

TABLE

in vapor	$T^{\circ}\text{K}$	$P$ , mm Hg	in liquid	$T^{\circ}\text{K}$	$P$ , mm Hg
$X_v = 97\% \text{ He}^3$	1.497	46	$X_l = 93.6\% \text{ He}^3$	1.464	42.4
	1.652	66.7		1.483	45.7
	1.953	127		1.571	57.2
	2.048	151.5		1.657	70
$X_v = 94\% \text{ He}^3$	1.509	42		1.794	94.3
	1.648	61		1.861	108.2
	1.963	115		1.915	119.7
	2.048	140		1.975	135.2

the pressure on the basis of the results of the first set of experiments. From the relatively weak dependence of the vapor pressure on the concentration in the liquid in the concentration region studied (1 mm at 0.5%  $\text{He}^3$  for  $2^{\circ}\text{K}$ ), the values of the pressure, obtained from experiments with several different amounts of liquid and, consequently, different concentrations (from 93.8% to 93.4%  $\text{He}^3$ ), lay at the limits of accuracy of the pressure measurement on one curve. For this same reason it was possible to neglect the change in the vapor concentration by diffusion (during the course of the experiment) in the conducting tubes.

The temperature of the apparatus was determined from the vapor pressure of  $\text{He}^4$  in the tank (taking into account the corrections mentioned above for the first set of experiments) to within  $\pm 0.001^{\circ}\text{K}$  (the 1949 tables were used).

The results that we obtained are shown in the Table. Phase diagrams were constructed from these for the mixtures of  $\text{He}^3$ - $\text{He}^4$  for the temperature and concentration region under investigation (Fig. 2). Plotted in the same graph are the data of Esel'son and Beresniak<sup>3</sup>. As is evident from the graphs, our results and those of Ref. 3 are in satisfactory agreement.

The separation coefficient

$$k = X_v(1 - X_l) / (1 - X_v)X_l$$

for the values of concentration studied is equal to 3.0 ( $\pm 20\%$ ) and (with the accuracy pointed out) does not depend on the temperature in the interval  $1.5$ - $2.0^{\circ}\text{K}$ .

The authors take this opportunity to express their thanks to A. I. Filimonov for his help in carrying out the experiments.

<sup>1</sup> V. P. Peshkov, J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 850 (1956); Soviet Phys. JETP 3, 706 (1956); J. Tech. Phys. (U.S.S.R.) 26, 664 (1956).

<sup>2</sup> H. S. Sommers, Phys. Rev. 88, 113 (1952).

<sup>3</sup> B. N. Esel'son and P. G. Beresniak, J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 628 (1956); Soviet Phys. JETP 3, 568 (1956).

<sup>4</sup> Abraham, Osborne and Weinstock, Phys. Rev. 80, 366 (1950).

Translated by R. T. Beyer  
153

### Ionizational Slowing Down of High-Energy Electron Positron Pairs

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(Submitted to JETP editor July 4, 1956)  
J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 722-723  
(October, 1956)

THE components of a high energy electron-positron pair of energy  $E$  separate with the small angle  $\theta \sim mc^2/E$ . The interference of the electron and positron fields at a small distance from the point of creation leads to a smaller ionization than that which would be caused by two electrons. This phenomenon was theoretically examined by Chudakov.<sup>1</sup> The aim of the present note is to give another derivation of the equations of the ionizational slowing down of a pair and to clarify the limitations of such a development.

We make use of a method first shown by Landau (Ref. 2, p. 145) for the derivation of the equations of ionization loss at high energies (in the region of the polarization effect). If the positron and electron are at a particular moment of time  $t$  at

points  $\mathbf{r}_1(t)$  and  $\mathbf{r}_2(t)$  then  $T_1$  the energy loss of the pair in a unit of time, is

$$T = ec \{ \mathbf{v}_1 \mathbf{E}(\mathbf{r}_1, t) - \mathbf{v}_2 \mathbf{E}(\mathbf{r}_2, t) \}, \quad (1)$$

where  $cv_1$  and  $cv_2$  are the speeds of the positron and electron, and  $E$  is the electric field of the pair. The last can be considered as a field in the macroscopic realm caused by charge densities  $\rho$  and currents  $j$ ,

$$\begin{aligned} \rho &= e\delta(\mathbf{r} - \mathbf{r}_1(t)) - e\delta(\mathbf{r} - \mathbf{r}_2(t)); \\ \mathbf{j} &= ec \{ \mathbf{v}_1 \delta(\mathbf{r} - \mathbf{r}_1(t)) - \mathbf{v}_2 \delta(\mathbf{r} - \mathbf{r}_2(t)) \} \end{aligned}$$

and can be presented in the form of the following Fourier integral

$$\mathbf{E}(\mathbf{r}, t) = (ie/2\pi^2) \int d\mathbf{k} \{ e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}_1)} [\mathbf{v}_1(kv_1) - \mathbf{k}/\epsilon(kv_1)] / [k^2 - (kv_1)^2 \epsilon(kv_1)] - e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}_2)} [\mathbf{v}_2(kv_2) - \mathbf{k}/\epsilon(kv_2)] / [k^2 - (kv_2)^2 \epsilon(kv_2)] \}, \quad (2)$$

where  $\epsilon(\omega)$  is the dielectric constant of the medium corresponding to a frequency  $\omega$ . In the derivation of the (2) the trajectories of the particle being scattered in the medium can be considered as straight line segments for regions in which slowing down is still important.

Substituting Eq. (2) into Eq. (1) we obtain  $T = 2T_0 - T_1$ , where  $T_0$  is the ionizational slowing down of a single electron, and  $T_1$  is the interference term

$$T_1 = \frac{ice^2}{2\pi^2} \int d\mathbf{k} \left\{ \frac{(\mathbf{v}_1 \mathbf{v}_2)(\mathbf{v}_1 \mathbf{k}) - (kv_2)/\epsilon(kv_1)}{k^2 - (kv_1)^2 \epsilon(kv_1)} e^{i\mathbf{k}(\mathbf{r}_2 - \mathbf{r}_1)} + \frac{(\mathbf{v}_1 \mathbf{v}_2)(\mathbf{v}_2 \mathbf{k}) - (kv_1)/\epsilon(kv_2)}{k^2 - (kv_2)^2 \epsilon(kv_2)} e^{-i\mathbf{k}(\mathbf{r}_2 - \mathbf{r}_1)} \right\}.$$

In the calculation of  $T_1$  it is important that the transverse component of the pair separation be larger than the parallel component. Indeed the last is proportional to  $v_1 - v_2 \sim (mc^2/E)^2$  while the first is determined by the angle of separation of the pair  $\theta \sim mc^2/E$  and by the multiple scattering angle. Therefore, having selected the  $Z$  axis along the direction of  $\mathbf{v}_1$  or  $\mathbf{v}_2$ , we can substitute  $k_x s$  for  $\mathbf{k}(\mathbf{r}_2 - \mathbf{r}_1)$  in the exponent, where  $s = (x_2 - x_1)$ . After this the  $k_z$  integration can be carried out in the same way as done by Landau in the calculation of  $T_0$ . It turns out that the limiting expression for  $\epsilon$  for high frequencies is of importance in the integral,  $\epsilon = 1 - \lambda^2 c^2 / \omega^2$  where  $\lambda^2 = 4\pi n c^2 / mc^2$ , and  $n$  is the number of electrons in the unit volume. We obtain

$$T_1 = \frac{ce^2 \lambda^2}{\pi} \int \frac{\cos k_x s}{k_x^2 + k_y^2 + \lambda^2} dk_x dk_y = 2e^2 c \lambda^2 K_0(s\lambda), \quad (3)$$

where  $k_0$  is the corresponding cylindrical function.

The convergence of this integral is shown by the fact that the interference effects depend on large distances for which the macroscopic viewpoint is valid. The analogous integral for  $T_0$ , as is known, diverges and must be limited by some maximum value of the transverse wave vector  $K_n$  which is related to the energy  $E_n$  transmitted to the atomic electron.

For large  $s$  ( $s\lambda \gg 1$ ) the interference term disappears, as can be seen from Eq. (3). For small  $s$  ( $s\lambda \ll 1$ ), one can use the relationship  $k_0(z) = \ln(2/\gamma z)$ , where  $\gamma = e^C = 1.781$ . Then

$$T_1 = 2e^2 c \lambda^2 \ln(r_{\max}/s),$$

where  $r_{\max} = 2/\gamma\lambda$ .

If  $T_0$  is written in analogous form (see, for example, Ref. 2)

$$T_0 = ce^2 \lambda^2 \ln(r_{\max}/r_{\min}), \quad r_{\min} = a(\hbar/mc) \sqrt{mc^2/E_m}$$

( $a = 1.85$ ), then  $T$  can be written in the following form obtained in Ref. 1.

$$T = 2T_0 \ln(s/r_{\min}) / \ln(r_{\max}/r_{\min}). \quad (4)$$

Entering into  $r_{\min}$  is the quantity  $E_m$  which is the maximum energy transmitted to an atomic electron as determined from experimental data. We would like to express our thanks to L. D. Landau for discussion of the results.

<sup>1</sup> A. E. Chudakov, *Izv. Akad. Nauk SSSR, Ser. Fiz.* 19, 650 (1955).

<sup>2</sup> N. Bohr, *Passage of atomic particles through matter.*

Translated by G. L. Gerstein  
154

### Diffraction Scattering of High-Energy Photons by Nuclei

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(Submitted to JETP editor July 4, 1956)

*J. Exptl. Theoret. Phys. (U.S.S.R.)* 31, 723

(October, 1956)