points $\mathbf{r}_1$ (t) and $\mathbf{r}_2$ (t) then T, the energy loss of the pair in a unit of time, is

$$T = ec\{\mathbf{v}_1 \mathbf{E}(\mathbf{r}_1, t) - \mathbf{v}_2 \mathbf{E}(\mathbf{r}_2, t)\}, \quad (1)$$

where $c\mathbf{v}_1$ and $c\mathbf{v}_2$ are the speeds of the positron and electron, and E is the electric field of the pair. The last can be considered as a field in the macroscopic realm caused by charge densities ρ and currents j,

$$\rho = e\delta(\mathbf{r} - \mathbf{r}_1(t)) - e\delta(\mathbf{r} - \mathbf{r}_2(t));$$
$$\mathbf{j} = ec\{\mathbf{v}_1 \delta(\mathbf{r} - \mathbf{r}_1(t)) - \mathbf{v}_2 \delta(\mathbf{r} - \mathbf{r}_2(t))\}$$

and can be presented in the form of the following Fourier integral

$$\mathbf{E}(\mathbf{r}, t) = (ie/2\pi^2)\int d\mathbf{k}\,\{e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}_1)}[\mathbf{v}_1(\mathbf{k}\mathbf{v}_1) - \mathbf{k}/\varepsilon(\mathbf{k}\mathbf{v}_1)]/[k^2 - (\mathbf{k}\mathbf{v}_1)^2\varepsilon(\mathbf{k}\mathbf{v}_1)] - e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}_2)}[\mathbf{v}_1(\mathbf{k}\mathbf{v}_2) - \mathbf{k}/\varepsilon(\mathbf{k}\mathbf{v}_2)]/[k^2 - (\mathbf{k}\mathbf{v}_2)^2\varepsilon(\mathbf{k}\mathbf{v}_2)]\}, \quad (2)$$

where $\epsilon(\omega)$ is the dielectric constant of the medium corresponding to a frequency ω. In the derivation of the (2) the trajectories of the particle being scattered in the medium can be considered as straight line segments for regions in which slowing down is still important.

Substituting Eq. (2) into Eq. (1) we obtain $T = 2T_0 - T_1$, where T_0 is the ionizational slowing down of a single electron, and T_1 is the interference term

$$T_1 = \frac{ice^2}{2\pi^2}\int d\mathbf{k}\left\{\frac{(\mathbf{v}_1\mathbf{v}_2)(\mathbf{v}_1\mathbf{k}) - (\mathbf{k}\mathbf{v}_2)/\varepsilon(\mathbf{k}\mathbf{v}_1)}{k^2 - (\mathbf{k}\mathbf{v}_1)^2\varepsilon(\mathbf{k}\mathbf{v}_1)} e^{i\mathbf{k}(\mathbf{r}_2-\mathbf{r}_1)} + \frac{(\mathbf{v}_1\mathbf{v}_2)(\mathbf{v}_2\mathbf{k}) - (\mathbf{k}\mathbf{v}_1)/\varepsilon(\mathbf{k}\mathbf{v}_2)}{k^2 - (\mathbf{k}\mathbf{v}_2)^2\varepsilon(\mathbf{k}\mathbf{v}_2)} e^{-i\mathbf{k}(\mathbf{r}_2-\mathbf{r}_1)}\right\}.$$

In the calculation of T_1 it is important that the transverse component of the pair separation be larger than the parallel component. Indeed the last is proportional to $v_1 - v_2 \sim (mc^2/E)^2$ while the first is determined by the angle of separation of the pair $\theta \sim mc^2/E$ and by the multiple scattering angle. Therefore, having selected the Z axis along the direction of $\mathbf{v}_1$ or $\mathbf{v}_2$, we can substitute $k_x s$ for $\mathbf{k}(\mathbf{r}_2 - \mathbf{r}_1)$ in the exponent, where $s = (x_2 - x_1)$. After this the k_z integration can be carried out in the same way as done by Landau in the calculation of T_0. It turns out that the limiting expression for ϵ for high frequencies is of importance in the integral, $\epsilon = 1 - \lambda^2 c^2/\omega^2$ where $\lambda^2 = 4\pi nc^2/mc^2$, and n is the number of electrons in the unit volume. We obtain

$$T_1 = \frac{ce^2\lambda^2}{\pi}\int \frac{\cos k_x s}{k_x^2 + k_y^2 + \lambda^2}\,dk_x\,dk_y = 2e^2c\lambda^2 K_0(s\lambda), \quad (3)$$

where k_0 is the corresponding cylindrical function.

The convergence of this integral is shown by the fact that the interference effects depend on large distances for which the macroscopic viewpoint is valid. The analogous integral for T_0, as is known, diverges and must be limited by some maximum value of the transverse wave vector K_n which is related to the energy E_n transmitted to the atomic electron.

For large s $(s\lambda >> 1)$ the interference term disappears, as can be seen from Eq. (3). For small s $(s\lambda << 1)$, one can use the relationship $k_0(z) = \ln(2/\gamma z)$, where $\gamma = e^C = 1.781$. Then

$$T_1 = 2e^2c\lambda^2 \ln(r_{max}/s),$$

where $r_{max} = 2/\gamma\lambda$.

If T_0 is written in analogous form (see, for example, Ref. 2)

$$T_0 = ce^2\lambda^2 \ln(r_{max}/r_{min}),\quad r_{min} = a(\hbar/mc)\sqrt{mc^2/E_m}$$

$(a = 1.85)$, then T can be written in the following form obtained in Ref. 1.

$$T = 2T_0 \ln(s/r_{min}) / \ln(r_{max}/r_{min}). \quad (4)$$

Entering into r_{min} is the quantity E_m which is the maximum energy transmitted to an atomic electron as determined from experimental data. We would like to express our thanks to L. D. Landau for discussion of the results.

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Diffraction Scattering of High-Energy Photons by Nuclei

V. B. BERESTETSKII AND E. V. KUZNETSOV

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THE properties of the nucleus with a respect to photons of high energy (for $kR << 1$, where k is the wave number of the photon, R is the atomic radius) can be characterized by a complex index of refraction $n + i\varkappa/k$, where n is approximately 1 and $\varkappa R << 1$. The magnitude of the absorption coefficient $\varkappa$ can be expressed in terms of general formulas in terms of the experimentally determined cross section for photo-meson production on nuclei:

$$\varkappa R = 3\sigma_c^2 / 4\pi R^2. \qquad (1)$$

The existence of absorption must lead to strong scattering of photons. Using general diffraction relations for polarized nuclei[1] it is easy to show that the cross section for scattering σ_s is

$$\sigma_s = 9\sigma_c^2 / (32\pi R^2). \qquad (2)$$

The scattering amplitude at small angles θ is

$$f(\theta) = ik\varkappa \int_0^R J_0 (k\theta \sqrt{R^2 - s^2})\, s^2\, ds,$$

from which we find for the differential cross section

$$d\sigma_s / do = {}^1/_2\, \sigma_s (kR)^2\, \Phi^2 (kR\theta), \qquad \Phi(x) = x^{-2} (x^{-1} \sin x - \cos x). \qquad (3)$$

In agreement with the experimental data[2] at photon energies of the order of 300 mev, σ_c is approximately $10^{-28} A$ cm^2. In this case the scattering cross section must be

$$\sigma_s = 10^{-30}\ \text{см}^2 \text{ for Be}, \quad \sigma_s = 0{,}9\cdot 10^{-28}\ \text{см}^2 \text{ for U}.$$

Let us compare the diffraction scattering with scattering of photons by a Coulomb field. The cross section of the last σ_γ for $E >> mc^2$ is equal[3] to

$$\sigma_\gamma = 8{,}5\cdot 10^{-35} Z^4\ \text{cm.}^2$$

Thus the ratio σ_s / σ_γ changes from 50 for Be to 10^{-2} for U, that is for heavy nuclei the diffraction scattering is considerably smaller than the coherent scattering by the charge. Nevertheless, it must appear as a consequence of a different angular distribution. In agreement with Eq. (3), diffraction scattering is effective at an angle $\theta_s \sim 1/kR$ while scattering by the Coulomb field is concentrated in the region $\theta_\gamma \sim mc^2/E$. Therefore, for $E = 300$ mev, the differential cross sections for U are comparable for $\theta = 0.015$, after which $d\sigma_\gamma / d\theta$ rapidly decreases, while $d\sigma_s / d\theta$ remains in this region at a constant value which is equal to 0.8 mb ($\theta_s = 0.09$).

We would like to express appreciation to K. A. Ter-Martirosian for discussing this problem.

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Concerning the Impulse Approximation

I. M. SHMUSHKEVICH
Leningrad Physico-Technical Institute

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BRUECKNER[1] has examined the problem of the scattering of a particle by a system of two scatterers with zero-range forces [the scattering from each of these centers is spherically symmetric and is characterized by the amplitude $\eta = (1/k) \times \sin\delta\, e^{i\delta}$, where δ is the phase of the S-wave at infinity]. For the scattered amplitude in this problem, we obtain the following expression:

$$f(\vartheta) = \left(1 - \eta^2 \frac{e^{2ikR}}{R^2}\right)^{-1} \left[\eta \left(e^{i(\mathbf{k}_0 - \mathbf{k})\mathbf{r}_A} + e^{i(\mathbf{k}_0 - \mathbf{k})\mathbf{r}_B}\right) + \eta^2 \frac{e^{ikR}}{R^2} \left(e^{i(\mathbf{k}_0\mathbf{r}_A - \mathbf{k}\mathbf{r}_B)} + e^{i(\mathbf{k}_0\mathbf{r}_B - \mathbf{k}\mathbf{r}_A)}\right)\right], \qquad (1)$$

where $\mathbf{k}_0$ and $\mathbf{k}$ are the wave vectors before and after scattering, $\mathbf{r}_A$ and $\mathbf{r}_B$ are the radius vectors of the scattering centers, and $R = |\mathbf{r}_A - \mathbf{r}_B|$.

From this expression, Brueckner, using a well-known theorem relating the imaginary part of the scattering amplitude in the forward direction with the total cross section, obtains the latter. Comparing this expression for the total cross section with the corresponding one obtained with the aid of the impulse approximation, the author shows that the difference between these two expressions becomes insignificant not for $R \sim \infty$, but for $\delta \to 0$ (for simplicity, it is assumed that the amplitude η is the same for both scatterers). From this the conclusion is reached that the use of the impulse approximation without taking account of multiple scattering is valid only when the Born approximation is applicable.