pi$ mesons by protons in $S$, $P$ and $D$ states with isotopic spin $T = 3/2$ is known to be characterized by five phase shifts $\alpha_3$, $\alpha_{31}$, $\alpha_{33}$, $\delta_{33}$, and $\delta_{35}$, corresponding to the states $S$, $P_{1/2}$, $P_{3/2}$, $D_{3/2}$ and $D_{5/2}$ respectively. Ashkin and Vosko[5] have shown that the mathematical formulation of the problem of finding four solutions for the phase of $\pi^+$-p scattering by $SP$ analysis. However, many physical considerations and the requirement of a continuous and smooth variation of the phase shifts with energies make it possible to select a solution in which $\alpha_{33}$ has a resonance character[6], i. e., passes through 90°. Taking this circumstance into account we have sought in our phase analysis of the experimental data only that solution corresponding to a resonance character of the phase $\alpha_{33}$.

The optimum phase shifts can be determined in a general case only with electronic computers or special mechanical phase analyzers. However, under many simplifying conditions the phase analysis can be carried out in a sufficiently simple way. This happens for example, when a differential scattering cross section can be expressed in the form

$$d\sigma/d\Omega = a_0 + a_1\cos\vartheta + a_2\cos^2\vartheta.$$

If it is assumed that only the $S$ and $P$ states participate in the scattering, the phase shifts corresponding to these states can be determined in a given case graphically[5]. The phase shifts calculated graphically using Eq. 1 are:

$$\alpha_{33} = 146°;\quad \alpha_{31} = -14°;\quad \alpha_3 = -31°. \quad (A)$$

Phase analysis which takes into account contributions from $S$, $P$ and $D$ states ($SPD$ analysis) was carried out with a mechanical phase analyzer[7].

The following phase shifts were obtained

$$\alpha_{33} = 143°;\ \alpha_{31} = -5°;\ \alpha_3 = -14°; \qquad \text{(B)}$$
$$\delta_{33} = 10°;\ \delta_{35} = -13°.$$

The differential cross sections calculated using the above set of phases are shown in Figure 2,

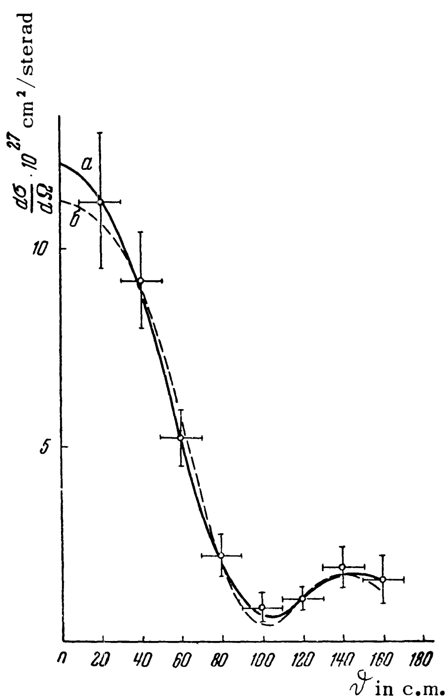


FIG. 2. Differential cross section for elastic scattering of mesons. Curves *a* and *b* are computed from phase sets *B* and *C*.

curve *a*. From the quoted values of the phase shifts it is evident that the magnitudes of the phase $\alpha_{33}$ are practically equal both for *SP* and for *SPD* analyses. The dependence of this phase on the meson momentum

$$(\eta^3/\omega)\operatorname{ctg}\alpha_{33} = 9{,}3\,(1 - \omega/2{,}15),$$

as proposed by Chew and Low[8], does not hold at meson energies of 360 Mev ($\eta$ is the momentum of the meson in the center of mass system in $\mu c$ units; $\omega$ is the total energy in the center of mass system after subtracting the proton rest energy, in $\mu c^2$ units). A deviation from this dependence is already noticeable for meson energies above 240 Mev[2]; for 360 Mev mesons this deviation is approximately 20°. The values of the phase $\alpha_{33}$ given above agree well with the expression

$$\eta^3\operatorname{ctg}\alpha_{33} = 4{,}3 + 0{,}6\,\eta^2 - 0{,}8\,\eta^4,$$

proposed by Mukhin et al[2].

The magnitude of the phase $\alpha_3$ depends strongly on whether the *D* state is taken into account in the phase analysis. *SPD* analysis yields almost half the value of $|\alpha_3|$ obtained by *SP* analysis. This observation evidently holds for a broad range of meson energies[2]. The quantity $\alpha_3 = -14°$ obtained in the analysis which included the *D* state agrees well with the linear dependence of $\alpha_3$ on the meson momentum in the center of mass system: $\alpha_3 = -0.11\mu^9$.

Mukhin et al[2] proposed the following dependence of the phases $\delta_{33}$ and $\delta_{35}$ on the momentum:

$$\delta_{33} \approx -\delta_{35} = 0{,}20°\,\eta^5.$$

Taking these expressions into consideration and assuming that $\alpha_3$ depends on the momentum linearly, $\alpha_3 = -6.3°\eta$, it is possible to calculate the phase shifts $\alpha_{33}$ and $\alpha_{31}$ from the condition that the total scattering cross section determined from the phase shifts must agree with the experimental value:

$$\sigma_t = 4\pi\lambda^2[\sin^2\alpha_3 + \sin^2\alpha_{31}$$
$$+ 2(\sin^2\alpha_{33} + \sin^2\delta_{33}) + 3\sin^2\delta_{35}].$$

The phase shifts calculated in this manner are:

$$\alpha_{33} = 148°;\ \alpha_{31} = -4°;$$
$$\alpha_3 = -15°;\ \delta_{33} = -\delta_{35} = 15°. \qquad \text{(C)}$$

The differential scattering cross sections corresponding to this set of phases is shown in Figure 2 (curve *b*). It is evident from the figure that curve *b* fits the experimental data as well as curve *a*. It must be noted, taking into account the statistical accuracy of the results, that both sets of phases are almost identical.

It is interesting to consider the results from the point in view of an application of the causality condition to the meson-nucleon scattering[10]. The energy dependence of the real part of the forward scattering amplitude was previously obtained[11, 12] from the data on the total meson-nucleon scattering cross section. It is possible to compare the magnitude of the real part of the forward scattering cross section obtained in this manner with the amplitude obtained by means of the above sets of phases (*A*, *B*, *C*). If we denote by $D^+$ the real part of the forward scattering amplitude in the laboratory system of coordinates and by $\lambda$ and $\lambda_{cm}$ the meson wavelengths in the laboratory and the center of mass systems respectively, the relationship between the real part of the forward scattering amplitude and its value in terms of the phase shifts assumes the form:

$$2\lambda\lambda_{cms}^{-2} D_{+} = \sin 2\alpha_3 + \sin 2\alpha_{31} + 2(\sin 2\alpha_{33} + \sin 2\delta_{33}) + 3\sin 2\delta_{35} + \ldots$$

Substituting into this expression the magnitudes of the phase shifts from sets $A$, $B$, $C$, we obtain $-3.20$, $-3.22$, and $-2.96$ for the right hand side and $-3.16$ for the left hand side. This result points out the good agreement between the real part of the forward scattering amplitude calculated from the causality condition and the values calculated using phase shifts obtained from angular distribution. A similar deduction was made in a study of the scattering of mesons in a broad energy range up to 310 Mev. The results obtained in the present work give grounds for assuming that this deduction can be extended to meson energies up to 360 Mev.

It is necessary to note in conclusion that the experimental data presented still do not afford a sufficiently strong proof that it is necessary to include $D$ states in a phase analysis of the results. It is even less possible to make any conclusions concerning the dependence of the phase shifts $\delta_{33}$ and $\delta_{35}$ on the meson momentum. However, if it is assumed that the inclusion of the $D$ states is justified, it is possible to deduce conclusively from the results of the $SPD$ analysis for 307 and 360 Mev mesons that the phase shifts for the $D_{3/2}$ and $D_{5/2}$ states are approximately equal, opposite in sign, and have a tendency to increase with the meson energy.

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