$$E_1' = E_1 - \sum_{f,m} \frac{A_f^2 |(e_f)_{1,m}|^2}{E_m - E_1 + E_1(f)} \cdot \qquad (14)$$

For the same reason the altered exciton mass $\mu_{\rm eff}$ is equal to

$$\frac{1}{\mu_{\rm eff}} \qquad (15)$$

$$= \frac{1}{m}\left[1 - \frac{2}{3}\frac{\hbar^3}{m}\sum_{f,m}\frac{A_f^2 f^2 |(e_f)_{1,m}|^2}{[E_m - E_1 + E_1(f)]^3}\right].$$

An approximate evaluation of the sum for the case $\mu_1 = \mu_2$ gives

$$E_1' = E_1\left(1 + \frac{5}{2} cx \frac{\hbar\omega}{|E_1|}\right); \qquad (16)$$

$$\mu_{\rm eff} = m\left(1 + \frac{2}{3} cx \frac{\hbar\omega}{|E_1|}\right),$$

where $E_1 = -\mu e^4/2\hbar^2 x^2$.

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Correlation between the Planes of Production and Decay of Λ^0-Particles

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1. RECENTLY, correlations between planes of production and decay of hyperons have been found in a series of experiments[1-3]. These correlations consist in the fact that the angle between these planes is usually small. A theoretical analysis of the effect of correlation was carried out[4] and conclusions were drawn about the large spin of the Λ^0-particle and about the spin of the θ^0-meson being different from zero. It is not without interest, we believe, to consider another approach to this problem in addition to the method of Ref. 4.

2. We consider the z-axis to be perpendicular to the plane of production of the Λ^0-particle, and the direction of motion of the Λ^0-particle is taken as the y-axis. Then the angle between the planes of production and decay will be equal to the azimuthal angle φ. For calculation we now go to the system where the Λ^0 is at rest. it is not difficult to see that the angle φ does not change in this transition.

Let $\Phi_J^M(x, q)$ be the wave function of the system $\Lambda^0 + \theta^0$ describing the state with total angular momentum J and component $\hbar M$ along the z-axis; x and q denote dynamical variables of the Λ^0 - and θ^0-particles, respectively.

The Λ^0-particle as a sub-system of the system $\Lambda^0 + \theta^0$ is characterized by the density matrix $\rho(x', x)$[5]:

$$\rho(x', x) = \int \Phi_J^{M*}(x', q)\, \Phi_J^M(x, q)\, dq. \qquad (1)$$

The $\rho(x', x)$ can be expanded in the wave functions of the Λ^0-particle, which we denote by $\Lambda_m(x)$ (in which m can be considered the quantum number of the spin projection on the z-axis)

$$\rho(x', x) = \sum_{m_1, m_2} \rho_{m_1 m_2} \Lambda^*_{m_1}(x')\, \Lambda_{m_2}(x). \qquad (2)$$

3. We now turn to the calculation of the distribution of angles φ between the planes of production and decay of the Λ^0-particle.

The total moment of the proton and π^--mesons which are formed in the decay at rest of the Λ^0-particle, coincides with its spin j. If we designate by $\Psi_j^m(\theta, \varphi)$ the angular (and spin) part of the wave function of the stationary state of the system $p + \pi^-$, then

$$\Psi_j^m(\theta, \varphi) = c_1^m \chi^{+1/2} P_l^{m-1/2}(\theta)\, e^{i(m-1/2)\varphi} \qquad (3)$$

$$+ c_2^m \chi^{-1/2} P_l^{m+1/2}(\theta)\, e^{i(m+1/2)\varphi},$$

where c_1^m and c_2^m are Clebsch-Gordan coefficients, $\chi^{+1/2}$ are the spin wave functions of the proton, P_l^n is the spherical harmonic and $l = j + 1/2$ (depending on the relative parity of the Λ^0). In order to obtain the angular distribution, $f(\theta, \varphi)$, of the products of the Λ^0-decay it is obviously necessary to substitute in Eq. (2) the functions $\Psi_j^{m^*_1}(\theta, \varphi)$, $\Psi_j^{m_2}(\theta,\varphi)$ from Eq. (3) instead of $\Lambda^*_{m_1}$ and Λ_{m_2}, and carry out the summation over spin variables

$$f(\theta, \varphi) = \sum_{m_1, m_2 = \pm j, \pm(j-1), \ldots} \rho_{m_1 m_2} \Psi_j^{m_1^*}(\theta, \varphi) \Psi_j^{m_2}(\theta, \varphi). \tag{4}$$

The probability density for angle φ is $W(\varphi) = \int_0^\pi f(\theta,\varphi) \sin\theta \, d\theta$. If we substitute here from Eqs. (4) and (3), then one can see that $W(\varphi)$ takes the form

$$W(\varphi) = \sum_{k=0}^{(2j-1)/2} (a_k e^{2ik\varphi} + a_k^* e^{-2ik\varphi}), \tag{5}$$

where the coefficients a_k for $k \neq 0$ are linear combinations of off-diagonal elements only of the desnity matrix ($\rho_{m_1 m_2}$). Further, it must be noted that angle φ in the experiment is always measured in such a way that $\varphi < 90°$ (that is, the angles φ, $\pi \pm \varphi$ and $2\pi - \varphi$ are considered as equivalent in the measurements). If we designate the probability density of the angle φ, defined in this way, through $\omega(\varphi)$, then we have $\omega(\varphi) = W(\varphi) + W(\pi + \varphi) + W(\pi - \varphi) + W(2\pi - \varphi)$ or

$$\omega(\varphi) = \sum_{k=0}^{(2j-1)/2} b_k \cos 2k\varphi, \tag{6}$$

$$b_k = 8 \,|\, a_k \,|\, \cos(\arg a_k).$$

From Eq. (6) it follows that if the off-diagonal elements are zero, then the distribution of angles φ is isotropic and, consequently, there is no correlation between the planes of production and decay, even if $j \geq 3/2$.

Since $\omega(\varphi)$ satisfies the normalization condition

$$\int_0^{\pi/2} \omega(\varphi) \, d\varphi = 1,$$

then, substituting here from Eq. (6), we obtain the value of the coefficient b_0:

$$b_0 = 2/\pi \approx 0{,}637.$$

4. We consider in more detail the possible correlation effects, assuming that the spin of the Λ^0-particle is $j = 3/2$. In this case

$$\omega(\varphi) = 2/\pi + b_1 \cos 2\varphi. \tag{7}$$

We designate by $P(\varphi < \alpha)$ the probability that φ takes on a value less than α. From Eq. (7) we have

$$P(\varphi < \pi/4) = (1 + b_1)/2; \tag{8}$$

$$P(\pi/8 < \varphi < 3\pi/8) = 0{,}5.$$

On the other hand, we must have $\omega(\varphi) \geq 0$ for any $0 \leq \varphi \leq \pi/2$. This is possible only in case $b_1 \leq b_0 \approx 0.64$, i.e., $P(\varphi < \pi/4) \leq 0.82$. Thus, if the spin of the Λ^0-particle is equal to 3/2, then the probability that $\varphi > 45^0$ must be less than 18%, and the probability that $22.5° \leq \varphi \leq 67.5^0$ should equal 50%. If this condition is not fulfilled (when correlation is present), then the spin of the Λ^0-particle j is greater than 3/2.

If we collect all experimental data relating to the angle φ given in Refs. 1-3 (including also the data of Table 8 in Ref. 3), we can conclude that the number of cases in which $\varphi > 45^0$ constitute 26%, and the number of cases in which $22.5° \leq 67.5°$ is 42%, which is comparatively near to 50%. Taking into account the fact that the statistics are still insufficient (there are data for only 19 decays) and that the accuracy of measurement is not high (this relates to the data of Table 8 in Ref. 3), one can say that the experimental data to date do not exclude the possibility that the spin of the Λ^0-particle is equal to 3/2. It must be noted that these data also do not exclude the possibility $j > 3/2$.

5. From the above development it follows simply that the spin of the θ^0-meson is different from zero. In fact, we shall assume the opposite. Then the wave function Φ_J^M of the $\Lambda^0 + \theta^0$ system defined in Sec. 2 can be expressed as

$$\Phi_J^M = \sum_m c_m \Lambda_m \varphi^{M-m}, \tag{9}$$

where φ^n is the wave function of the θ^0-meson and $n = M - m$ is the quantum number of the projection of its orbital moment. Substituting from Eq. (9) in Eq. (1), and taking account of the fact that functions φ^n with different n are orthogonal, we obtain an expression of the type Eq. (2), but with the off-diagonal elements of $\rho_{m_1 m_2}$ equal to zero. According to the preceding, correlation cannot be observed in this case, which contradicts experiment. Consequently, the spin of the θ^0-meson is different from zero. This result was obtained earlier[4] in a different way.

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Renormalization in the Equations of the New Tamm-Dancoff Method

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1. IT is well known (see, for example, Ref. 1) that the equations of the new Tamm-Dancoff method for scattering of a meson by a nucleon, including only three virtual particles, contain divergent self-energy kernels of the meson and the nucleon and finite kernels corresponding to scattering with the meson being first absorbed ("graph with absorption") and scattering with the meson first emitted ("graph with emission"). We shall not consider renormalization of the self-energy kernels of the meson and nucleon and the difficulties arising with this[1], but shall assume that these kernels are corrections to the propagator of the system, which can be neglected in first approximation. We omit also all two-particle amplitudes containing antiparticles, assuming that in first approximation their influence on the amplitude " +meson, +nucleon" is small.

As a result, in first approximation an equation for one amplitude, " +meson, +nucleon" without self-energy kernel remains. In the solution of this equation, infinities of a vertex and self-energy type occur because of the kernel corresponding to the "graph with absorption." The renormalization of these divergences was studied recently by Dalitz and Dyson[2]; they came to the conclusion that the removal of these divergences was connected with the introduction of two renormalized charges into the theory (respectively, for the $s_{1/2}$ and $p_{1/2}$ states with isotopic spin $T = 1/2$) in addition to the fundamental charge g^2. Renormalization of these divergences was studied independently by us by a method used in Ref. 3* and it was shown that, contrary to the results of Dalitz and Dyson, the removal of divergences is connected with a finite charge renormalization and does not lead to any new constants.

In this note we give briefly our results and clarify the contradiction between them and the results of Ref. 2. For brevity of exposition and convenience in making a comparison, the notation of Ref. 2 will be used where possible.

2. The general solution of the integral equation for the amplitude "+meson, +nucleon" in state α (state α is characterized by angular momentum j and the parity of the system) has the form**

$$c_\alpha(p, l) = g_\alpha(p, l) + \frac{Qg^2}{8\pi^2}\sqrt{\frac{(E_p-\eta m)(E_l-\eta m)}{E_p\omega_p E_l\omega_l}}\,\frac{V_\alpha(p,\varepsilon)\,V_\alpha(l,\varepsilon)}{\varepsilon-\eta m-QS_\alpha(\varepsilon)} \quad (1)$$

and is connected with the phase δ_α by the formula

$$\operatorname{tg}\delta_\alpha = -(\pi E_l\omega_l l\ \ \varepsilon)\,c_\alpha(l, l).$$

In Eq. (1) $E_p = \sqrt{p^2+m^2}$, $\omega_p = \sqrt{p^2+\mu^2}$, $\epsilon = E_l + \omega_l$, $\eta = 1$ or -1, respectively, for even and odd states, $Q = 3$ or 0, respectively, for $T = 1/2$ and $3/2$; $g_\alpha(p, l)$ is the solution corresponding to the "graph" with emission." The integral equation for $g_\alpha(p, l)$ is obtained from the integral equation for $c_\alpha(p, l)$ if the term corresponding to the "graph with absorption" is omitted in the latter. Thus $g_\alpha(p, l)$ does not contain divergences. All divergences due to the "graph with absorption" are contained in the vertex function $V_\alpha(p, \epsilon)$ and the self-energy of the nucleon $S_\alpha(\epsilon)$. The divergence contained in the vertex function can be separated in