

for  $\rho > 2\rho_k$   $(\partial\varphi/\partial T)_\rho > 0$ ,

for  $\rho < 2\rho_k$   $(\partial\varphi/\partial T)_\rho < 0$ .

It may be suggested that the change in the character of the temperature dependence of the viscosity of nitrogen at a density near  $2\rho$  is connected with a change in the mechanism of the viscosity. For  $\rho > 2\rho_c$  the mechanism (liquid-like) prevails, while a different (gas-like) mechanism prevails for  $\rho < 2\rho_c$ .

In conclusion, I regard it as a pleasant duty to

thank B. I. Verkin and I. S. Rudenko for directing this work.

<sup>1</sup> B. I. Verkin and I. S. Rudenko, J. Exptl. Theoret. Phys. (U.S.S.R.) 20, 523 (1950).

<sup>2</sup> A. Michels and R. O. Gibson, Proc. Roy. Soc. (London) A134, 288 (1931).

<sup>3</sup> N. F. Golubev and V. A. Petrov, *Nitrogen Handbook*, ONTIKhL, 1944.

<sup>4</sup> N. S. Rudenko, J. Exptl. Theoret. Phys. (U.S.S.R.) 18, 1123 (1948).

<sup>5</sup> I. I. Frenkel, *Kinetic Theory of Liquids*.

Translated by S. D. Elliott

2

## Production of Positive Pions in ( $p-p$ ) Collisions at 660 Mev

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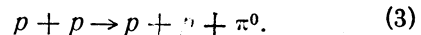
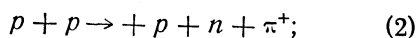
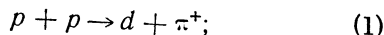
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J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 178-187 (August, 1956)

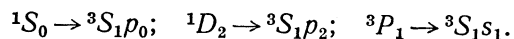
Using nuclear emulsions the energy spectra of positive pions produced in the reaction  $p + p \rightarrow \pi^+ + \{ \begin{smallmatrix} d \\ p+n \end{smallmatrix} \}$  at an energy of  $657 \pm 8$  mev have been measured at angles,  $\theta_\pi$  equal to  $60^\circ, 75^\circ, 90^\circ, 105^\circ$  and  $120^\circ$  in the laboratory system. The angular dependence of the cross section  $d\sigma/d\omega(\theta^*)$  in the center-of-mass system of the colliding nucleons and the total cross section for the reactions:  $p + p \rightarrow \pi^+ + \{ \begin{smallmatrix} d \\ p+n \end{smallmatrix} \}$  and  $p + p \rightarrow p + n + \pi^+$  has been determined. The results are consistent with the assumption that the mesons are produced chiefly in  $p$ -states. In addition to transitions to the  $S$ -state of the final ( $n-p$ ) system, it is found that an important contribution is made by transitions in which the final state is a  $P$ -state.

### 1. INTRODUCTION

IN ( $p-p$ ) collisions at an incident energy of 660 mev there is a strong probability for meson production in addition to elastic scattering. Experiments in which elastic proton-proton scattering has been studied<sup>1</sup> and measurements of the total cross section for ( $p-p$ ) interactions<sup>2</sup> indicate that in this energy region the probability for meson production is only slightly smaller than that for elastic scattering. Meson production can occur via the three following reactions:



Experimental studies in which (1) has been investigated<sup>1</sup> yield an angular dependence\* of the cross section in the form  $A + B \cos^2 \theta^*$  which remains substantially constant over the energy range  $E_p = 460-660$  mev<sup>3,4</sup>. Possible transitions to the  $^3S_1$  state of the final ( $n-p$ ) system are



(Here we are using the nomenclature given by Rosenfeld<sup>5</sup> in designating the final state of the system and it is assumed that the mesons are emitted only in  $s$ -states and  $p$ -states.) Analysis

\* The asterisk denotes quantities measured in the center-of-mass system (c.m.) of the colliding nucleons.

of the angular distribution and the energy dependence of the meson yield shows that these are consistent with the assumption that in (1) the mesons are produced chiefly in the  $p$ -state and that the transition of most importance is  $^1D_2 \rightarrow ^3S_1 P_2$ .

In (2), at an incident proton energy  $E_p = 660$  mev, in addition to those indicated above, transitions to the  $P$ -state of the final nucleon system are energetically possible. The angular and energy distributions associated with the mesons will be determined to a considerable extent by the relative intensities of the corresponding transitions.

It is well known that the probability for neutral pion production in  $(p-p)$  collisions is small close to threshold; at 460 mev the cross section  $\sigma(p+p \rightarrow \pi^0)$  is as much as 3-4 times smaller than the cross section  $\sigma(p+p \rightarrow d+\pi^+)$ . However, as the energy of the incident proton is decreased<sup>6,7</sup>, a sharp rise is observed in the quantity  $\sigma(p+p \rightarrow \pi^0)$ . The extremely small cross section for the production of neutral pions close to threshold has been explained by the fact that the production of a pseudoscalar meson in an  $Sp$ -state is strictly forbidden. At the higher energies the  $Pp$ -state becomes available and this restriction no longer holds.

Thus, in meson-production, processes in which the relative nucleon energy is small at the end of the reaction the nature of the reaction is determined to a large extent by the nucleon final states. The present paper is devoted to an experimental investigation, using nuclear emulsions, in which the energy spectra and angular distributions for positive pions produced in reactions (1) and (2) were examined.

## 2. EXPERIMENTAL METHOD

This work on the production of mesons in hydrogen by protons was carried out at the synchrotron of the Institute for Nuclear Problems, Academy of Sciences, USSR. Because of the high intensity of the external proton beam it was possible to make the measurements behind a four-meter concrete shield at a considerable distance from the machine; this arrangement resulted in extremely favorable background conditions. The proton beam was passed through a three-meter steel collimator by means of which a beam 120 mm in diameter was defined. The proton energy was  $657 \pm 8$  mev.<sup>8</sup> A container filled with liquid hydrogen was placed in the beam. The diameter of the liquid hydrogen container was 12 cm. The positive pions produced in the  $(p-p)$  collisions were emitted from

the target through the glass walls of the container, which were 2 mm thick. Using a system of slit-collimators 20 cm long and  $1 \times 3$  cm in cross section cut into the copper block, it was possible to select mesons emitted from given points of the target at angles,  $\theta_\pi$ ,  $60^\circ$ ,  $75^\circ$ ,  $90^\circ$ ,  $105^\circ$ , and  $120^\circ$  with respect to the beam; these mesons were recorded in photographic emulsions. The detector angular resolution was  $1.5^\circ$ . The experimental arrangement is shown in Fig. 1. The emulsions were disposed in a copper block at an angle of  $10^\circ$  with respect to the incident mesons. The meson energy was determined by the range in the emulsion. The path length in the copper block was measured from the end of the meson track. The developed emulsions were scanned under a microscope with a magnification of 450 x. The positive pions stopped in the emulsion were identified by the character of the  $\pi \rightarrow \mu$  decay. The background was determined by exposing the plates under these same geometrical conditions.

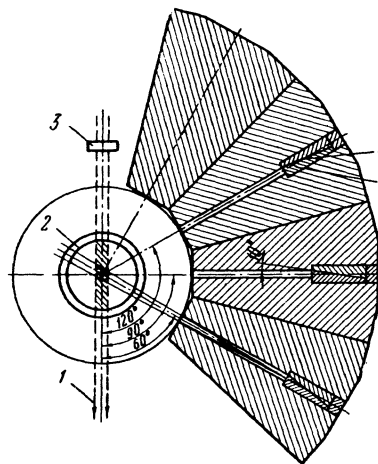


FIG. 1. Experimental set-up: 1 - proton beam, 2 - container with liquid hydrogen, 3 - monitor, 4 - photographic emulsion, 5 - copper absorber.

In plotting the positive pion spectra approximately 2000 ( $\pi \rightarrow \mu$ ) decays were studied at each angle.

## 3. DETERMINATION OF THE CROSS SECTION

The differential cross section for pion production is given by the expression

$$\frac{d^2\sigma}{d\omega dE} = \frac{N}{nN_p t [dE/dR]_E} \frac{r^2}{S} \frac{\eta_1 \eta_2}{\eta_3},$$

where  $N_p$  is the number of protons which traverse the target, measured in the direction of the incident beam;  $N$  is the number of pions found in an area of the emulsion  $S$ ;  $r$  is the distance from the target to the emulsion surface;  $t$  is the thickness of the emulsion;  $[dE/dR]_E$  is the energy lost in the emulsion by mesons having an energy  $E$ ;  $\eta_1$  is a factor which takes into account mesons which are deflected from the beams by nuclear reactions;  $\eta_2$  is a factor which denotes the mesons undergoing  $\pi \rightarrow \mu$  decay before stopping in the emulsion;  $\eta_3$  is the scanning efficiency for  $\pi - \mu$  decay.

The proton number  $N_p$  was determined within an accuracy of 3 percent by means of a calibrated ionization chamber. The meson energy loss was determined from the expression<sup>9</sup>

$$[dE/dR]_E = 0.067R_\pi^{0.419},$$

which is derived from the range-energy relation for protons. The thickness of the emulsion

prior to development,  $t^*$ , was determined within an accuracy of 6 percent. In calculating the factor  $\eta_1$  the mean path-length for mesons in copper was taken to be  $104.4 \text{ gm/cm}^2$  and its value varied within the limits 1.02 to 2.35 for meson energies ranging from 30 meV to 175 meV. The correction for the decay of positive pions in flight was small; the factor  $\eta_2$  varied within the limits 1.04 and 1.10. The scanning efficiency was estimated by repeated scannings and found to be  $\sim 0.92$ .

#### 4. EXPERIMENTAL RESULTS

In Fig. 2 are shown the meson energy distributions at angles of  $60^\circ$ ,  $75^\circ$ ,  $90^\circ$ ,  $105^\circ$ ,  $120^\circ$  in the laboratory system (l.s.). For all angles, with the exception of  $60^\circ$ , the energy distributions have clearly defined maxima in the hard part of the spectrum. The locations of these maxima correspond to values obtained from kinematic considerations in the reaction  $p + p \rightarrow d + \pi^+$ .

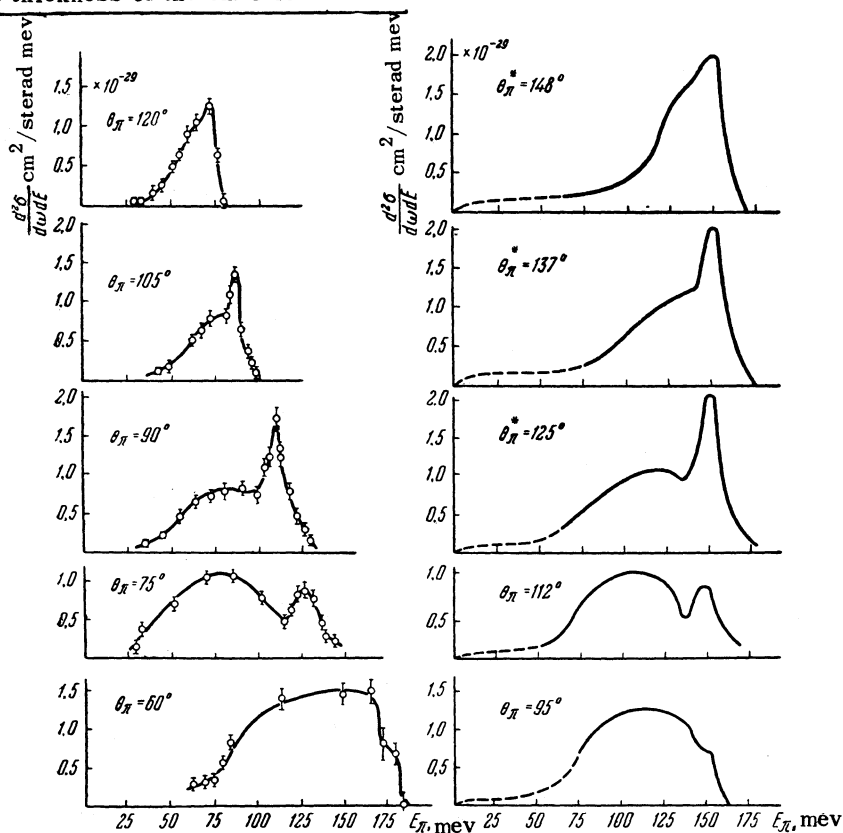


FIG. 2. Energy distributions for positive pions produced by 657 mev protons in hydrogen;  $\theta_\pi$  refers to the laboratory system,  $\theta_\pi^*$  refers to the c.m. system.

In these cases the spread about a monoenergetic line appears as a result of the following: a) limited angular resolution of the detector, b) energy spread in the proton beam, c) multiple scattering of mesons, d) spread due to fluctuations in the ionization losses. The calculated values for the energy spread are in agreement with the experimental results.

The smooth spectrum in the region of lower energy is attributed to mesons produced in the reaction

$p + p \rightarrow p + n + \pi^+$ . It was found possible at angles of  $75^\circ$ ,  $90^\circ$ ,  $105^\circ$  and  $120^\circ$  (in the laboratory system) to separate out the contribution due to mesons produced in the reaction  $p + p \rightarrow d + \pi^+$ ; the remainder is taken to be the energy distribution for the mesons from the reaction  $p + p \rightarrow p + n + \pi^+$ .

TABLE I

$\frac{d\sigma}{d\omega} \cdot 10^{28}$ in $\text{cm}^2/\text{sterad}$ for the reaction	$\theta$				
	60	75	90	105	120
$pp \rightarrow \pi^+ + \begin{cases} d \\ pn \end{cases}$	$14.6 \pm 1.6$	$8.9 \pm 0.9$	$6.4 \pm 0.7$	$3.5 \pm 0.5$	$3.2 \pm 0.4$
$pp \rightarrow d\pi^+$	$1.4 \pm 0.1^*$	$1.7 \pm 0.4$	$1.8 \pm 0.3$	$1.4 \pm 0.3$	$1.2 \pm 0.2$
$pp \rightarrow pn\pi^+$	$13.2 \pm 1.6$	$7.2 \pm 1.3$	$4.6 \pm 1.0$	$2.1 \pm 0.8$	$2.0 \pm 0.6$
$\frac{(d\sigma/d\omega)_{pp \rightarrow pn\pi^+}}{(d\sigma/d\omega)_{pp \rightarrow d\pi^+}}$	9.4	4.2	2.6	1.5	1.7

\* The value  $(d\sigma/d\omega)_{pp \rightarrow d\pi^+}^{60^\circ} = (1.4 \pm 0.1) \cdot 10^{-28} \text{ cm}^2/\text{sterad}$  is taken from Ref. 4.

In Table 1 are shown the cross sections for positive pion production at the indicated angles in the laboratory system. At  $60^\circ$ , because of the excessive energy spread, it was not possible to distinguish the contribution due to the reaction  $p + p \rightarrow d + \pi^+$  and in the present experiment it was possible to measure only the combined cross section  $d\sigma/d\omega (p + p \rightarrow \pi^+ + \begin{cases} d \\ p+n \end{cases})$  at this angle. Combining the magnitude of the cross section  $d\sigma/d\omega (p + p \rightarrow \pi^+ + \begin{cases} d \\ p+n \end{cases})$  at  $60^\circ$  with the results of Ref. 4 for reaction (1) we find that the magnitude of the cross section  $d\sigma/d\omega (p + p \rightarrow d + \pi^+)$  is approximately 10 percent of the cross section  $d\sigma/d\omega \times (p + p \rightarrow \pi^+ + \begin{cases} d \\ p+n \end{cases})$ . Hence, it may be assumed that the spectrum measured at  $60^\circ$  is determined chiefly by mesons produced in reaction (2).

##### 5. TRANSFORMATION OF THE ENERGY DISTRIBUTIONS FROM THE LABORATORY SYSTEM TO THE CENTER-OF-MASS SYSTEM

In order to make a theoretical interpretation of these results it is necessary to find the meson

energy distribution and the cross section in the center-of-mass system (c. m. s.) of the colliding nucleons.

In converting the cross section from the laboratory system to the cm system use is made of the relation

$$\frac{d^2\sigma^*}{d\omega^* dE^*} = \frac{P^*}{P} \frac{d^2\sigma}{d\omega dE},$$

where  $d^2\sigma^*/d\omega^* dE^*$ ;  $P^*$  and  $d^2\sigma/d\omega dE$ ;  $P$  are the differential cross sections and the meson momenta in the c. m. system and laboratory system respectively. The ratio  $P^*/P$  is determined from the formula

$$E = E^*\gamma_c + P^*\beta_c\gamma_c \cos\theta^*,$$

where  $\gamma_c = 1/(1 - \beta_c^2)^{1/2}$  and  $\cos\theta^*$  are determined from the following relations<sup>10</sup>

$$\cos\theta^* = \frac{1}{1 - \beta_c^2 \cos^2\theta} \left\{ -\gamma_\pi \sin^2\theta + (1 - \beta_c^2) \times \left( 1 - \frac{\gamma_\pi^2 - \gamma_c^2}{1 - \beta_c^2} \sin^2\theta \right)^{1/2} \cos\theta \right\}.$$

In Fig. 3a is shown the ratio  $D^*/D$  as a function of meson energy at angles  $\theta$  of  $60^\circ$ ,  $75^\circ$ ,  $90^\circ$ ,  $105^\circ$  and  $120^\circ$  for  $E_p = 657$  mev.

The expression given for  $\cos \theta^*$  indicates that mesons emitted at a given angle, which have different energies, correspond to mesons emitted at different angles. In Fig. 3b is shown the dependence of the meson emission angle  $\theta^*$  on the kinetic energy  $E^*$  for the experimental angles in the laboratory system. It is apparent from this curve that mesons with a kinetic energy  $E^*$  in the region from 30 to 150 mev and emitted, for example, at an angle  $\theta = 90^\circ$ , correspond to mesons emitted in the region  $125^\circ < \theta^* < 135^\circ$  in the center-of-mass system. Thus the angular spread in the c. m. system will cause a substantial distortion of the results in the conversion relation unless the mesons are concentrated in a narrow energy interval or the angular dependence of  $d^2\sigma/d\omega^*dE^*$  is close to isotropic. Since the mesons have a broad energy distribution and the angular dependence at different energies is not known beforehand, the situation noted above had to be taken into account in transforming the energy distributions into the center-of-mass system. For this purpose the values  $d^2\sigma/d\omega dE$  multiplied by the appropriate factor  $P^*/P$  are plotted in Fig. 3b. The

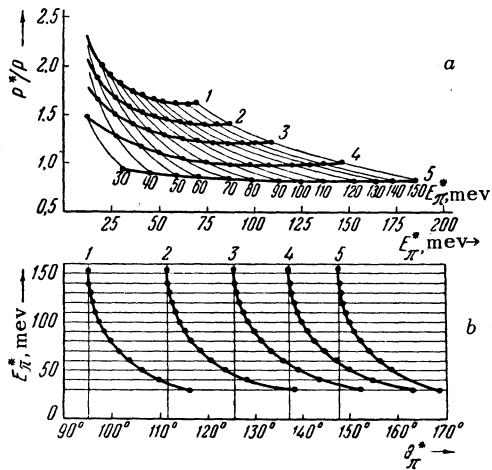


FIG. 3. a—the ratio  $P^*/P$  as a function of meson energy for the following values of  $\theta$ : 1— $120^\circ$ , 2— $105^\circ$ , 3— $90^\circ$ , 4— $75^\circ$  and 5— $60^\circ$ ; b—dependence of the meson emission angle  $\theta_\pi^*$  on kinetic energy  $E_\pi^*$  for the same angles in the laboratory system.

points corresponding to the same value of meson kinetic energy  $E^*$  are connected by the smooth curves over the entire range of  $\theta^*$ . In plotting

the spectra in the center-of-mass system the values used were for  $d\sigma^*/d\omega^*dE^*$  at angles  $\theta^*$  of  $5^\circ$ ,  $112^\circ$ ,  $125^\circ$ ,  $137^\circ$ , and  $148^\circ$ . These angles correspond to mesons produced in the reaction  $p + p \rightarrow d + \pi^+$  with kinetic energies  $E^* = 149$  mev and emitted at corresponding angles  $\theta$  of  $60^\circ$ ,  $75^\circ$ ,  $90^\circ$ ,  $105^\circ$  and  $120^\circ$  in the laboratory system.

## 6. MESON ANGULAR DISTRIBUTIONS IN THE CENTER-OF-MASS SYSTEM

In Figs. 2 and 4 are shown energy spectra for mesons produced in reactions (1) and (2) for incident proton energies  $E_p = 657$  mev at the c.m. angles indicated above. In Table II are shown the cross sections for positive pion production, obtained by integrating the spectra, and the ratio for positive-pion yield produced in reactions (1) and (2). The total pion-production cross sections and the ratio of the total cross sections for reactions (1) and (2) are shown at the top.

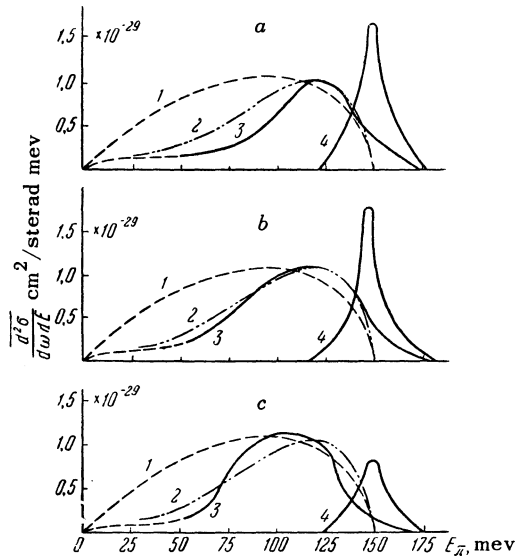


FIG. 4. Energy distributions for positive pions produced in  $(p-p)$  collisions at proton energy  $E_p = 657$  mev. a— $\theta_\pi^* = 137^\circ$ ; b— $\theta_\pi^* = 125^\circ$ ; c— $\theta_\pi^* = 112^\circ$ . Curve 1 is a theoretical curve taken from Ref. 12; curve 2 is a theoretical curve taken from Ref. 13; curve 3 is the experimental curve for the reaction  $p + p \rightarrow d + \pi^+$ .

TABLE II

$\frac{d\sigma^*}{d\omega^*} \cdot 10^{28}$ in $\text{cm}^2/\text{sterad}$ for the reaction	$\theta^*$ in degrees				
	95	112	125	137	148
$pp \rightarrow \pi^+ + \begin{cases} d & \dots \\ pn & \dots \end{cases}$	$10.5 \pm 1.3$	$9.7 \pm 1.1$	$11.3 \pm 1.2$	$10.7 \pm 1.6$	$11.0 \pm 1.3$
$pp \rightarrow d\pi^+$	$1.0 \pm 0.07$	$1.9 \pm 0.5$	$3.0 \pm 0.5$	$3.5 \pm 0.7$	$4.2 \pm 0.6$
$pp \rightarrow pn\pi^+$	$9.5 \pm 1.3$	$7.8 \pm 1.2$	$8.3 \pm 1.3$	$7.2 \pm 1.7$	$6.8 \pm 1.4$
$\frac{(d\sigma^*/d\omega^*)_{pp \rightarrow pn\pi^+}}{(d\sigma^*/d\omega^*)_{pp \rightarrow d\pi^+}}$	9.5	4.1	2.8	2.1	1.6

The data obtained in the present work indicates that the angular distribution of positive pions produced in ( $p$ - $p$ ) collisions at a proton energy  $E_p = 657$  mev is virtually isotropic in the angular region from  $95^\circ$  to  $148^\circ$  in the c.m. system. The dependence of  $d\sigma^*/d\omega^*(p+p \rightarrow \pi^+ + \begin{smallmatrix} d \\ p+n \end{smallmatrix})$  on the function  $\cos^2\theta^*$  is shown in Fig. 5. Expressing the experimental results in the form of an empirical relation of the form  $A(1 + B \cos^2\theta^*)$ , using a least-squares fit, we find the angular distribution is given by the expression

$$\begin{aligned} (d\sigma^*/d\omega^*)_{pp \rightarrow \pi^+ + \begin{cases} d \\ pn \end{cases}} = & (10.3 \\ & \pm 1.8) [1 + (0.1 \pm 0.2) \cos^2\theta^*] \\ & \times 10^{-28} \text{ cm}^2/\text{sterad}. \end{aligned}$$

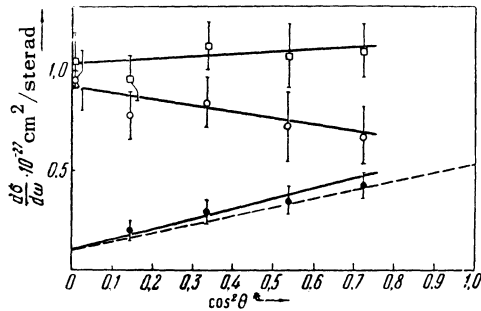


FIG. 5. Angular distributions for positive pions produced in the following reactions:  $-p + p \rightarrow \pi^+ + \begin{cases} d \\ n+p \end{cases}$ ;  $\bullet - p + p \rightarrow d + \pi^+$ .

Using a least-squares fit for mesons produced in the reaction  $p + p \rightarrow d + \pi^+$ , the following angular dependence is found:  $d\sigma^*/d\omega^*(p + p \rightarrow d + \pi^+) = (4.7 \pm 1.3)[(0.23 \pm 0.13) + \cos^2\theta^*] \cdot 10^{-28} \text{ cm}^2/\text{sterad}$  which is in agreement, within experimental errors, with the results in Ref. 4 (the dotted line in Fig. 5 indicates the angular distribution for mesons from the reaction  $p + p \rightarrow d + \pi^+$ , as measured in Ref. 4).

Subtracting the mesons due to reaction (1) from the above quantity we obtain the meson angular distribution due to reaction (2)

$$\begin{aligned} (d\sigma^*/d\omega^*)_{pp \rightarrow pn\pi^+} \\ = (9.2 \pm 1.8) [1 - (0.4 \pm 0.3) \cos^2\theta^*] \\ \times 10^{-28} \text{ cm}^2/\text{sterad} \end{aligned}$$

In interpreting the results we shall consider the reaction  $p + p \rightarrow p + n + \pi^+$  in detail. In the following we will assume that the important role is played by  $s$  and  $p$  meson states with respect to the two-nucleon system. Moreover, we will assume that at the energies being considered here both  $S$  and  $P$  nucleon states are involved at the end of the reaction. In this case the following are possible as final states for the system consisting of the two nucleons and the pion:

- 1)  $^1S_0$ ;  $^3S_1$  ( $Ss$ );
- 2)  $^1S_0$ ;  $^3S_1$ ;  $^5S_2$  ( $Ps$ );
- 3)  $^1P_1$ ;  $^3P_{0,1,2}$  ( $Sp$ );
- 4)  $^1P_1$ ;  $^3P_{0,1,2}$ ;  $^5P_{1,2,3}$  ( $Pp$ ).

Here states 1) and 2) correspond to pions with orbital momentum  $l = 0, 3$  and 4) orbital momentum  $l = 1$ . The final nucleon and meson states (in the nomenclature of Ref. 5) are shown in brackets.

According to Ref. 11, if a polarized proton beam is incident on an unpolarized target, the initial state of the system is described by the wave functions

$$\begin{aligned} \psi_1 = & q_1 \chi_{11} (\varphi_{10} + \varphi_{30} + \dots) \\ & + (q_2 / \sqrt{2}) \chi_{10} (\varphi_{10} + \varphi_{30} + \dots) \\ & + (q_2 / \sqrt{2}) \chi_{00} (\varphi_{00} + \varphi_{20} + \dots); \\ \psi'_1 = & q_2 \chi_{1-1} (\varphi_{10} + \varphi_{30} + \dots) \\ & + (q_1 / \sqrt{2}) \chi_{10} (\varphi_{10} + \varphi_{30} + \dots) \\ & - (q_1 / \sqrt{2}) \chi_{00} (\varphi_{00} + \varphi_{20} + \dots), \end{aligned}$$

where  $q_1 = \cos(\vartheta/2) \exp(-i\delta/2)$ ,  $q_2 = \sin(\vartheta/2) \exp(i\delta/2)$  ( $\vartheta, \delta$  are the polarization angles);  $\chi_{s\mu}$  is the spin wave function and  $\varphi_{lm}$  is the spherical function. Combining the orbital and spin moments, we obtain the initial states for the system

$$\begin{aligned} {}^3P_{1,2}; \quad {}^3F_{2,3} & \quad (M = \pm 1); \\ {}^3P_{0,2}; \quad {}^3F_2 & \quad (M = 0); \\ {}^1S_0; \quad {}^1D_2 & \quad (M = 0). \end{aligned}$$

Here we have given only those states of the initial system which can feed the final states being considered. The possible transitions in the reaction  $p + p \rightarrow p + n + \pi^+$  are shown in Table III.

The amplitude of the final state is given by the expression

$$f = \sum A_{lsjM} I_{lsjM},$$

TABLE III

Type	Initial state of the system	Final state of the system	Transition	Projection of total momentum
<i>Sp</i>	${}^1S_0$	${}^3S_1 p_0$	${}^1S_0 \rightarrow {}^3P_0$	0
<i>Sp</i>	${}^1D_2$	${}^3S_1 p_2$	${}^1D_2 \rightarrow {}^3P_2$	0
<i>Ss</i>	${}^3P_1$	${}^3S_1 s_1$	${}^3P_1 \rightarrow {}^3S_1$	$\pm 1$
<i>Ss</i>	${}^3P_0$	${}^1S_0 s_0$	${}^3P_0 \rightarrow {}^1S_0$	0
<i>Ps</i>	${}^1S_0$	${}^3P_0 s_0$	${}^1S_0 \rightarrow {}^1S_0$	0
<i>Ps</i>	${}^1D_2$	${}^3P_2 s_2$	${}^1D_2 \rightarrow {}^5S_2$	0
<i>Pp</i>	${}^3P_0$	${}^3P_1 p_0$	${}^3P_0 \rightarrow {}^3P_0$	0
<i>Pp</i>	${}^3P_1$	${}^3P_1 p_1$	${}^3P_1 \rightarrow {}^3P_1$	$\pm 1$
<i>Pp</i>	${}^3P_1$	${}^3P_0 p_1$	${}^3P_1 \rightarrow {}^1P_1$	$\pm 1$
<i>Pp</i>	${}^3P_1$	${}^3P_2 p_1$	${}^3P_1 \rightarrow {}^5P_1$	$\pm 1$
<i>Pp</i>	${}^3P_2$	${}^3P_1 p_2$	${}^3P_2 \rightarrow {}^3P_2$	$\pm 1; 0$
<i>Pp</i>	${}^3P_2$	${}^3P_2 p_2$	${}^3P_2 \rightarrow {}^5P_2$	$\pm 1; 0$
<i>Pp</i>	${}^3F_2$	${}^3P_1 p_2$	${}^3F_2 \rightarrow {}^3P_2$	$\pm 1; 0$
<i>Pp</i>	${}^3F_2$	${}^3P_2 p_2$	${}^3F_2 \rightarrow {}^5P_2$	$\pm 1; 0$
<i>Pp</i>	${}^3F_3$	${}^3P_2 p_3$	${}^3F_3 \rightarrow {}^5P_3$	$\pm 1$

where the summation is carried out over all final states being considered, ( $A_{lsjM}$  is the transition probability,  $I_{lsjM}$  are the spherical functions).

The differential cross section  $d\sigma^*/d\omega^*(\theta^*)$  is expressed in terms of the amplitude of the final state by

$$d\sigma^*/d\omega^* = f^+ f. \quad (I)$$

Substituting the expression for  $f$  in Eq. (1) we obtain the meson angular distribution

$$d\sigma^*/d\omega^* = a' + b' \cos^2 \theta^* + c' \sin^2 \theta^*,$$

where  $a'$ ,  $b'$  and  $c'$  are expressed in terms of  $A_{lsjM}$  or

$$d\sigma^*/d\omega^* = a + b \cos^2 \theta^*,$$

where  $a = a' + c'$ ,  $b = b' - c'$ .

We may note that terms containing  $\sin^2 \theta^*$  in the angular distribution arise from transitions of the  $P_p$  class and from interference between the transitions  ${}^1S_0 \rightarrow {}^3S_1 p_0$  and  ${}^1D_2 \rightarrow {}^3S_1 p_2$ . The meson angular distribution in reaction (1) is almost the same as the theoretical distribution for the transition  ${}^1D_2 \rightarrow {}^3S_1 p_2$  and indicates that the contribution of

terms proportional to  $\sin^2 \theta^*$  due to interference of the indicated transitions, is small. It is also possible that this applies in reaction (2) since at an energy  $E_p = 440$  mev<sup>12</sup> there was no noticeable change in the angular distribution with regard to an increase in the constant term for reactions (1) and (2) as compared with reaction (1).

Thus we see that the angular distributions for the reactions  $p + p \rightarrow d + \pi^+$  ( $Sp$  and  $Ss$  transitions) and  $p + p \rightarrow p + n + \pi^+$  ( $Sp$ ;  $Ss$ ;  $Pp$  and  $Ps$  transitions) can be given in the form  $a + b \cos^2 \theta$ ; however, if transitions of the  $Pp$  type participate in the meson production process and their contribution cannot be neglected, the coefficient  $b = b' - c'$  in reaction (2) may be smaller. Under these conditions the value of the coefficient  $a$  in the same reactions may become larger; this increase, however, may be due to other causes as well as  $Pp$  transitions.

The angular distributions for positive pions produced in reactions (1) and (2), measured in the present work, indicate that the coefficients  $b'$  and  $c'$  are approximately equal; hence, it would seem that a considerable contribution is made by transitions of the  $Pp$  type in reaction (2) for proton energies of 657 mev. This effect is also found in the angular distributions in the reaction  $p + p \rightarrow p + n + \pi^+$  in which the coefficient  $b$  becomes negative.

## 7. ANALYSIS OF THE POSITIVE PION PRODUCTION IN THE REACTION $p + p \rightarrow p + n + \pi^+$

From kinematic considerations it follows that mesons produced in the reaction  $p + p \rightarrow p + n + \pi^+$  have a continuous distribution in energy over the range from zero to 150 mev in the center-of-mass system. The experimental energy spectra, however (cf. Figs. 2 and 4) extend to an energy of 175 mev; this may be explained by errors in the detector (some mesons may traverse only part of the path in the emulsion or in the wall). The energy distributions measured at angles of  $95^\circ$ ,  $112^\circ$  and

$125^\circ$  would seem to indicate the meson spectra associated with reaction (2) have maxima in the region 100-125 mev. An analysis of the energy distributions in the measured regions of angle leads one to believe that mesons emitted at angles close to  $\theta^* = 180^\circ$  are concentrated in the higher energy region. Because of this deformation in the spectra, the angular distributions for different energy integrals have the form

$$E_\pi = 75 \div 100 \text{ mev}, \quad d\sigma^*/d\omega^* = 1 - \cos^2 \theta^*;$$

$$E_\pi = 125 \div 150 \text{ .mev}, \quad d\sigma^*/d\omega^* = 0.2 + \cos^2 \theta^*.$$

This result is in agreement with the assumption of important contributions due to transitions of the  $Pp$  type in the production of positive pions at high energies. As a matter of fact, it was indicated above that  $Pp$  transitions tend to reduce the coefficient  $b$  in the expression for the angular dependence of the meson yield and to increase the constant term. This effect should be more noticeable for low meson energies which leave a considerable energy in the relative motion of the nucleons. In the region of high meson energy,  $Pp$  transitions should play a diminishing role and the angular dependence in the meson yield should be determined chiefly by  $Sp$  transitions; hence, in this energy region we should have a more intense production of mesons at angles close to  $180^\circ$  as compared with angles around  $90^\circ$ .

Thus the shape of the positive-pion energy distributions at different angles in the center-of-mass system as well as the angular distributions leads us to the conclusion that transitions of the  $Pp$  type make a large contribution to the cross section for meson production at  $E_p = 657$  mev.

For comparison with the experimental results, Fig. 4 shows theoretical curves which describe the meson spectrum in reaction (2). Curves 1 and 2 are taken from Ref. 13 and correspond to the case  $H \sim \text{const}$  and  $H \sim P^*$ , where  $H$  is the matrix element for the interaction. The statistical weight was computed for the case in which all particles appearing in the final state are relativistic. Comparison with the experimental data indicates that a better approximation results if the assumption is made that the matrix element depends linearly on meson momentum (curve 2). The dependence indicated for the matrix element may indicate that in the reaction being considered mesons are produced with an angular momentum  $l = 1$ . Better agreement for the theoretical curve 2, with the experimental data was observed at an angle  $\theta^* = 125^\circ$ . At other angles there was a discrepancy which was



attributed to the deformation of the positive pion spectrum with changing angle. The change in the shape of the spectrum, as was indicated above, occurs because in the final state the nucleons can have an angular momentum  $l = 0$  or  $l = q$  and the relative contributions of the associated transitions are different at different angles. Consequently, one expects that satisfactory agreement between theory and experimental results can be found at all angles only when account is taken of the nucleon interaction in the final states.

### 8. TOTAL CROSS SECTION FOR POSITIVE-PION PRODUCTION IN $(p - p)$ COLLISIONS

The total cross section for positive pion production in  $(p - p)$  collisions at an energy of 675 mev is

$$\sigma_{pp \rightarrow \pi^+ + \int_{pn}^d} = (13.4 \pm 2.2) \cdot 10^{-27} \text{ cm}^2.$$

It is apparent that among the inelastic processes which occur in proton collisions at energies of 660 mev, an important part (80 percent) is played by the production of positive pions. Using the value

$\sigma_{pp}^{\text{total}} = (41.4 \pm 0.6) \text{ mb}$  taken from Ref. 2 and  $\sigma_{pp}^{\text{elas}} = (24.7 \pm 1.2) \text{ mb}$  from Ref. 14, we have for  $\sigma_{pp}^{\text{inelas}}$  the value  $(16.7 \pm 1.3) \text{ mb}$ . Hence, it may be considered that  $\sigma(p + p \rightarrow p + \pi^0) = (3.3 \pm 2.5) \text{ mb}$  which is in agreement with the value for this quantity as measured in Ref. 6.

The value  $\sigma(p + p \rightarrow p + n + \pi^+) = (10.1 \pm 2.3) \text{ mb}$  is corroborated by Refs. 1 and 2. For the ration of the cross sections of these reactions we find

$$\sigma_{pp \rightarrow pn\pi^+}^* / \sigma_{pp \rightarrow d\pi^+}^* = 3.1.$$

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