fact, the functions $f_0(y)$ and $\Phi(y)$ can be expressed in terms of each other, since they are connected by the condition

$$d(e_0^2, 0) = 1, \ (\xi = L),$$

which, by Eq. (4) gives

$$e_0^2 = \Phi\left[\frac{e_0^2}{1 - e_0^2 f_0(e_0^2)}\right] = \Phi\left[\frac{1}{e_0^{-2} - f_0(e_0^2)}\right]$$

or

$$f_0(y) = 1/y - [\Phi^{-1}(y)]^{-1}, \tag{7}$$

where $\Phi^{-1}(y)$ is the function inverse to $\Phi(y)$. If (6) is satisfied, it follows that $\Phi^{-1}(y) \to y$ for $y \to 0$. But the function inverse to $\Phi^{-1}(y) \approx y$ will be $\Phi(y) \approx y$; i.e., we get Eq. (2). In an analogous way, Eq. (6) follows at once from (2) and (7).

From Eqs. (4) and (2) it follows at once that $e_c^2 \to 0$ for $L \to \infty$. Indeed, let us suppose that e_0^2 has a fixed (and arbitrary) value, and $L - \xi \to \infty$. Then, by Eq. (4), $\lambda_0(\xi) \to 3\pi/(L-\xi)$, i.e., according to Eq. (2),

$$e_0^2 d(e_0^2, L-\xi) \approx 3\pi/(L-\xi)$$

with increasing accuracy as $L - \xi$ is made larger. Since $e_c^2 d_c \equiv e_0^2 d$,

$$e_c^2 d_c(e_c^2, \xi) \approx 3\pi/(L-\xi), \ L - \xi \to \infty.$$

For $\xi \to 0$, when $d_c \approx 1$, this equation gives

$$e_c^2 \approx 3\pi/L \to 0, \ L \to \infty,$$

which was to be proved.

We note that if in Eq. (4) we regard e_0^2 as dependent on L, (as Taylor indeed assumed), the proof does not go through, since as L increases the quantity $e_0^2(L) f_0[e_0^2(L)]$ in Eq. (4) can change in such a way that λ_0 will not decrease, and will in general not be small for $L \to \infty$.

The writer expresses his gratitude to B. L. Ioffe and A. D. Galanin for discussion of the manuscript and valuable remarks.

* Cf. Ref. 3, Eq. (5.6). Account is taken of the relation $(-k^2/m^2)\Phi(e_c^2) = \exp(-3\pi/\lambda_c)$ if the function f_c of Eq. (1) is related to the function φ of Ref. 3 in the following way: $f_c = (1/e_c^2) + \ln\varphi(e_c^2)$ (the quantities k^2 and e^2 of Gell-Mann and Low are here denoted by $-k^2$ and e_c^2).

** For example, as Landau has remarked, for the function $[\ln(1 + e^{1/\lambda})]^{-1}$ the relation (2) holds, but the condition (2′) does not: for $\lambda \to -0$

$$[\ln(1 + e^{1/\lambda})]^{-1} \approx e^{1/|\lambda|}.$$

*** Cf. Eq. (B.11) of Ref. 3; note that

$$\frac{-k^2}{\lambda^2} G(e_0^2) = \exp(-3\pi/\lambda_0),$$

if in Eq. (4) $f_0(e_0^2) = e_0^{-2} + \ln G(e_0^2)$. Therefore,

$$F\left[\frac{-k^2}{\lambda^2} G(e_0^2)\right] \equiv \Phi(\lambda_0),$$

if $\Phi(y)$ is determined by the function F of Ref. 3 by the equation $\Phi(y) = F(e^{-3\pi/y})$.

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Translated by W. H. Furry
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A Polarization Method for Measuring the Velocities of Particles with Intrinsic Magnetic Moment

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(Submitted to JETP editor March 13, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 512-513
(September, 1956)

MEASUREMENT of the velocities of particles in a beam (spectrometry) is a typical problem of physical experiment. A polarization method can be used for particles which possess an intrinsic magnetic moment. The spectrometer resembles the type which is used to determine the magnetic moments of individual particles. In the path of the beam there is placed a polarizer, a device for rotating the plane of polarization, an analyzer and, finally, a particle detector. The device for spin rotation can be constructed in such a way as to change the orientation only for particles which possess a given energy. The analyzer removes the remaining particles. The detector readings correspond to the number of particles of the given energy in the beam spectrum.

For neutral atoms with spin it is technically possible to construct the spectrometer in the following manner (for definiteness we shall discuss particles with spin ½). The particles enter a nonuniform Stern-Gerlach magnetic field which splits the beam into two components. Then the reorienting intrument (which will be described in detail below) flips the spins of atoms possessing a given velocity. Then the particles traverse a Stern-Gerlach field which is similar to the first field. This field further separates the two components of the beam, with the exception of the atoms which were reoriented. Instead, the latter are focused at a slit behind which the detector is placed. When necessary, on the straight line connecting the source with the slit the analyzer may contain a screen (filament) which prevents direct entry by spinless atoms in the detector. The detector can be a vacuum pressure gauge or a thermocouple.

A neutron spectrometer can contain Bloch ferromagnetic polarizers, such as are used for the measurement of the neutron magnetic moment. The sensitivity of the detectors should be determined from sources with a known spectrum. A detector of very simple geometry which permits an exact calculation of the sensitivity can also be used as a standard.

The most important part of the spectrometer is the reorienting device. This consists of a series of conductors, the current in which excites a periodic magnetic field along the particle trajectory. The field reverses its sign along the trajectory. Alternating current is used. It is evident that some particles possess such velocity that during their entire transit they will be in a field which does not change sign. The magnetic moment of such a particle precesses in the field and the field strength can be chosen to provide a spin rotation of 180° at the exit point, that is, reversal. Particles of other velocities enter a field which changes its sign. When the sign of the field changes, the direction of precession changes. As a result, during their passage the spins of such particles cannot be rotated through any appreciable angle. It is easily seen, however, that there is more than one resonance velocity, namely:

$$v_{\text{res}} = a\omega/(\pi + 2n\pi).$$

Here a is the half period of the field, ω is the current frequency, $n = 0, 1, 2 \ldots$ is the order of the maximum. The reversing system can be designed in such a way as to completely eliminate all maxima above the zeroth order. Moreover, they can be cut off by screens placed properly in the analyzer. During the measurements the resonance velocity changes with variation of ω. It is clear from elementary considerations that the resolving power of the spectrometer is approximately given by the total number N of field periods in the reorienting system and is independent of other properties of the system.

The author has calculated a few variants of the reorienting system. The quantum equation for the spin functions was solved. The particles were assumed to have a classical trajectory. For a field with a single component (in the region of the beam) when a sinusoidal current is used the probability of reversal close to resonance is

$$W(v) = {}^1/_2 - {}^1/_2 J_0 (A \sin \mu/\mu).$$

Here v is the particle velocity, J_0 is the zeroth order Bessel function, $\mu = N\omega a(1/v - 1/v_{\text{res}})$, A is a coefficient which is determined by the specific type of system. The maximum probability for reversal is 0.7. This is less than unity as a result of averaging over the current phases.

We shall take as an example the following technical realization of a reorienting system. Two parallel wires are extended in zigzag fashion along the particle trajectory so that the particle moves between them. Current from the generator is sent through these wires.

A second type of reorienting device uses constant fields. The system of conductors which produces the periodic field is supplied with direct current. In addition, an electromagnet excites a strong uniform field parallel to the magnetic moment of the particle. The reorienting mechanism is the same as in apparatus for the measurement of the magnetic moments of individual particles. The resonance velocity changes with the uniform field strength.

It is also possible to use combinations which unite characteristics of both types of reorientation devices.

With this atomic spectrometer it is possible to investigate various collision processes between atoms and molecules, chemical kinetics, the behavior of statistical systems and recoil atoms in nuclear physics. (Radicals possessing a magnetic moment can, of course, also be analyzed by the spectrometer.) The atomic spectrometer enables us to investigate processes which take place at temperatures of hundreds of thousands of degrees (and, in particular, to measure such temperatures) by analyzing the velocities of a beam of neutral atoms leaving the heated region. It is thus possible that the spectrometer can be used in work on thermonuclear reactions, since hydrogen, deuterium and tritium atoms possess magnetic moments. With a neutron spectrometer it is possible to measure the cross sections of neutron interactions with nuclei and polarization effects, and to obtain the spectra of neutron sources. The measurable energy range goes from thermal ranges to the order of 10 mev. For large energies the size of the apparatus and the power of the generator are increased, but pulsed operation is possible.

Translated by I. Emin
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Transformation Properties of the Electron-Positron Field Amplitudes

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(Submitted to JETP editor, June 8, 1956)
J. Exptl.Theoret. Phys. (U.S.S.R.) **31**, 535-536 (September, 1956)

ONE of the conditions for the relativistic invariance of a theory is that the state vectors Φ of the field transform according to some representation of the Lorentz group. Since any vector Φ can be obtained by operating on the vacuum state Φ_0 with creation and annihilation operators, the transformation of Φ reduces to the transformations of Φ_0 and of the creation operators, that is, of the field amplitudes. It will be shown that the field amplitudes do not transform according to the spinor representations of the Lorentz group, and that the amplitudes corresponding to electron and positron states transform independently of each other according to the same representations. We shall consider the inhomogeneous Lorentz group $\mathfrak{L}$, including space reflections but not time reflections. The question of time reflections is more complicated and requires special consideration. It is clear that the vacuum state Φ_0 is invariant under the group $\mathfrak{L}$. In order to derive the transformation properties of the field amplitudes we shall write the field operator $\psi(x)$ in the interaction representation in the following relativistically invariant form (in the following we make use of Feynman's[1] notation and set $\hbar = c = 1$)

$$\psi(x) = (2\pi)^{-3/2} \times \int \{u(p)\, a(p)\, e^{-ipx} + v(p)\, b^+(p)\, e^{ipx}\}\, d\Gamma. \quad (1)$$

The integration in (1) is taken over the hypersurface given by $p^2 = m^2$, $p_4 \equiv \epsilon > 0$; $d\Gamma = (m/\epsilon)\, d^3p$ is the invariant element of the hypersurface; $u(p) \equiv \| u_\mu^\alpha(p) \|$ is a matrix of four rows and two columns given by the two solutions of the Dirac equation for positive energy, normalized according to the condition $\bar{u}^\alpha u^\beta = \delta_{\alpha\beta}$; $v(p)$ is the charge conjugate matrix; $a(p) \equiv \binom{a_1(p)}{a_2(p)}$ is the column operator consisting of electron annihilation operators, and $b^+(p)$ is the analogous column operator of positron creation operators. The operators a, b are normalized by the invariant δ-function on Γ, that is, they satisfy the anticommutation relations

$$[a(p), a^+(q)]_+ = \Delta(p-q), \text{ и т. д. } (\Delta(p) = (\varepsilon/m)\,\delta(\mathbf{p})).$$

Let us first consider the homogeneous Lorentz transformation. (Similar, though somewhat simpler, considerations hold for translations.) Under a transformation L the spinor field $\psi(x)$ transforms according to

$$\psi(x) \to \psi'(x) = S_L \psi(L^{-1}x), \quad (2)$$

where S_L is the spinor representation of the Lorentz group, which satisfies the condition $S_{L_1} S_{L_2} = S_{L_1 L_2}$. It is easy to see that the transformation (2) is equivalent to the following system of transformations in p-space:

$$u(p) \to S_L u(L^{-1}p); \qquad a(p) \to a(L^{-1}p) \quad (3)$$

with similar systems of transformations for v and b^+. From the relativistic invariance of the Dirac equation, however, it follows that

$$S_L u(L^{-1}p) = u(p)\, Z_L(p), \quad (4)$$

where the second degree matrix $Z_L(p)$ is determined by the relation $Z_L(p) = \bar{u}(p)\, S_L\, u(L^{-1}p)$.

The following properties of the matrix $Z_L(p)$ are easily established:

1) $Z_L(p)$ generates a representation of the Lorentz group, that is,

$$Z_{L_1}(p)\, Z_{L_2}(L_1^{-1}p) = Z_{L_1 L_2}(p). \quad (5)$$

2) The representation (5) is unitary

$$Z_L^+(p) = Z_L^{-1}(p) = Z_{L^{-1}}(L^{-1}p).$$