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The Dynamical Magnetic Moment of the Deuteron

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The dynamical magnetic moment of the deuteron is considered on the basis of the pseudoscalar meson theory with the pseudoscalar type of coupling , in the fifth order of perturbation theory. Exchange currents in the deuteron make an essential contribution to the dynamical magnetic moment.

INTRODUCTION

THE well-known experimental result that the constant magnetic moment of the deuteron differs from the sum of the magnetic moments of the neutron and proton is commonly explained phenomenonologically by the existence of a tensor interaction between nucleons. Because of this the ground state of the deuteron consists of an *S*-state with an admixture of a *D*-wave, which on one hand leads to the existence of the quadrupole moment of the deuteron, and on the other to the nonadditivity of the magnetic moments in the deuteron[1].

Although such an interpretation is not in qualitative contradiction with experiment, it still does not correspond exactly to the effect, since it does not take into account the existence of meson exchange currents in the deuteron, which are shown by experiment to have an essential effect on the electromagnetic properties of the deuteron, particularly on radiative effects in the neighborhood of the energy threshold for production of π-mesons.

Unfortunately there is at present no consistent phenomenological theory of exchange currents. Of the attempts in this direction most deserving of attention, mention must be made of the work of Sachs[2], and a paper of Villars[3] is devoted to the meson-field treatment of this effect.

An essential difficulty in the treatment of the magnetic moment of the deuteron on the basis of the meson theory of nuclear forces arises from the circumstance that within the framework of this theory the relativistic problem of two nucleons is at the present time unsolved, and the magnetic moments of the separate nucleons are explained only qualitatively by the theory with weak interaction between the nucleon and meson fields, so that any theoretical investigations in this subject are as yet only of a qualitative nature. Nevertheless, it can turn out that the perturbation theory to a certain extent gives a correct indication of the general tendencies in the behavior of the two-nucleon system in interaction with high-energy γ-ray quanta.

The present paper is devoted to a consideration of the dynamical magnetic moment of the deuteron on the basis of the pseudoscalar meson theory with the pseudoscalar type of coupling. In interaction with the meson field of the vacuum the nucleons making up the deuteron can emit and then absorb virtual mesons, so that the real nucleon can be thought of as surrounded by a certain stationary cloud. If the charged meson clouds of the neutron and proton overlap, exchange meson currents arise, flowing from one nucleon to the other. The interaction of the meson field surrounding the deuteron with the electromagnetic field can be interpreted as a supplementary direct electromagnetic interaction of the deuteron itself.

In the expression obtained for this supplementary interaction one can single out the terms that represent the energy of a certain additional magnetic moment in the electromagnetic field. The size of this magnetic moment will depend on the frequency of the electromagnetic field. Thus the energy of the supplementary electromagnetic interaction will have the form

$$(e\hbar/2Mc)(\sigma_{\mu\nu}F_{\nu\mu})[\mu_c + f(k)],$$

where $F_{\nu\mu}$ is the electromagnetic field tensor, $\sigma_{\mu\nu}$ is the spin operator, μ_c is the static magnetic moment of the deuteron, and $f(k)$ is a certain function of k that goes to zero for $k \to 0$. The quantity in square brackets represents the dynamical magnetic moment of the deuteron.

We shall consider the supplementary interaction of the deuteron with the electromagnetic field, including effects of exchange currents, on the basis of the symmetric pseudoscalar meson theory with pseudoscalar coupling, by the use of the relativistically invariant method of Feynman and Dyson[4,5].

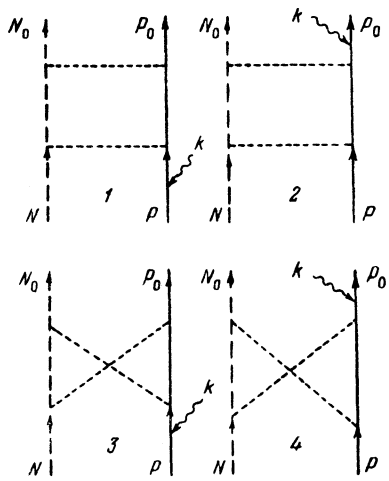

FIG. 1

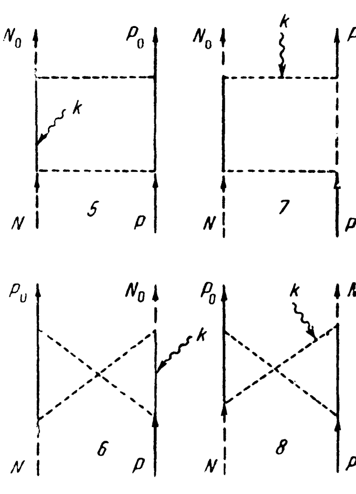

FIG. 2

In the case of the bound state we cannot apply directly the relativistically invariant method, which has been systematically developed only for the free states of two nucleons. Therefore, following the method of Brueckner[6], we develop the space factor of the initial state wave function into a Fourier integral, and in finding the matrix element we operate with one component of the expression, which, we may suppose, describes a "quasi-free state", and afterwards carry out the integration over the entire momentum space. The use of this method leads to the possibility of writing the matrix element for a radiative transition in the simple form

$$H_{if} = \Psi_d(0) M_{if}(0); \tag{1}$$

M_{if} is the matrix element, which is constructed by the method given in Refs. 4 and 5, and in which the momentum in the initial state is taken equal to zero*.

Figures 1 — 3 show the Feynman diagrams relating to the process under consideration.

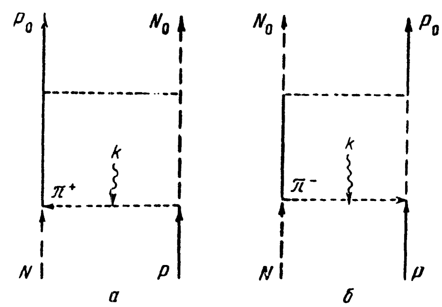

FIG. 3

In the expressions obtained for the supplementary interaction energy of the deuteron with the electromagnetic field we single out the terms having the form $i\gamma_\mu\gamma_\nu k_\mu A_\nu \mu(k)$ in the momentum representation (γ_ν are the Dirac matrices, A_ν the vector potential of the electromagnetic field, and k_μ the energy-momentum vector of the γ-quantum). In the

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* The form of the expression (1) is due to the circumstance that in the absorption of a short-wavelength γ-quantum by the deuteron the nucleons in the deuteron must be separated only by small distances of the order

$$\lambda \sim h/p_0 \sim (\hbar/Mc)(Mc^2/2E_k)^{1/2}.$$

This means that we must take the wave function of the deuteron for some effective value of r, smaller than λ, but since $\Psi_d(r)$ is weakly dependent on r at small distances $r \lesssim h/Mc$, we may take $r_{\text{eff}} \sim 0$[6,8].

coordinate representation this term has the form $(\sigma_{\mu\nu}F_{\nu\mu})\mu(k)$, i.e., it represents the energy of the magnetic moment of the system in the electromagnetic field[7].

It can be shown that in the case in which the nucleons exchange an odd number of mesons the magnetic terms add up to zero. We first consider the case of the exchange of one meson. This corresponds to a process of third order in perturbation theory (exchange meson currents), and of the fifth order (the effect of the meson cloud surrounding the deuteron)[8].

In this case the terms in the magnetic interaction energy will have the form

$$\gamma_5'\gamma_5''\hat{k}'\hat{A}'\mu_1(k) + \gamma_5'\gamma_5''\hat{k}''\hat{A}''\mu_2(k). \tag{2}$$

The primes $'$ and $''$ indicate whether the Dirac matrices γ_5 and γ_μ refer to the first or the second particle.

If we denote the matrix-vectors for the initial and final states of the proton and neutron by $\hat{P}$, $\hat{N}$ and $\hat{P}_0$, $\hat{N}_0$, respectively, then on the basis of the Dirac equation we can write:

$$(\hat{P} - M)\,u(P) = 0; \quad (\hat{N} - M)\,u(N) = 0; \tag{3}$$

$$\bar{u}(P_0)(\hat{P}_0 - M) = 0; \quad \bar{u}(N_0)(\hat{N}_0 - M) = 0.$$

Moreover, the four-dimensional matrix-vectors satisfy the following relations

$$\hat{a}\hat{b} + \hat{b}\hat{a} = 2(ab), \tag{4}$$

$$(ab) = a_4b_4 - a_1b_1 - a_2b_2 - a_3b_3, \tag{5}$$

$$\gamma_5\hat{a} = -\hat{a}\gamma_5. \tag{6}$$

Thereupon, taking into account the law of conservation of energy in the initial and final states of the system, $k = P_0 + N_0 - P - N$, the transversality condition for free γ-quanta, which gives $(AP) = (AN) = (Ak) = 0$, and the relations (1), (3) and (4), we find that the matrix element of the operator (2) becomes zero. In the case of interchange of $(2n+1)$ mesons by the nucleons, we can use Eq. (6) to bring all of the matrices γ_5 together, and since $\gamma_5^2 = -1$, the expression for the magnetic interaction energy again takes the form (2), and consequently is equal to zero.

This result is due to the specific form of meson theory used (pseudoscalar meson theory with pseudoscalar and pseudovector couplings). For other types of coupling these magnetic terms can be different from zero. When the nucleons exchange an even number of mesons the magnetic moment does not reduce to zero, and thus we begin our consideration with the next, the fifth, order of perturbation theory (case of exchange of two mesons). In the approximation in question all diagrams can be divided into two types: diagrams relating to the internal motion of the nucleons, and exchange nucleon and meson currents.

INTERNAL MOTION OF THE NUCLEONS

To the first type belong the diagrams (cf. Fig. 1) describing the absorption of the γ-quantum by a nucleon. Diagrams *1* and *2* relate to the case in which the emission and absorption of the first meson precede the emission and absorption of the second meson (nonintersecting meson lines). Diagrams *3* and *4* represent the case of intersecting meson lines, when the meson first emitted by a nucleon is absorbed after the emission and subsequent absorption of the second meson.

The exchange of the two mesons brings about a change of the internal motion of the nucleons in such a way that to the ground S-state of the deuteron there are also added states with other orbital angular momenta. When all approximations of the perturbation theory are taken into account, this should in principle give the experimentally observed combination of the S- and D-waves in the ground state of the deuteron.

To obtain the matrix elements corresponding to diagrams *1-4*, we choose a coordinate system connected to the center-of-mass of the system deuteron + γ-quantum. Then, according to Eq. (1), the initial four-vector momenta of the proton and neutron are equal to each other and have the components $P_4 = M$, $P = -k/2$, and the final momenta N_0 and P_0 of neutron and proton have the components $P_{04} = N_{04} = E_0$, $\overline{N_0} = -\overline{P_0}$.

If the second meson emitted is absorbed before the first (intersecting meson lines of diagrams *3* and *4*), then there also exist two possibilities of realization of the process in question: absorption of the γ-quantum by the proton before and after the exchange of the two mesons between the two nuclei (matrix elements M_3 and M_4).

For the symmetric meson theory of nuclear forces there are four matrix elements corresponding to diagrams *1* and *2*, differing from each other in that the exchange between the two nucleons is made by two charged mesons, two neutral mesons, or one charged and one neutral meson. These matrix elements either are equal to M_1 (or M_2), or else they differ from M_1 (or M_2) by the interchange of the indices of the particles. The same can be said with regard to diagrams *3* and *4*, for each of which there exist three matrix elements.

The calculation of the integrals occurring in the matrix elements M_1 to M_4 is rather cumbersome. But if we restrict ourselves just to terms of order k^2/M^2, the result can be obtained in analytical form. Taking into account all matrix elements characterizing the internal motion of the nucleons, and using the method of parametric integration[4], we obtain the following expression for the radiative transition matrix element M_n characterizing the internal motion of the nucleons in the deuteron*.

$$M_{\text{н}} = -\frac{eg^4}{4\pi}\left\langle\frac{\hat{k}''\hat{A}''}{M^4}[F_1-(\gamma'_\mu\gamma''_\mu)\,G_1]\right\rangle+\frac{eg^4}{4\pi}\left\langle\frac{(\hat{k}'\hat{A}'+\hat{k}''\hat{A}'')}{M^4}[F_2-(\gamma'_\mu\gamma''_\mu)G_2]\right\rangle \tag{7}$$
$$-\frac{eg^4}{4\pi}\left\langle\frac{(^1/_2\hat{k}'A'+\hat{k}''\hat{A}'')}{M^4}[F_4^- - G_4^-(\gamma'_\mu\gamma''_\mu)]\right\rangle+\frac{eg^4}{4\pi}\left\{\left\langle\frac{\hat{k}'\hat{A}'}{M^4}[F_3^+ - G_3^+(\gamma'_\mu\gamma''_\mu)]\right\rangle\right.$$
$$\left.+\left\langle\frac{\hat{k}''\hat{A}''}{2M^4}[F_3^- - G_3^-(\gamma'_\mu\gamma''_\mu)]\right\rangle\right\},$$

where

$$F_1 = \left\{\frac{Mk+\mu^2}{M^2+2Mk}\left(\ln\frac{\mu^2}{M^2}+1\right)+\sqrt{\frac{k^2-\mu^2}{4M^2}}\ln\frac{[2(M^2+Mk)+M\sqrt{k^2-\mu^2}][2k+\sqrt{k^2-\mu^2}]}{[2(M^2+Mk)-M\sqrt{k^2-\mu^2}][2k-\sqrt{k^2-\mu^2}]}\right.$$
$$+\frac{Mk}{(M^2+Mk)}+\frac{Mk-2\mu^2}{2M^2}\ln\frac{k}{2M}-2\sqrt{\frac{k^2-4\mu^2}{M^2}}\operatorname{arctg}2\sqrt{\frac{M^2}{k^2-\mu^2}}$$
$$\left.+\sqrt{\frac{k}{M}}\operatorname{arctg}4\sqrt{\frac{M}{k}}\right\}\frac{M^2}{(Pk)};$$

$$G_1 = \left[\frac{Mk}{2(M^2+Mk)}+\frac{Mk-2\mu^2}{4M^2}\ln\frac{k}{2M}-\sqrt{\frac{|k^2-4\mu^2|}{M^2}}\operatorname{arctg}2\sqrt{\frac{M^2}{k^2-\mu^2}}\right.$$
$$\left.+\sqrt{\frac{k}{M}}\operatorname{arctg}4\sqrt{\frac{M}{k}}\right]\frac{M}{(P_0k)};$$

$$F_2^- = \left[1-\ln\frac{2M}{\mu}+\frac{\mu}{M}\operatorname{arctg}\frac{M}{\mu}-\frac{5}{24}\ln 2\right]\frac{M^2}{(P_0k)};$$

$$G_2^- = \left[\frac{\mu}{M}\operatorname{arctg}\frac{M}{\mu}+1-\ln\frac{2M}{\mu}\right]\frac{M^2}{(P_0k)};$$

$$F_3^+ = \left\{\left[\frac{2(M^2-(P_0k))}{M^2-3(P_0k)}+\frac{6}{5}\frac{(P_0k)}{M^4}\ln\frac{k}{M}+\frac{2}{3}\frac{(P_0k)}{M^2}\right.\right.$$
$$\left.+\frac{4}{3}\frac{(P_0k)}{M^2-3(P_0k)}\frac{M^2}{\lambda}T(k)\right\}\frac{M^2}{(Pk)};$$

$$G_3^+ = \left\{\frac{(P_0k)\,M}{(M^2-3(P_0k))k}\ln\frac{M}{k}+\frac{\lambda M}{(M^2-3(P_0k))k}T(k)+\frac{2}{9}\right\}\frac{M^2}{2(Pk)};$$

$$F_4^- = \frac{M^2+\beta^2}{2(M^2-2\beta^2)}\left\{\frac{(M^2-2\beta^2)}{2(M^2-\beta^2)}+\frac{9}{4}\frac{\beta^2M^2}{(M^2-\beta^2)^2}+\frac{2\beta^2}{d}\frac{(M^2-\beta^2)}{(M^2+\beta^2)}\ln\frac{2\beta^2-d}{2\beta^2+d}\right.$$
$$\left.+\frac{2M^2-5\beta^2}{M^2}\ln\frac{M}{\mu}\right\}\frac{M^2}{(P_0k)};$$

$$G_4^- = \left\{\frac{\mu}{M}\operatorname{arctg}\frac{M}{\mu}-\frac{\beta^2}{6M^2}\right\}\frac{M^2}{(P_0k)};$$

$$T(k) = \left[\operatorname{arctg}\frac{2(M^2-2(P_0k))}{\lambda}-\operatorname{arctg}\frac{2(P_0k)}{\lambda}\right];$$

$$\lambda = \sqrt{4M^2\beta^2-9(P_0k)^2};\quad d=2\sqrt{M^2\mu^2-\beta^4};\quad \beta=(Mk-\mathbf{P}_0\mathbf{k});$$
$$(P_0k) = (E_0+P_0\cos\theta)\,k.$$

* In the expression (7) terms in the electric interaction $\sim(AP_0)$ are not included, since an estimate showed them to be small.

In Eq. (7), F_i^+, G_i^+ differ from F_i^-, G_i^- by the interchange of P_0 and N_0 in the scalar products (P_0k) and (N_0k) on which these functions depend; in other words, they differ by the sign of the spatial momentum of the nucleon.

NUCLEON AND MESON EXCHANGE CURRENTS IN THE DEUTERON

Diagrams 5-8 describe the absorption of the γ-quantum by the deuteron by nucleon (diagrmas 5 and 6) and meson (diagrams 7 and 8) exchange currents in the deuteron, which arise in the exchange between the nucleons of two mesons, of which at least one must be charged.

We have calculated the anomalous dynamic magnetic moment due to meson exchange currents in the deuteron (non-intersecting meson lines) when the second meson emitted by the nucleon absorbs the γ-quantum in the intermediate state. There exist, however, still other matrix elements that characterize the absorption of γ-quanta by meson exchange currents, but do not contribute to the effect. These are the matrix elements that are due to the absorption of the γ-quantum by the first meson exchanged, and also to the matrix elements corresponding to the diagrams with intersecting meson lines. To this case there correspond two matrix elements: a) with the second meson neutral (cf. Fig. 3), and b) with the secon meson charged. Then the absorption of the γ-quantum occurs in case a) by a π^+, and in case b) by a π^- meson, for the given direction of motion of the mesons from the final state neutron to the proton. Thus we have for these matrix elements $M_a = M_b$ [$P = N$ on the basis of (1)], and their sum is zero.

Similarly it can be shown that matrix elements of the type M_8 make a small contribution to the effect, and we neglect them. The matrix element for the electromagnetic transition arising from the existence of nucleon and meson exchange currents finally takes the form:

$$M_0 = -eg^4/4\pi M^4 < (\hat{k}'\hat{A}' + \hat{k}''\hat{A}'')\,[(F_5 - {}^1/_2F_7) - (\gamma'_\mu\gamma''_\mu)\,G_5] >$$
$$+ \frac{eg^4}{4\pi M^4} < \hat{k}'\hat{A}'\,(F_6^- - (\gamma'_\mu\gamma''_\mu)G_6^-) > + \frac{eg^4}{2\pi M^4} < \hat{k}''\hat{A}''\,(F_6^+ - (\gamma'_\mu\gamma''_\mu)\,G_6^+) >, \quad (8)$$

$$F_5 = -\frac{1}{8}\left[\frac{E_0^2}{E_0^2 - P_0^2\cos^2\theta} + 1\right]; \quad G_5 = -\frac{2}{5}\left[\frac{\mu^2}{7M^2} + \frac{k}{29\,M} - \frac{E_0k}{72M^2}\right]; \qquad (8a)$$

$$F_6^+ = \Big\{\frac{M}{\mu}\left(\frac{3}{4} - \frac{\beta^2}{M^2} + \frac{10\,\mathbf{P}_0\mathbf{k}}{M^2}\right)\operatorname{arctg}\frac{M}{\mu} - \left(2 + \frac{10\mathbf{P}_0\mathbf{k}}{M^2} + \frac{\beta^2}{M^2}\right)\ln\frac{M}{\mu}$$
$$-\left(\frac{3}{4} + \frac{10\,\mathbf{P}_0\mathbf{k}}{M^2}\right)\ln 2 + \frac{3}{8} + \left[\frac{3}{4}\sqrt{\frac{\beta^2 + 4\mu^2}{4\beta^2}} + \frac{\sqrt{\beta^4 + 4\mu^2\beta^2}}{M^2}\right.$$
$$\left.+\left(\frac{3}{2}\frac{k}{M} - \frac{5}{8}\frac{(P_0k)}{M^2}\right)\frac{M^2}{\sqrt{\beta^4 + 4\mu^2\beta^2}}\right]\ln\frac{\beta^2 + \sqrt{\beta^4 + 4\beta^2\mu^2}}{\beta^2 - \sqrt{\beta^4 + 4\beta^2\mu^2}}\Big\};$$

$$G_6 = \left\{\left(1 + \frac{4}{3}\frac{\beta^2}{M^2}\right)\frac{M}{\mu}\operatorname{arctg}\frac{M}{\mu} - \left(1 + \frac{\beta^2}{4M^2}\right)\ln\frac{2M}{\mu} - \frac{3}{4}\frac{\beta^2}{M^2}\ln 2\right\};$$

$$F_7 = \Big\{\frac{(M^2 + \beta^2)\ln 4}{M^2 - 2(N_0k)} + \frac{3\pi}{8}\frac{(N_0k)}{M^2} + \frac{M^2}{2(M^2 - (N_0k))} - \frac{M^2 + 2(N_0k)}{M^2 - 2(N_0k)}\ln\frac{M^2}{(N_0k)}$$
$$-\frac{M}{\beta}\left[\ln\frac{(2M^2 - \beta^2 - 2M\beta)\,|\,\beta^2 - 2M\beta\,|}{(2M^2 - \beta^2 + 2M\beta)\,|\,\beta^2 + 2M\beta\,|}\right] + \frac{(M^2 + \beta^2)}{4\,(M^2 - 2\,(N_0k))}\frac{M}{\sqrt{(N_0k)}}$$
$$\times\ln\frac{[2M^2 + \beta^2 - 2M\sqrt{(N_0k)}\;|\,\beta^2 + 2M\sqrt{(N_0k)}\,|}{[2M^2 + \beta^2 + 2M\sqrt{(N_0k)}\;|\,\beta^2 - 2M\sqrt{(N_0k)}\,|} - \frac{1}{4}\ln\frac{M}{\mu}$$
$$-\frac{c_0M^3}{(N_0k)^{3/2}}\ln\frac{M - \sqrt{(N_0k)}}{M + \sqrt{(N_0k)}} + \frac{(P_0k)}{2(N_0k)}\left[9c_1 + \frac{3}{2}\ln\frac{M}{\mu} + \frac{M^2 + 2\beta^2}{2M\beta}\ln\frac{M}{\mu}\right]\ln\frac{M - \beta}{M + \beta}$$
$$+\frac{4c_1M}{\sqrt{(N_0k)}}\ln\frac{M - \sqrt{(N_0k)}}{M + \sqrt{(N_0k)}}\Big\}; \quad c_0 = 0.93; \;\; c_1 = 0.65.$$

It must be noted that the functions F_i appear in the matrix elements M_n and M_0 in an unsymmetrical way, so that the angular dependence for the radiative transitions must have an asymmetry in the direction of small angles (when $\cos\theta \sim 1$).

THE DYNAMICAL MAGNETIC MOMENT OF THE DEUTERON

The additional interaction energy of the deuteron with the electromagnetic field, which appears because of the existence of nucleon and meson exchange currents in the deuteron and the change of the internal motion of the nucleons owing to them, can be written by (1), (7) and (8) in the following form:

$$H_{if} = \frac{eg^4}{4\pi}\frac{\Psi_d(0)}{M^4}\{<\hat{k}'\hat{A}'(\mathfrak{F}' - (\gamma'_\mu\gamma''_\mu)\mathfrak{G}')>_{if} \quad (9)$$

$$+ <\hat{k}''\hat{A}''(\mathfrak{F}'' - (\gamma'_\mu\gamma''_\mu)\mathfrak{G}'')>_{if}$$

$$+ <(\hat{k}'\hat{A}' + \hat{k}''\hat{A}'')(\mathfrak{F} - (\gamma'_\mu\gamma''_\mu)\mathfrak{G})>_{if}\}.$$

Here the new functions $\mathfrak{F}$ and $\mathfrak{G}$ are connected with the old functions F_i and G_i [cf. Eqs. (7a) and (8)] in the following way:

$$\mathfrak{F}' = \left(F_3^+ + F_6^- + \frac{1}{2}F_4^-\right); \quad (9a)$$

$$\mathfrak{F}'' = \left(2F_6^+ + \frac{1}{2}F_3^- - F_1\right);$$

$$\mathfrak{F} = \left(F_2^- - F_4^- - F_5 + \frac{1}{2}F_7\right);$$

$$\mathfrak{G}' = \left(G_3^+ + G_6^- + \frac{1}{2}G_4^-\right);$$

$$\mathfrak{G}'' = \left(2G_6^+ + \frac{1}{2}G_3^- - G_1\right);$$

$$\mathfrak{G} = (G_2^- - G_4^- - G_5).$$

Let us consider the form of the operators appearing in Eq. (9) in the x-representation. Since by the law of conservation of energy and momentum

$$k_\mu = P_{0\mu} + N_{0\mu} - P_\mu - N_\mu = \Delta P_\mu \rightarrow i\partial/\partial x_\mu,$$

we can write the operator $\hat{k}\hat{A}$ in the following form:

$$\hat{k}\hat{A} = \gamma_\mu\gamma_\nu k_\mu A_\nu = \sigma_{\mu\nu}\mathfrak{F}_{\nu\mu},$$

where $\sigma_{\mu\nu} = (i/2)(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$ is the four-dimensional spin vector and $\mathfrak{F}_{\mu\nu}$ is the electromagnetic field tensor. If we introduce the total spin of the system, $S_{\mu\nu} = (\sigma'_{\mu\nu} + \sigma''_{\mu\nu})$, Eq. (9) takes the form:

$$H_{if} = (eh/2Mc)(g^2/hc)^2\,\Psi_d(0)/2\pi M^3 \quad (10)$$

$$<\{(\sigma'_{\mu\nu}\mathfrak{F}_{\nu\mu})(\mathfrak{F}' - (\gamma'_\mu\gamma''_\mu)\mathfrak{G}')$$

$$+ (\sigma''_{\mu\nu}\mathfrak{F}_{\nu\mu})(\mathfrak{F}'' - (\gamma'_\mu\gamma''_\mu)\mathfrak{G}'')$$

$$+ (S_{\mu\nu}\mathfrak{F}_{\nu\mu})(\mathfrak{F} - (\gamma'_\mu\gamma''_\mu)\mathfrak{G})\}>_{if}.$$

Now H_{if} has the form of the energy of the dynamical magnetic moment in the electromagnetic field.

By means of the expression just obtained one can consider various radiative processes for the system neutron-proton, for example, the photodisintegration of the deuteron, the production of γ-quanta in collisions of high-energy neutrons with protons, and the Compton effect with the deuteron.

Let us consider as an illustration the photodisintegration of the deuteron. In the Pauli approximation, H_{if} takes the form

$$H_{if} = -\left(\frac{eh}{2Mc}\right)\left(\frac{g^2}{\hbar c}\right)^2\frac{\Psi_d(0)}{2\pi M^3c^3} \quad (11)$$

$$\times <\{(\mathbf{SH})\left[F + \frac{F' + F''}{2} - \left(1 + \frac{k}{Mc^2}\right)\left(\mathfrak{G} + \frac{\mathfrak{G}' + \mathfrak{G}''}{2}\right)\right]$$

$$+ (\mathbf{CH})\left[\frac{\mathfrak{F}' - \mathfrak{F}''}{2} - \left(1 + \frac{k}{Mc^2}\right)\left(\frac{\mathfrak{G}' - \mathfrak{G}''}{2}\right)\right]>_{if},$$

$$\mathbf{S} = (\sigma' + \sigma''), \quad \mathbf{C} = \sigma'_1 - \sigma''_2,$$

The first term in Eq. (11) describes the triplet-triplet transitions, and the second, the triplet-singlet transitions. For the triplet-triplet transitions the differential cross section for the photodisintegration of the deuteron can be written in the following form:

$$d\sigma_\Phi = \frac{\pi^2}{32}\left(\frac{e^2}{\hbar c}\right)\left(\frac{g^2}{\hbar c}\right)^4\frac{\Psi_d^2(0)}{M^3c^3}\left(\frac{k}{Mc^2}\right)^{3/2}D^2(k)\,d\Omega, \quad (12)$$

$$D(k) = \left(\mathfrak{F} + \frac{\mathfrak{F}' + \mathfrak{F}''}{2}\right) - \left(1 + \frac{k}{Mc^2}\right)\left(\mathfrak{G} + \frac{\mathfrak{G}' + \mathfrak{G}''}{2}\right).$$

The function $D(k)$ increases slowly with the energy of the absorbed γ-ray.

With increasing γ-ray energy the effective cross section for the photodisintegration of the deuteron, Eq. (12), increases as $k^{3/2}$. The angular distribution of the photonucleons is isotropic in the range of energies in which the magnitude of the spatial momentum of the nucleon can be neglected in comparison with Mc, and the anisotropy increases with increase of the energy of the absorbed γ-quantum. Such a picture agrees qualitatively with the experimental data[9-11].

In conclusion, the writer regards it as his pleasant duty to express his gratitude to M. A. Markov for suggesting this problem and for valuable discussions.

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The Production of Charged Mesons by the Bombardment of Beryllium and Carbon with 660 MEV Protons

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The energy spectra of positive and negative pions released in p + Be and p + C collisions was measured with a magnetic spectrometer at an angle 24° to a 660 mev proton beam. The π^+—meson spectrum has a clearly defined maximum at an energy of about 210 mev in the laboratory system, whereas the spectrum for the π^-—mesons varied only slightly over a range from 60 to 250 mev. The probability of positive pion formation when protons collide with protons bound in Be and C nuclei was discovered to be at least three times less than where protons act on free protons. The maximum of the π^+—meson spectrum in the center-of-mass coordinate system is situated near 100 mev. The ratio of positive to negative pion emission was determined for Be and C over the whole spectral range. The ratio of total emission of positive to negative pions for these two elements is equal, respectively, to 5.3 ±0.6 and 7.0 ±0.8.

1. INTRODUCTION

THE present article is devoted to research on the energy spectra of positive and negative pions released during the bombardment of beryllium and carbon with protons whose energy was sufficient to excite one of the colliding nucleons to the state with an angular momentum of 3/2 and an isotopic spin of 3/2 (i.e., a $P_{3/2,3/2}$ state). The proton energy was not so high that the process of forming two pions in a single collision occurred to any appreciable degree.

2. THE EXPERIMENTAL PROCEDURE

A magnetic spectrometer was used to obtain the energy distributions of the pions. Those pions which are emitted at an angle 24° to the proton beam pass through the spectrometer and are recorded with a telescope of three scintillation counters. Information on the proton beam and magnetic spectrometer was presented in a previous paper[1]. The method for determining the contamination of the pion beam by μ—mesons and electrons upon exit from the spectrometer was described in the same paper. The influence of pion absorption in the target, as well as in the crystals and filters, was evaluated in the light of current data on total cross sections of pion-nucleus interaction[2-5]. For the