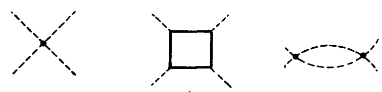
most general case, when λ_0 and g_0^2 are of the same order of magnitude. The solution obtained in this case is also valid when one of these quantities is small with respect to the other ($g_0^2 \ll \lambda_0$ or $\lambda_0 \ll g_0^2$), since one may then simply neglect the small quantity with respect to the large one. We shall limit ourselves to the asymptotic solutions in the zero approximation, i.e., we shall consider all quantities as functions of only $g_0^2(\xi - L)$ and $\lambda_0(\xi - L)$ or, what amounts to the same thing, $g_0^2 \times (\xi - L)$ and g_0^2/λ_0. Perturbation theory calculations indicate that α, β, and d do not contain any terms in $\lambda_0(\xi - L)$, and we can therefore apply the results of Ref. 2 which give F_1, F_2, F_3 and the asymptotic forms of α, β and d, and the effective charge $g^2(\xi)$. Perturbation theory yields for $\lambda_0 \mathcal{P}$:

$$\lambda_0 \mathcal{P} = \lambda_0 - (g_0^4/\pi^2)(\xi - L) + {}^{11}/_2 \lambda_0^2 (\xi - L).$$



a b c

The three terms of this formula correspond to the diagrams *a, b, c*. In order to obtain the function F_4 it is necessary to compute $\mathcal{P}'/\mathcal{P}|_{\xi \to L}$ in the last formula and in the resulting expression substitute g_0^2 for g^2 and λ_0 for λ; this yields

$$F_4 = 11\lambda/2 - g^2/\pi^2\lambda. \qquad (4')$$

Applying Eq. (4′) and Eqs. (3′) and (4) of Ref. 2 to Eq. (5) one finds

$$\frac{\lambda'}{\lambda} = \frac{11}{2}\lambda - \frac{g_0^4}{\pi^2 \lambda Q^2} + \frac{2g_0^2}{\pi Q}, \qquad Q = 1 + \frac{5g_0^2}{4\pi}(L - \xi). \qquad (6)$$

Carrying out the substitution

$$\lambda(\xi) = (g_0^2/4\pi)\, \mathcal{P}(x)\, d^2(x), \quad x = Q^{3/5}, \qquad (7)$$

we obtain the equation for the meson-meson scattering amplitude P.

$$dP/dx = {}^{16}/_3 - {}^{11}/_6 (P/x)^2$$

with the boundary condition $P(1) = 4\pi\lambda_0 g_0^2$. The solution of Eq. (7) is

$$P = \frac{16}{11} x \frac{B - x^{-19/3}}{B + (8/11)\, x^{-19/3}}; \qquad (8)$$

$$B = \left(1 + \frac{1}{2}\frac{4\pi\lambda_0}{g_0^2}\right) \Big/ \left(1 - \frac{11}{16}\frac{4\pi\lambda_0}{g_0^2}\right),$$

which coincides with the results obtained in Ref. 3 for $\lambda_0 = 0$ ($B = 1$).

The present investigation may be used to compute asymptotic solutions to any order in g^2, λ from perturbation theory results to the same order.

In conclusion, the author would like to thank I. Ia. Pomeranchuk and V. B. Berestetskii.

*The symmetric theory is considered here.

**The expressions for the logarithmic derivatives of renormalized quantities are obtained from Eq. (1).

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Excitation of Nuclear Vibrational Levels by Scattering of Fast Neutrons

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IT is shown in Ref. 1 that there exists a large group of even-even nuclei in the interval $36 \leq N \leq 88$, whose first two excited levels apparently have vibrational character; this group property may be explained by the theory of A. Bohr[2] for the case of weak or intermediate coupling between the nuclear surface and the motion of individual nucleons. We shall consider here the excitation of the first vibrational level as it occurs during scattering of fast neutrons by a black nucleus and we shall limit ourselves to the zero coupling case.

The nuclear surface is characterized by the equation

$$r(\vartheta, \varphi) = R\left[1 + \sum_{\mu} \alpha_\mu Y_{2\mu}(\vartheta, \varphi)\right], \qquad (1)$$

the coordinates A_μ are correlated with creation and annihilation operators β^*_μ and β_μ: $\alpha_\mu = \sqrt{\hbar/2B\omega}$ $\times[\beta_\mu + (-1)^\mu \beta^*_{-\mu}]$. The wave functions for the ground and first excited states are given by χ^0_{00} (no phonons) and $\chi^1_{2\mu}$ (one phonon with angular momentum 2 and z-component μ.) The matrix elements for the operators β_μ and β^*_μ are

$$(2\mu \mid \beta^*_{\mu'} \mid 00) = \delta_{\mu\mu'}, \qquad (2\mu \mid \beta_{\mu'} \mid 00) = 0. \qquad (2)$$

The energy of the surface vibration is given by the expression $\epsilon_n = \hbar\omega(5/2 + n)$, where n is the number of phonons. The energy of the first excited level, $\epsilon_1 = \hbar\omega$, is limited to the interval $0.3 \leq \epsilon_1 \leq 2.0$ mev for the nuclei in question.

In the ground state, the nucleus performs zero order vibration of frequency ω. For sufficiently fast neutrons, the problem may be solved in the adiabatic approximation by computing the scattering of neutrons by a fixed surface nucleus. In this case, it is necessary that the period of zero order vibration, $T \approx 2\pi/\omega$, be much longer than the collision time, $\tau \approx R/v$, where R is the nuclear radius and v the neutron velocity. This leads to the condition $(\epsilon_1/4\pi E)kR \ll 1$, which is quite well satisfied for $\epsilon_1 \approx 1$ mev and for a neutron energy of several tens mev.

The amplitude for scattering by a black nucleus is obtained from the well-known formula

$$f(\theta, \alpha_\mu) = (ik/2\pi)\int e^{-i\mathbf{k}'\rho} dS, \qquad (3)$$

where $\mathbf{k}$ and $\mathbf{k}'$ are the wave vectors of the incident and scattered neutrons, and the integration is to be carried out over a projected area of the nucleus whose surface is defined by the parameters α_μ, the projection being taken on a plane perpendicular to the incident vector $\mathbf{k}$. The scattering cross section in the θ direction for an excited state where there is one phonon with angular momentum 2 and component μ, is computed from the formula (see Ref. 3)

$$\sigma_\mu(\theta) = |(2\mu \mid f(\theta, \alpha_\mu) \mid 00)|^2. \qquad (4)$$

Integral (3) may be calculated after carrying out a transformation of the variables which changes the surface of an ellipsoid into the surface of a sphere; the region of integration becomes a circle instead of an ellipse, the projection of an ellipsoid. Carrying out the computation, one obtains the following expression for the scattering amplitude (accurate to within terms linear in α_μ):

$$f(\theta, \alpha_\mu) = ikR^2 \left[\left(1 - \sqrt{\frac{5}{4\pi}}\alpha_0\right)\frac{J_1(kR\theta)}{kR\theta} - \frac{3}{4}\sqrt{\frac{5}{6\pi}}\left(\alpha_2 + \alpha_{-2} - \sqrt{\frac{2}{3}}\alpha_0\right) J_2(kR\theta)\right]. \qquad (5)$$

The α_μ's are now replaced by the operators β_μ and β^*_μ, and the application of the expression for the matrix elements of these operators leads to the required cross section

$$\sigma_0(\theta) = pR^2[(J_1(kR\theta)/\theta) - {}^1/_2\, kRJ_2(kR\theta)]^2, \quad \sigma_{\pm 1}(\theta) = 0, \qquad (6)$$
$$\sigma_{\pm 2}(\theta) = {}^3/_8\; p(kR)^2R^2J_2^2(kR\theta),$$

where $p = (5/8\pi)\hbar/B\omega$. The ratio of the inelastic to the elastic scattering cross section depends mainly on the quantity p. Hydrodynamical calculations give $B = (3/8\pi)MR^2$, where M is the mass of the nucleus; it is shown in Ref. 4, however, that in order to bring about agreement with experiment, this value must be multiplied by a factor of approximately five. Choosing such a value of B, the quantity p is of the order of 10^{-2}, i.e., the cross section for the process under consideration is two orders of magnitude lower than the cross section for elastic scattering.

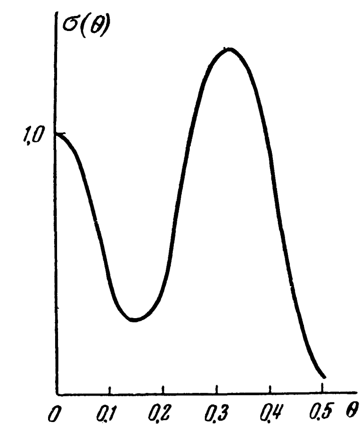

FIG. 1.

Comparison with experiment can be carried out first of all by measuring the angular distribution of inelastically scattered neutrons and comparing it with the angular distribution predicted by Eq. (6) summed over μ, i.e., with $\sigma(\theta) = \sigma_0(\theta) + 2\sigma_2(\theta)$. Fig. 1 shows $\sigma(\theta)$(in units of $pk^2R^4/4$) when $kR = 10$. Comparison with experiment can also be achieved by measuring the angular correlation between inelastically scattered neutrons and γ-rays produced during nuclear transitions to the ground state. The photons here carry two units of angular momentum; applying the well-known formula for the angular distribution of photons, one obtains the following exrpession for the transition cross section, wherein the neutrons are inelastically scattered at an angle θ and the photon makes an angle ϑ with the direction k

$$\sigma(\theta, \vartheta)\, d\Omega_n d\Omega_\gamma \quad (7)$$

$$= \frac{5}{8\pi}\left[3(x^2 - x^4)\,\sigma_0(\theta) + (1 - x^4)\,\sigma_2(\theta)\right] d\Omega_n d\Omega_\gamma,$$

where $x = \cos\vartheta$.

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Density of the Normal Component for Solutions of the Isotopes of Helium

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IN accordance with current ideas[1] there are to be associated with atoms of He^3 in solution He II elementary excitations characterized by the following dependence of energy upon momentum:

$$E = E_0 + (p - p_0)^2 / 2\mu, \quad (1)$$

where μ is the effective mass of the impurity atom. The minimum of the energy corresponds either to $p_0 = 0$ or to $p_0 \neq 0$. The contribution of the impurity to the density of the normal component of the He II depends critically upon the form of the energy spectrum, and can be written in the following manner[2]:

$$\rho_n = \rho_{n0} + (\rho/m)\,\mu x \quad \text{for} \quad \rho_0 = 0, \quad (2)$$

$$\rho_n = \rho_{n0} + (\rho/m)\,(p_0^2 / 3kT)\,x \quad \text{for} \quad p_0 \neq 0, \quad (3)$$

where x is the molar concentration of the He^3 in the solution. From this it is immediately evident that an experimental determination of the temperature dependence of the normal component density for solutions of He^3 and He^4 will make it possible to establish the form of the energy spectrum.

For this purpose an experiment was performed to determine ρ_n for solutions of the helium isotopes by means of the customary method, i.e., the measurement of the period of oscillation Θ of a stack of discs immersed in the solution[3,4]. From this the ratio of ρ_n to the density ρ_λ of the solution for the corresponding λ-point was computed. The results thus obtained* are presented in Fig. 1.

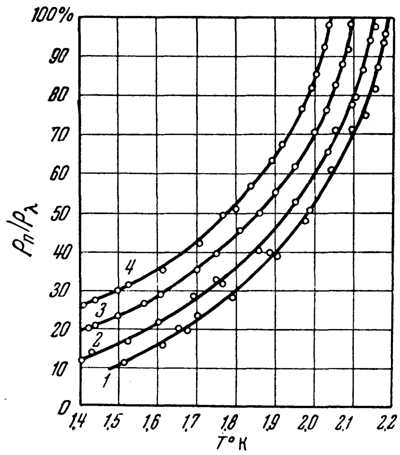

FIG. 1. Dependence of ρ_n / ρ_λ upon temperature: *1*–pure He^4 ; *2*–3.0% He^3 ; *3*–5.6% He^3 ; *4*–8.2% He^3

Using the indicated values of ρ_n / ρ_λ, and determining ρ_λ from the known densities of He^4 and He^3 [6-7], assuming them to be additive, it is possible to derive the dependence of ρ_n upon x for various temperatures, as shown in Fig. 2.

It is clearly evident that this dependence is linear down to a temperature of 1.8° K, the isotherms thus obtained being parallel to one another. A relation of this sort among ρ_n, T and x, in accordance with Eq. (2), testifies to the fact that the