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160

Concerning a Certain Generalization of a Renormalization Group

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The basic goal of the present note is the generalization of the Lie equations for the Green functions for the case of an arbitrary longitudinal coupling of photons.

1. We shall work with finite Green functions and shall assume that all divergences which exist in the theory have been eliminated by means of the subtraction formalism[2]. However, there still remain, even after the elimination of the infinites, final ambiguities which are associated with the finite counter-terms introduced into the Lagrangian of the same operator which occurs in the elimination of the divergences[2]. In the case where the zeroth coupling of the photon contains an admixture of an arbitrary longitudinal coupling,

$$\mathfrak{G}^0_{mn}\, d(k^2) = \frac{i}{k^2}\left(g^{mn} - \frac{k_m k_n}{k^2}\right) + \frac{i}{k^2}\frac{k_m k_n}{k^2}\,\omega^2 d_l(k^2),$$

the calculation of the introduced counter-terms introduces a multiplicative renormalization factor for the transverse part of the photon coupling but does not change its longitudinal part. Insofar as the introduced finite counter-terms (for the transverse photon coupling) led to a renormalization of the Green function and of the electron charge, then, in the present problem, because of the circumstance shown above regarding the "incomplete renormalization" of the zeroth coupling, we shall work with the following transformation group:

$$G_1 \to G_2 = Z_2 G_1;\quad \mathfrak{G}_1 \to \mathfrak{G}_2 = Z_3 \mathfrak{G}_{,1};\quad \Gamma_1 \to \Gamma_2 = Z_1^{-1}\Gamma_1;$$

$$e_1^2 \to e_2^2 = Z_3^{-1} e_1^2;\quad \omega_1^2 \to \omega_2^2 = Z_3\omega_1^2;\quad Z_1 = Z_2. \quad (1)$$

This transformation group is a generalization of the transformation group[1] for the case of an arbitrary longitudinal photon coupling. The significance of these transformations is that the set of quantities (G_1, $\mathfrak{G}_1$, Γ_1, e_1, ω_1) and (G_2, $\mathfrak{G}_2$, Γ_2, e_2, ω_2) can be used in the same form for the specification of particles with their mass and charge equal to the experimental values.

2. The transformation group obtained above permits one to derive the equations for the Green functions. If we represent the Green functions in the form

$$G(k) = i\,\frac{\hat{k}a(k^2) + mb(k^2)}{k^2 - m^2}, \quad (2)$$

$$\mathfrak{G}_{mn}(k) = \frac{i}{k^2}\left(g^{mn} - \frac{k_m k_n}{k^2}\right) d(k^2) + \frac{i}{k^2}\frac{k_m k_n}{k^2}\,\omega^2 d_l(k^2)$$

and if we reason as in Ref. 1, we obtain with no difficulty the following functional equations,

$$d(x, y, e^2) = d(-t, y, e^2)\, d(x\ \ t, y/t, e^2 d(-t, y, e^2)),$$

$$t > 0, \quad (3)$$

$$s'(x, y, \omega^2, e^2) = \frac{s'(x/t,\ y/t,\ \omega^2 d^{-1},\ e^2 d(-t, y, e^2))}{s'(-1/t,\ y/t,\ \omega^2 d^{-1},\ e^2 d(-t, y, e^2))},$$

where

$$k^2\ \ \lambda_2^2 = x,\quad m^2/\lambda_2^2 = y,$$

$$s'(x, y, \omega^2, e^2) = s(x, y, \omega^2, e^2)\, s^{-1}(-1, y, \omega^2, e^2).$$

The first of these equations agrees with the analogous equation for d in Ref. 2, insofar as the transverse part of the zeroth coupling.

Differentiating each of the above equations with respect to x and setting $t = -x$, we obtain,

$$\frac{\partial}{\partial x}\ln d(x, y, e^2) \quad (4)$$

$$= -\frac{1}{x}\left[\frac{\partial}{\partial \xi} d\left(\xi - \frac{y}{x},\ e^2 d(x, y, e^2)\right)\right]_{\xi=-1},\quad x < 0,$$

$$\frac{\partial}{\partial x}\ln s'(x, y, \omega^2 e^2)$$

$$= -\frac{1}{x}\left[\frac{\partial}{\partial \xi} s'\left(\xi, -\frac{y}{x},\ \omega^2 d^{-1},\ e^2 d(x, y, e^2)\right)\right]_{\xi=-1}.$$

The functions $s'(x, y, \omega^2, e^2)$ and $d(x, y, e^2)$ are real in the region $x < 0$. In order that we deal only with real functions in the region $x > 0$, we can write the Lie equations separately for the real and imaginary parts of the Green functions:

$$\frac{\partial}{\partial x}\ln d_R(x, y, e^2) \tag{5}$$
$$= \frac{1}{x}\left[\frac{\partial}{\partial \xi}\ln d_R\left(\xi, \frac{y}{x}, e^2 d(-x, y, e^2)\right)\right]_{\xi=1},$$

$$\frac{\partial}{\partial x}\ln d_J(x, y, e^2)$$
$$= \frac{1}{x}\left[\frac{\partial}{\partial \xi}\ln d_J\left(\xi, \frac{y}{x}, e^2 d(-x, y, e^2)\right)\right]_{\xi=1},$$

$$\frac{\partial}{\partial x}\ln s'_R(x, y, \omega^2, e^2)$$
$$= \frac{1}{x}\left[\frac{\partial}{\partial \xi}\ln s'_R\left(\xi, \frac{y}{x}, \omega^2 d^{-1}, e^2 d(-x, y, e^2)\right)\right]_{\xi=1},$$

$$\frac{\partial}{\partial x}\ln s'_J(x_1, y, \omega^2, e^2)$$
$$= \frac{1}{x}\left[\frac{\partial}{\partial \xi}\ln s'_J\left(\xi, \frac{y}{x}, \omega^2 d^{-1}, e^2 d(-x, y, e^2)\right)\right]_{\xi=1},$$

where $d = d_R + id_J$; $s' = s'_R + is'_J$.

3. Let us consider the asymptotic region of large momenta, i.e., for $m^2 \ll |k^2|$. According to excitation theory we find:

$$a_c\left(\frac{k^2}{m^2}, \omega_0^2, e_0^2\right) = 1 - \frac{e_0^2\omega_0^2}{4\pi}\int\limits_{\ln(-k^2/m^2)}^{\infty} d_l(z)\,dz, \tag{6}$$

$$b_c\left(\frac{k^2}{m^2}, \omega_0^2, e_0^2\right)$$
$$= 1 - \frac{e_0^2}{4\pi}\left[3\ln\left(-\frac{k^2}{m^2}\right) + \omega_0^2\int\limits_{\ln(-k^2/m^2)}^{\infty} d_l(z)\,dz\right],$$

where $d_l(z)$ is a slowly varying function of z and, a_c and b_c are counterparts of the real electron Green function. If we consider the obvious relation between the function $s(x, y, \omega^2, e^2)$ and the real electron function $s_c(x/y, \omega_0^2, e_0^2)$

$$s'(x, y, \omega^2, e^2) = \frac{s_c(x/y, \omega_0^2, e_0^2)}{s_c(-1/y, \omega_0^2, e_0^2)}, \tag{7}$$

$$\omega^2 = \omega_0^2 d_c^{-1}\left(-\frac{1}{y}, e_0^2\right),\quad e^2 = e_0^2 d_c\left(-\frac{1}{y}, e_0^2\right),$$

then we obtain

$$a'(x, y, \omega^2, e^2) = 1 - \frac{e^2\omega^2}{4\pi}\int\limits_{\ln(-x/y)}^{-\ln y} d_l(z)\,dz, \tag{8}$$

$$b'(x, y, \omega^2, e^2)$$
$$= 1 - \frac{e^2}{4\pi}\left[3\ln(-x) + \omega^2\int\limits_{\ln(-x/y)}^{-\ln y} d_l(z)\,dz\right].$$

If we insert these expressions into Eq. (4), we have

$$a'(x, y, \omega^2, e^2) = \exp\left[-\frac{e^2\omega^2}{4\pi}\int\limits_{\ln(-x/y)}^{-\ln y} d_l(z)\,dz\right], \tag{9}$$

$$b'(x, y, \omega^2, e^2)$$
$$= \left[1 - \frac{e^2}{3\pi}\ln(-x)\right]^{9/4}\exp\left[-\frac{e^2\omega^2}{4\pi}\int\limits_{\ln(-x/y)}^{-\ln y} d_l(z)\,dz\right].$$

Let us consider the region $k^2 \sim m^2$, where the functions a' and b' possess an "infrared characteristic."

If we represent s' in the form

$$s'\left(\frac{k^2}{\lambda^2}, \frac{m^2}{\lambda^2}, \omega^2, e^2\right) \tag{10}$$
$$= S'\left(-\frac{m^2 - k^2}{m^2 + \lambda^2}, -\frac{m^2 + \lambda^2}{m^2}, \omega^2, e^2\right),$$

we can transform Eq. (4) to

$$\ln s'(x, y, \omega^2, e^2) = \int\limits_{-1}^{x}\frac{dx}{x - y}\left[\frac{\partial}{\partial \eta}S'\left(\eta, \frac{x}{y} - 1, \omega^2 d^{-1}, e^2 d(x, y, e^2)\right)\right]_{\eta=-1},$$

from which, by means of the formulas of excitation theory,

$$\left.\frac{\partial A'(\eta, x-1, \omega^2, e^2)}{d\eta}\right|_{\eta=-1} \tag{11}$$
$$= -\frac{e^2}{4\pi}\left[-\frac{6 + \omega^2 d_l^0}{x} + \frac{2\omega^2 d_l^0}{x^2} + \frac{(1-x)(2\omega^2 d_l^0 - 6)\ln(1-x)}{x^3} + \omega^2 d_l^0\right],$$

$$\left.\frac{\partial B'(\eta, x-1, \omega^2, e^2)}{\partial \eta}\right|_{\eta=-1}$$
$$= \frac{e^2}{4\pi}(3 - \omega^2 d_l^0)\left[\frac{1-x}{x}\ln(1-x) + \frac{x+1}{x}\right],$$

we obtain

$$a' \sim a_0' \left(1 - \frac{k^2}{m^2}\right)^{-(e^2/2\pi)(3-d_l^0)}, \tag{12}$$

$$b' \sim b_0' \left(1 - \frac{k^2}{m^2}\right)^{-(e^2/2\pi)(3-d_l^0)}.$$

Equations (9) and (12) which we have obtained as a qualitative illustration of the method of renormalization groups agrees with those results obtained earlier[3] by means of a summation of a series of Feynman "primary diagrams."

In conclusion, I wish to express my deepest thanks to Academician N. N. Bogoliubov under whose guidance this work was completed and, in addition, to D. V. Shirokov for discussion of this work.

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161

On the Mass of the Photon in Quantum Electrodynamics

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IN the present quantum theory of fields, point (local) interaction is often considered as the limit of smeared (nonlocal) interaction[1,2]; this permits one to operate with finite expressions in the intermediate calculations. For this purpose a scalar smearing function F, a form-factor which contains a cut-off parameter Λ, is introduced into the interaction Lagrangian. This factor converges to unity when $\Lambda \to \infty$. In this point of view the interaction has the form

$$S \sim e_1 \int F(p, k, p-k, \Lambda)\, \overline{\psi}(p)\, \hat{A}(k)\, \psi(p-k)\, d^4p\, d^4k + \text{charge interaction}. \tag{1}$$

In order that the smearing function does not lead to a violation of physical reality (i.e., the Hermitian character of the Lagrangian) it must satisfy the condition[3,4]

$$F(p, k, p-k) = F^*(p-k, k, p). \tag{2}$$

To each node in the Feynman diagram (or to each operator of the vertex portion) there will correspond a factor in F which depends on the momentum associated with the node.

It is known that the use of the simplest square-form smearing functions* leads to a violation of the gradient invariance; a violation which appears in the form of a nonzero photon mass. In this connection in Ref. 1, where a square-smearing function was used, the photon mass was eliminated by subtraction. In addition to this it was expressed by these authors (whose larger goal was the elimination of the divergences without the use of a subtractive procedure) that the assumption about the existence of such a smearing function leads automatically with its use to the falling-out of the photon mass. This note is devoted to a consideration of this question.

The mass of the photon corresponds to the value of the polarization operator at $k = 0$ (the symbolism of Ref. 1 is used here):

$$P_{\mu\nu}(0) = \frac{e_1^2}{\pi i} \mathrm{Sp} \int G(p)\Gamma_\mu(p, p, 0) G(p)\gamma_\nu |F(p, 0, p, \Lambda)|^2 d^4p. \tag{3}$$

The appearance of the square of the modulus of F is associated with the Hermitian character of the Lagrangian [Eq. (2)]; it specifies the presence of two vertex parts in the diagram of the polarization operator for which the momenta differ only in direction**.

To study Eq. (3) we shall use the asymptotic expression[1,5]

$$\widetilde{G}(p) = \hat{p}^{-1}, \widetilde{\Gamma}_\mu(p, p, 0) = \gamma_\mu, \tag{4}$$

which means we will consider the case where the longitudinal part of the photonic Green's function d_l is equal to zero. With the help of Eq. (4), we find

$$P_{\mu\nu}(0) = \widetilde{P}_{\mu\nu}(0) + [P_{\mu\nu}(0) - \widetilde{P}_{\mu\nu}(0)],$$
$$\widetilde{P}_{\mu\nu}(0) = \frac{e_1^2}{\pi i} \int \mathrm{Sp}\left(\frac{1}{\hat{p}}\gamma_\mu \frac{1}{\hat{p}}\gamma_\nu\right) |F|^2 d^4p. \tag{5}$$

If, as is customary (see Ref. 2), $F(p, 0, p, \Lambda) = f(p^2/\Lambda^2)$, then