$$U_{nl} = -\frac{4\pi r_0^3 V_0}{3M} \int \psi_{nl}\varphi(Q)\,\delta\rho\psi_k h_0(Q)\,d\tau\,dQ, \quad (5)$$

where $R = r_0 A^{1/3}$, $\psi_{n,l}$ and ψ_k are the wave functions of the captured and incident neutron, $h_0(Q)$ is the wave function of the oscillator ground state. Because of the rapid fall off of the latter, in the integration indicated in Eq. (5) small Q are important and therefore the wave function of an excited level of width γ is[2]

$$\varphi(Q) = \{\gamma/\pi[(\varepsilon - \varepsilon_0)^2 + \gamma^2]\}^{1/2} h_1(Q), \quad (6)$$

where $h_l(Q)$ is the wave function of the first excited state of the oscillator.

Carrying out the integration in Eq. (5) and using Eq. (2), we obtain

$$\sigma_{nl} = \frac{4\pi^4(2l+1)r_0^4V_0^2}{9A^{2/3}a\hbar^2MkK^2} \quad (7)$$

$$\times \frac{\cos\eta_l j_l(kR) + (-1)^l \sin\eta_l j_{-l-1}(kR)}{j_l(KR)}$$

$$\times \frac{\hbar\omega_0\gamma}{(E_k^{(0)} - E_{nl}^{(0)} - \hbar\omega_0)^2 + \gamma^2} F_{nl}^2(k),$$

where $\hbar k = \sqrt{2ME_k^{(0)}}$; $\hbar K = \sqrt{2M(E_k^{(0)} + V_0)}$ η_l is the lth phase shift from the potential U_0^3;

$$F_{nl}(k) = F_{nl}^{(1)}(k) + F_{nl}^{(2)}(k), \quad F_{nl}^{(i)}(k) \quad (8)$$

$$= \alpha_i\left[P_l\left(\frac{K^2 + \varkappa_{nl}^2 - q_i^2}{2K\varkappa_{nl}}\right) - \frac{\cos(q_i + \varkappa_{nl} - K)R}{\pi(q_i + \varkappa_{nl} - K)R}\right],$$

$$\hbar\varkappa_{nl} = \sqrt{2M(E_{nl}^{(0)} + V_0)},$$

where P_l is the Legendre polynomial of order l, α_i is a dimensionless constant of order unity, which can be expressed in terms of D and q and, consequently, depends only on the number of neutrons and protons in the nucleus. In carrying out the integration of Eq. (5) we use the relation[4]

$$\int_0^\infty j_0(qr)\,j_l(Kr)\,j_l(\varkappa r)\,r^2\,dr = \frac{\pi}{4K\varkappa q} P_l\left(\frac{K^2 + \varkappa^2 - q^2}{2K\varkappa}\right), \quad (9)$$

valid for $K - \varkappa < q < K + \varkappa$; q_iR is the order of several units; if[5] $V_0 = 40$ mev, the inequality is satisfied. The formula (7) is valid only for $l \ll \varkappa R$, which is also satisfied, in so far as in Eq. (4) only those terms are important for which $E_k^{(0)} - E_{nl}^{(0)} - \hbar\omega_0 \sim \gamma$ and so far as for large l there are no bound states.

Taking[1] $a = 3.4 \times 10^5$ cm^5 gm^{-1} sec^{-2} and considering the order of several mev, we obtain a cross section of order 10^{-26} cm^2.

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Characteristics of the Levels of Nonspherical Even-Even Nuclei

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IN the adiabatic approximation the state of motion of external nucleons in a nonspherical nucleus which possesses an axis and center of symmetry is defined by quantum numbers that are completely analogous to the quantum characteristics of electronic terms for a diatomic molecule containing identical nuclei.[1] In this case the sum Ω of the projections w_i of the nucleonic spins on the axis of symmetry of the nucleus and the parity of the state $P = \pm 1$ are integrals of the motion.

In the present note we consider the characteristics of levels with $\Omega = 0$. Analogous to the Σ terms of a diatomic molecule, in the case of $\Omega = 0$ there is an additional quantum number $\eta = \pm 1$ which characterizes the behavior of the wave function under reflection with respect to a plane passing through the nuclear axis of symmetry, i.e., for the transformation $r \to r$, $\vartheta \to \vartheta$, $\varphi \to 2\Phi - \varphi$, where r, ϑ, φ are the polar coordinates of a point and Φ is the azimuthal angle of the plane with respect to which the reflection is performed. This transforms the wave function $\psi_{\Omega P}$ into $\hat{I}_{\Phi}\psi_{\Omega P}=\psi_{\Omega P}$. Since the Hamiltonian of the system (neglecting rotation of the nucleus) is invariant with respect to the above reflection because of the axial symmetry of the entire problem, each term will be doubly degenerate in Ω. For $\Omega = 0$

$$\hat{I}_{\Phi}\psi_{0P\eta} = \eta\psi_{0P\eta},\ \eta = \pm 1, \qquad (1)$$

so that instead of the degeneracy there can be two levels with different values of η.*

The states with $\Omega=0$ can exist in nuclei with even A. Particularly in even-even nuclei (even Z, even A) levels with $\Omega=0$ surely are to be found. These include, for example, the ground state (total spin $J=0$, $P=\pm 1$) and the first excited level ($J=2$, $P=+1$) which is of rotational origin. The ground and first excited levels of even-even nuclei will be characterized by identical values of η [see (2), below] . The same refers to all levels which differ only in excitation of collective degrees of freedom. Levels which differ in the states of external nucleons can in principle have either identical or different η.

The following selection rule for radiative transitions is associated with η: transitions between levels with identical η can only be of an electric multipole type, whereas transitions accompanied by a change of the sign of η can only have magnetic character.

Since we are actually considering transitions between levels with $\Omega=0$ the projection w_r of the angular momentum of the quantum on the nuclear axis of symmetry must also be 0. The wave function of the quantum will therefore be the eigenfunction of the operator $\hat{I}_{\Phi}$ with eigenvalues $\eta_r = \pm 1$. On the other hand, it is easily shown that in the most general case

$$\eta = (-1)^J P. \qquad (2)$$

When we apply (2) to electric and magnetic 2^l-pole radiation we obtain

$$P_E = (-1)^l,\ \eta_E = +1;$$
$$P_M = (-1)^{l-1},\ \eta_M = -1. \qquad (3)$$

Since η is an integral of the motion the following relationship must be in effect between the quantum numbers η_i, η_t of the initial and final nuclear states:

$$\eta_i = \eta_r\eta_f. \qquad (4)$$

From Eqs. (3) and (4) it follows that $\eta_i = \pm\eta_t$ for electric (+) and magnetic (−) transitions, respectively.

The above-mentioned selection rules have several interesting consequences. Let us, for example, consider two levels: i, f with $\Omega=0$ and $|\Delta J| \lesssim l-1$, $P_i = (-1)^l P_f$ (we assume that $J_i \neq 0$ and $J_f \neq 0$). In this case electric 2^l-pole and magnetic 2^{l-1}-pole transitions are allowed simultaneously by the parity and spin selection rules. If, however, they are not specially forbidden, the magnetic transitions should predominate, as the ratio of the squares of the matrix elements of an electric 2^l-pole and a magnetic 2^{l-1}-pole transition is equal (in order of magnitude) to $[(l^2-1)/l(2l+1)](R/\lambdabar)^2/\beta^2$, where R is the nuclear radius, $\lambdabar$ is the radiation wave length divided by 2π and β is the ratio of nucleon velocity in the nucleus to the velocity of light. When the transition energy is of the order of a few hundred kev for a nucleus with $A = 100-200$, this ratio will be much below 1. The above selection rules for η can change the situation; for example, the matrix element of a magnetic transition between levels with $\eta_i = \eta_f$ can have a considerably reduced value. Strong forbiddenness of the magnetic transitions is, of course, not to be expected, as Ω and η are good quantum numbers only in the adiabatic approximation, which can be assumed to be valid for nuclei which are markedly nonspherical. When the nonsphericality is small we can expect only a certain reduction of magnetic transition probabilities leading to mixed radiation $[E(l) + M(l-1)]$.

In view of the above it is interesting to examine the experimental data for even-even nuclei. Among these are nuclei whose first two excited levels are characterized by $J=2$, $P=+1$ (expressed briefly as $2+$)[3-5] for which the $2+ \to 2^+$ transition is usually mixed ($E2 + M1$). An important point is that the farther the numbers Z and $N=A-Z$ are from the magic numbers the smaller the $M1$ contribution (see the table). Since the nonsphericality of nuclei increases with the number of nuclei outside closed shells the data of the table clearly show that the probability of $E2$ radiation from the $2^+ \to 2^+$ transition in even-even nuclei is augmented as the nucleus becomes less spherical. This is also supported by the position of the first excited (rotational) level, which is lowered with greater nonsphericality. Among the nuclei included in the table the lowest of the first excited levels is found in the isotopes of Pt, for which the $M1$ fraction in the $2^+ \to 2^+$ transition is also smallest. For Pt nuclei, in addition, $|N-N_{mag}| \geq 8$, $|Z-Z_{mag}| \geq 4$, whereas for the other nuclei in the table $|N-N_{mag}| \geq 2$, $|Z-Z_{mag}| \geq 2$. This fact can be explained by the selection rules which were referred to above. If, indeed, it is assumed that for the states being considered $\Omega = 0$, then by (3) $\eta_i = \eta_f = +1$, so that magnetic radiation will be forbidden.

$2^+ \to 2^+$ transitions in even-even nuclei

Nucleus	$N-N_{mag}$ $Z-Z_{mag}$	% $M1$ in transition	transition energy in kev	Position of first excited level in kev
$^{76}Se^{42}_{34}$	+2 −6	20—66	650	560
$^{122}Te^{70}_{52}$	−12 + 2	20	680	560
$^{114}Cd^{66}_{48}$	+16 − 2	95,6	710	530
$^{194}Pt^{116}_{78}$	− 8 − 4	5—6	330	360
$^{196}Pt^{118}_{78}$	−10 − 4	2	1480	330
$^{198}Hg^{118}_{80}$	− 8 − 2	30—50	680	410

In connection with these considerations it would be desirable to obtain the following experimental data: a) a comparison of the nonsphericality of the nuclei in the table through observation of the Coulomb excitation (at present data exist for Cd ^{114} [6-7]); b) more accurate information concerning the multipolarity of the $2^+ \to 2^+$ transition in Os^{186} (at present we have only a reference to a private communication by the author of reference [5]); c) a study of the level schemes and transition multipolarities of strongly nonspherical even-even nuclei of rare earths and heavy elements (in the majority of cases only the characteristics of the first excited levels are known at present; see Ref. 5).

*This is completely analogous to the Σ^+ and Σ^- terms of a diatomic molecule (see Ref. 2).

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Concerning the Correlation Function for Quantum Systems

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THE correlation function for a classical system of weakly interacting particles can be determined by an approximate solution of the equation for the binomial distribution function.[1] In this method the distribution function f_3 in the set of equations is approximately expressed in terms of the binomial distribution function f_2.

The method of Bogoliubov is used in the present letter to determine the correlation function of a quantum system of interacting particles. Instead of a set of classical equations for the distribution function used by Bogoliubov[1] we use a set of equations for the quantum distribution function.[2] The approximation of the quantum distribution function f_3 by a binomial quantum distribution