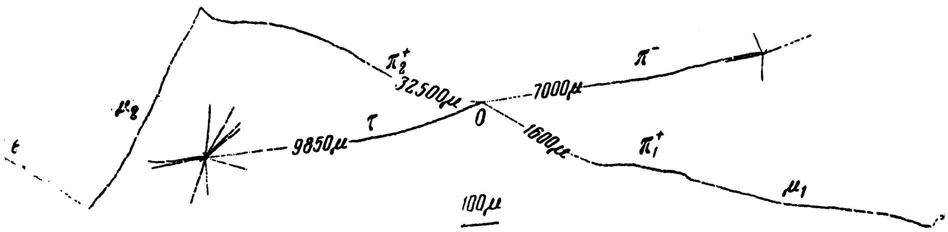

charged, the third, negatively. The tracks of the π-mesons lay in a plane to an accuracy of 2°. The decay energy was* $Q = (74.1 \pm 1.4)$ mev and the mass, determined from the decay scheme, $m_\tau = (964 \pm 3) m_e$. The first τ-meson came out at a very large angle to the plane of the emulsions, which made a direct measurement of its mass impossible. The track of the second τ-meson was gently sloping. A direct determination of its mass by multiple scattering and range led to the value $m_\tau = (1097 \pm 105) m_e$.

The comparatively small energy of one of the π^+-mesons produced in the decay of the second τ-meson stands out. Such a case supports, as is well known (see Ref. 1), the hypothesis that the τ-mesons and χ-mesons are different particles, and not different modes of decay of a single particle. Such a conclusion possible follows from the analysis of nuclear disintegrations in which heavy unstable particles are produced[2].

* In determination of the decay energies of both τ-mesons, several differences in the stopping power of emulsions of type R and type G-5 were taken into account. In order to carry out the corresponding corrections, we measured the track lengths of 110μ-mesons, produced in the decay of π-mesons which stopped. The mean length of track of these μ-mesons was $(584 \pm 2.4)\mu$, which differs by 2% from the value in G-5 emulsions.

[1] R. H. Dalitz, Phys. Rev. **94**, 1046 (1954).

[2] Gramenitskii, Zamchalova, Podgoretskii, Tret'iakova and Shcherbakova, J. Exptl. Theoret. Phys. (U.S.S.R.) **28**, 616 (1955); Soviet Phys. JETP **1**, 562 (1955).

Translated by G. E. Brown
199

On Mass Renormalization in the Tamm-Dancoff Method

D. A. KIRZHNITS
P. N. Lebedev Physical Institute, Academy of Sciences, USSR

(Submitted to JETP editor February 6, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 971-973 (May, 1956)

THE present article is concerned with the consistent application of the counterterm technique for mass renormalization in the Tamm-Dancoff method (abbreviated as the TD method). Because of the mathematical difficulties that arise, we also consider a renormalization method which does not require separation of the counterterms.

Mass renormalization in the TD method is usually accomplished by separation of the counterterms in the Hamiltonian[1]

$$H = H_0(M_1, \mu_1) + H' = H_0(M, \mu) + H' + \delta H',$$

$$H' = ig_1\overline{\psi}(x)\gamma_5\tau\varphi(x)\psi(x),$$

$$\delta H' = \delta M\overline{\psi}(x)\psi(x) + \delta\mu^2\varphi(x)\varphi(x),$$

where M_1 and μ_1 are bare masses, and M and μ are renormalized masses. The equations of motion for the amplitude A are then given by the relationship

$$i\delta A/\delta\sigma = [A, H'] + [A, \delta H'].$$

The state vector of the system is resolved into states with different numbers of free particles (but possessing renormalized mass), and all amplitudes with a number of particles smaller than the given number are taken into consideration in each approximation of the method.

I.E. Tamm has noted that there is usually some inconsistency in the manner of application of these amplitude selection rules. These rules are strictly adhered to in relation to terms arising from H' in the equation for A, whereas some of the terms associated with $\delta H'$ are simply dropped despite the fact that they contain terms which should be considered in a given approximation. This procedure corresponds to the Levy-Klein procedure (the retention in the integral equation kernels of only the lowest order terms in g_1^2) but is, strictly speaking, in contradiction with the TD method. Mass renormalization is considered below for the one-nucleon problem in the lowest TD approximation with the inclusion of all essential amplitudes.

Without dwelling on the details of the calculation (see Ref. 1) we shall present the final results (the meson counterterm can be neglected in this approximation):

$$\langle\psi(x)\rangle_t = \int S_R(x-x')\,[ig_1\gamma_5\tau\langle\psi(x')\varphi(x')\rangle_{t'} + \delta M\langle\psi(x')\rangle_{t'}]\,d^4x', \tag{1}$$

$$\langle\psi(x)\varphi(x)\rangle_t = g_1\int\Gamma(x-x')\langle\psi(x')\rangle_{t'}\,d^4x' + \delta M\int S_R(x-x')\langle\psi(x')\varphi(x)\rangle_{t'}\,d^4x'. \tag{2}$$

Here

$$S_R(x) = -\frac{1+\varepsilon(x)}{2}\,S(x),$$

$$\Gamma(x) = \frac{1}{2}\varepsilon(x)\,[S^+(x)\Delta(-x) - S(x)\Delta^+(-x)]\,\gamma_5\tau.$$

In the solution of this type of equation the terms with δM in the equations of intermediate amplitudes [in this case the last term in Eq. (2)] were usually dropped. The term with δM was retained only in the equation of the principal amplitude, where it compensated the divergence.

Retaining both terms with δM in (1) and (2) we first solve (2) with respect to $<\varphi(x)\psi(x)>_t$; this is easily accomplished because of the different kernel contained in (2). Substituting this solution in (1) we obtain

$$\langle\psi(x)\rangle_t = \int S_R(x-x')\left[\int\mathfrak{M}(x'-x'')\langle\psi(x'')\rangle_{t''}\,d^4x'' + \delta M\langle\psi(x')\rangle_{t'}\right]d^4x', \tag{3}$$

where the Fourier representation of the mass operator $\mathfrak{M}$ is

$$\mathfrak{M}(p) = 3g_1^2 i\int d^4k\gamma_5\left(1+\frac{\delta M}{i\gamma k + M - \delta M}\right)[(S^+\varepsilon)_k\Delta_{k-p} - (S\varepsilon)_k\Delta^+_{k-p}]\,\gamma_5. \tag{4}$$

A transition is easily made from (3) to the equation [where $\psi(p)$ is the Fourier transform of $<\psi(x)>_t$]:

$$[i\gamma p + M + \delta M + (2\pi)^4\,\mathfrak{M}(p)]\cdot\psi(p) = 0. \tag{5}$$

We now transform to the system in which the nucleon is at rest ($\mathbf{p} = 0$). For the mass renormalization δM must now be chosen to satisfy the relationship

$$\delta M + (2\pi)^4\,\mathfrak{M}(p_0) = 0 \text{ where } p_0 = -i\gamma_0 M. \tag{6}$$

When the term with δM in (2) and (4) is dropped, this gives the usual result (L is the cutoff momentum):

$$\delta M \sim g_1^2 M\ln(L/M). \tag{7}$$

When all terms with δM are taken into consideration (4) becomes

$$\int_0^L \frac{1+a(\delta M)^3}{k^2+b(\delta M)^4}\,k\,dk,$$

where a and b are constants and δM is given by the expression

$$\delta M \sim \sqrt{g_1^2}\,L. \tag{8}$$

Thus in the simplest case considered above, the inclusion of all essential amplitudes in mass renormalization does not give rise to any additional difficulties. But it does lead to a change in the character of the divergences of the theory; for this reason considerable care must be used in perturbation theory renormalization in the TD method.

When all amplitudes with δM are retained in a more complicated problem such as the scattering of mesons by nucleons the equations of the intermediate amplitudes are very complicated and cannot be solved explicitly.

We shall therefore consider another method of renormalization which is not associated with separation of the counterterms but which is based, instead, on the direct use of the relationships between the bare and renormalized constants (see

Ref. 2). The Hamiltonian remains expressed in terms of the bare masses; the state vector will therefore be resolved into states with different numbers of particles possessing bare masses. For this reason the solution of any problem concerning particle interactions will be expressed in terms of bare constants, and, in addition, relationships will be required between these constants and the renormalization constants in order to express the solution in terms of observed quantities. For this purpose it is essential to solve another problem along with the fundamental problem, that is, the problem of the "envelopment" of an isolated meson and nucleon, i.e., their interaction with their own field.

The latter problem may be reduced to the following equations (in the lowest approximation of the TD method):

$$(\gamma\nabla + M_1)\langle\psi(x)\rangle_t = -ig\gamma_5\tau\langle\varphi(x)\psi(x)\rangle_t = -\int \mathfrak{M}(x-x')\langle\psi(x')\rangle_{t'}\,d^4x', \tag{9a}$$

$$(\Box - \mu_1^2)\langle\varphi(x)\rangle_t = ig\tau(\gamma_5)_{\mu\nu}\langle\overline{\psi}_\mu(x)\psi_\nu(x)\rangle_t = \int P(x-x')\langle\varphi(x')\rangle_{t'}\,d^4x'.$$

Transforming to momentum space and using the definitions

$$\gamma\nabla\langle\psi(x)\rangle_t = -M\langle\psi(x)\rangle_t, \quad \Box\langle\varphi(x)\rangle_t = \mu^2\langle\varphi(x)\rangle_t,$$

we obtain equations which enable us to express M_1 and μ_1 in terms of M and μ:

$$-M + M_1 + (2\pi)^4\,\mathfrak{M}(M_1, \mu_1^2, M) = 0, \tag{10}$$

$$\mu^2 - \mu_1^2 - (2\pi)^4 P(M_1, \mu^2) = 0.$$

We note in explanation that the mass operators $\mathfrak{M}$ and the polarization operator P in (9) depend on bare masses, whereas their Fourier representations $\mathfrak{M}(p)$ and $P(k)$ enter into (10), in which we have put $i\gamma p = -M$, $k^2 = -\mu^2$. The divergent integrals in $\mathfrak{M}$ and P are cut off by the substitution

$$(p^2 + M_1^2)^{-2} \to 2\int_{M_1^2}^{L^2}(p^2 + x)^{-3}\,dx.$$

As a result of the simultaneous solution of Eqs. (10) the following relationships are obtained ($\lambda = g_1^2/4\pi^2$):

$$\mu_1^2 = \mu^2 + \lambda L^2 + O(\ln L^2); \tag{11}$$

$$M_1 = M\frac{(1-\lambda)(4-\lambda) + 3\lambda(4\lambda - 3)\ln\lambda}{4(1-\lambda)^2 + 6\cdot\lambda(1-\lambda)\ln\lambda} + O\left(\frac{1}{L^2}\right).$$

It is important to note that the bare nucleon mass and δM are finite and different from (7). This results from the fact that the meson distribution function appearing in the mass operator contains in its denominator the square of the bare meson mass, which increases with L:

$$\lim_{L\to\infty}\int_0^{L^2}\frac{dk^2}{k^2 + \lambda L^2 + \mu^2} = \int_0^1\frac{dx^2}{x^2+\lambda}, \quad x = \frac{k}{L}.$$

Passing now to the scattering of mesons by nucleons, we confine ourselves to a state with isotopic spin $I = 3/2$, since in the state with $I = 1/2$ the mass operator is more complicated than in (9). The meson-nucleon equation is[1]:

$$\langle\varphi(x_1)\psi(x_2)\rangle_t = \langle\varphi(x_1)\psi(x_2)\rangle_{-\infty} + \int(K_1 + K_2)\langle\varphi(x')\psi(x'')\rangle_{t'''}\,dx'\,dx''\,dx''', \tag{12}$$

where K_1 represents the self-energy kernels corresponding to (9), and K_2 represents the scattering kernels; both of these are expressed in terms of the bare masses.

The renormalization of (12) reduces simply to the substitution in (12) of M_1 and μ_1 expressed in terms of M and μ, using the relationships in (11). In some of the propagation functions which correspond to scattering, M_1 and μ_1 are simply replaced by M and μ due to "envelopment" of the corresponding lines. There will also appear, however, the propagation functions of the "undeveloped" particles to which the bare mass corresponds. To the last category belong, for example, the propagation function of a nucleon which has already emitted, but not yet absorbed, a meson; the inclusion of four-particle amplitudes would correspond to its "envelopment", i.e., a higher approximation of the TD method.

As a result of the use of the above-described renormalization method a solution is obtained which does not contain infinities but which is in general different from the usual solution[1], in which renormalized masses correspond to all propagation functions.

After the mass renormalization in the TD method it is still necessary to renormalize the charge; the associated difficulties are also inherent in the method here described. This hampers the comparison of the solution of (12) and of the expression derived therefrom for the scattering phases, etc., with the results obtained by the usual renormalization method[1].

I wish to express my profound gratitude to Academician I. E. Tamm and to his collaborators for their discussion of this note and for valuable suggestions.

---

[1] Silin, Tamm and Fainberg, J. Exptl. Theoret. Phys. (U.S.S.R.) **29**, 6 (1955); Soviet Phys. JETP **2**, 3 (1956).

[2] M. Neumann, Phys. Rev. **85**, 129 (1952).

Translated by I. Emin
201

Neutrino Induced Deuteron Disintegration

A. B. Govorkov

(Submitted to JETP editor February 12, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 974-975 (May, 1956)

IT has recently been clearly established that the β-interaction is a mixture of the scalar and tensor types. The scalar interaction constant has been determined by measuring ft for the β-decay of O^{14} to N^{14} [2]. The nuclear matrix element of β^+-decay of O^{14} can be obtained by an exact theoretical calculation since this is an $0+\to 0+$ transition with identical nucleonic wave functions in the initial and final nuclei.

The tensor interaction constant cannot be determined directly from β-decay since the matrix element for the tensor type cannot be calculated exactly. In this case the spin direction of the decaying nucleus is changed, and since spin-orbit coupling and the relative orientation of nucleonic spins play an important part in nuclear interactions the nucleonic wave functions of the initial and final nuclei will differ. But since we must know the form of the initial and final nucleonic wave functions in order to calculate the matrix element of the nucleus, whereas the exact form of a nucleonic wave function in the nucleus is unknown, the matrix element for the tensor type of β-interaction cannot be calculated exactly.

The only nucleus for which the nucleonic wave functions are known well is the deuteron, which is not β-active. However, it is possible to determine the tensor interaction constant directly and independently of the scalar interaction constant by measuring the cross section for the absorption of an antineutrons by a deuteron:

$$D+\tilde{\nu}\rightarrow n+n+e^+ \qquad \text{(I)}$$

(ν denotes a neutrino and $\tilde{\nu}$ an antineutrino). This process, as is indicated by an estimate given below, has a cross section which is smaller by an order of magnitude than the $p+\tilde{\nu}\rightarrow n+e^+$ cross section; this process has apparently been observed by the annihilation radiation of the positrons. The cross section is of the order of magnitude 10^{-44} cm^2.

The present note deals with the determination of the antineutrino absorption cross section for deuterons.

The following simple considerations show that the scalar interaction makes a small contribution to the cross section as compared with the tensor interaction. The neutrons emitted in process (I) will possess very small velocities since the larger part of the energy evolved in the interaction is borne off by the electron. We can therefore assume that both neutrons are formed in an S-state. By the Pauli principle the spin part of the wave function for the final state of the neutrons must be antisymmetric, i.e., the neutrons spins must be antiparallel. The spin part of the deuteron wave function is symmetric--the nucleonic spins in the deuteron are parallel,. Consequently, the relative direction of the spins must be changed. This is possibly only by a tensor interaction type and is impossible by a scalar type.

For the purpose of obtaining the differential cross section of process (I) the deuteron wave function was taken in the usual form; the wave function of the final state was made antisymmetrical in all variables of the neutrons.

For the differential cross sections, summed over all polarizations of the final states and averaged over all polarizations of the initial states of the particles, the following expressions are obtained:

$$d\sigma_S=\frac{2\pi\varkappa}{(2\pi)^5}G_S^2\left(\frac{1}{\varkappa^2+p_1^2}-\frac{1}{\varkappa_2+p_2^2}\right)^2\frac{E_eE_\nu-\mathbf{p}_e\mathbf{p}_\nu}{E_eE_\nu}\,\frac{p_1E_1\,do_1p_eE_e\,do_e\,dE_e}{1-(E_1p_2/p_1E_2)\cos\theta_{1,2}}; \qquad (1)$$

$$d\sigma_T=\frac{2\pi\varkappa}{(2\pi)^5}G_T^2\left\{\frac{2}{(\varkappa^2+p_1^2)^2}+\frac{2}{(\varkappa^2+p_2^2)^2}+\left(\frac{1}{\varkappa^2+p_1^2}-\frac{1}{\varkappa^2+p_2^2}\right)^2\right\} \qquad (2)$$

$$\times\frac{E_eE_\nu+{}^1/_3\,\mathbf{p}_e\mathbf{p}_\nu}{E_eE_\nu}\,\frac{p_1E_1\,do_1p_eE_e\,do_e\,dE_e}{1-(E_1p_2/p_1E_2)\cos\theta_{1,2}}.$$