linear term of its expansion in wave vectors of the external field $q/\hbar$ has the following form [$\hat{q} = q_\alpha \gamma_\alpha$; $q \simeq (q; iq_0)$]:

$$\Lambda_i = (\pi^2 / 2m) F (\gamma_i \hat{q} - \hat{q} \gamma_i). \quad (1)$$

The coefficient F serves to specify the radiative correction to the magnetic moment, i.e.,

$$\Delta\mu / \mu = (\alpha / 2\pi) F. \quad (2)$$

For $|\lambda_0| = \infty$, we have $F = 1$, and Eq. (2) is just the Schwinger formula. For finite λ_0, we can write $F = 1 - \delta F(\lambda_0)$, and hence

$$\Delta\mu / \mu = (\alpha / 2\pi) [1 - \delta F (\lambda_0)]. \quad (3)$$

F is expressed by integrals in momentum space of the form

$$J = \int \frac{d^4k\,[1; k_\sigma; k_\sigma k_\tau]}{(k^2 - 2p_1 k)(k^2 - 2p_2 k)\, k^2}$$

(p_1 and p_2 are the initial and final momenta of the meson and $p_2 - p_1 = q$). Instead of integrating over a finite region one can retain the infinite integration limits and introduce Feynman's[3] truncating factor $\lambda_0^2/\lambda_0^2 + k^2$. Then

$$J(\lambda_0) = J(\infty) - \delta J(\lambda_0),$$

where

$$\delta J(\lambda_0) = \int \frac{d^4k\,[1; k_\sigma; k_\sigma k_\tau]}{(k^2 - 2p_1 k)(k^2 - 2p_2 k)(k^2 + \lambda^2)}.$$

Continuing the calculation in the usual manner[3,4], we obtain for the apex the following expression

$$\Lambda_i(\lambda_0) = \Lambda_i(\infty) - \delta\Lambda_i(\lambda_0);$$

$$\delta\Lambda_i(\lambda_0) = \int_0^1 2x\,dx \int_0^1 dy\,\{(1 - y - xy)\,\hat{q}\gamma_i - (1 - x + xy)\,\gamma_i \hat{q}\} \times \frac{\pi^2 m x}{x^2 p_y^2 - (1 - x)\lambda_0^2}, \quad (4)$$

where

$$p_y = y p_1 + (1 - y) p_2.$$

Let us first perform the integration over y. Since we are only interested in terms linear in q, we can substitute $p^2 = -m^2$ for p_y^2 in the integrand. Then Eq. (4) assumes the form of Eq. (1), viz.,

$$\delta\Lambda_i = (\pi^2 / 2m)(\gamma_i \hat{q} - \hat{q}\gamma_i)\,\delta F(\lambda_0),$$

where

$$\delta F(\lambda_0) = 2\int_0^1 \frac{(1 - x)\,x^2}{x^2 + (\lambda_0 / m)^2 (1 - x)}\,dx \quad (5)$$

$$= 1 + 2\gamma - \gamma(\gamma + 2)\ln\frac{1}{\gamma} - \frac{\gamma^2 + 4\gamma + 2}{\sqrt{1 + 4/\gamma}} \ln\frac{1 + \sqrt{1 + 4/\gamma}}{1 - \sqrt{1 + 4/\gamma}}$$

($\gamma = \lambda_0^2/m^2$). With $\gamma \gg 1$, the value of the integral is

$$\delta F(\lambda_0) = 2m^2 / 3\lambda_0^2. \quad (6)$$

[1] L. Landau and I. Pomeranchuk, Dokl. Akad. Nauk SSSR 102, 489 (1955); I. Pomeranchuk, Dokl. Akad. Nauk SSSR 103, 1005 (1955); Dokl. Akad. Nauk SSSR 104, 51 (1955).

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Charged Particle Green's Function in the "Infrared Catastrophe" Region

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In the electrodynamics of the electron Abrikosov[1] has shown that the interaction with the electric field leads to the appearance in the Green's function of the electron in the infrared region ($|p^2 - m^2| \ll m^2$) of the additional singularity

$$\left(\frac{m^2}{p^2 - m^2}\right)^{(e^2/2\pi)\,[3 - d_l(0)]} \quad (1)$$

as compared with the simple pole for the Green's function of the free electron. An analogous investigation in the electrodynamics of spin zero[2] shows that the same singularity arises in the Green's function of a charged scalar particle, where the result obtained coincides exactly with (1). Hence, it seems worthwhile to present a result from which this effect would follow independently of the nature (spin) of the charged particle. The present letter is devoted to this question.

The Green's function of the particle is defined in the usual way:

$$G(x, x') = \langle (\psi(x), \bar{\psi}(x'))_+ \rangle_0$$

The brackets indicate a time-ordered average over the vacuum, $\psi(x)$ is the particle field operator, $\bar{\psi}(x)$ is the adjoint operator. The operations of conversion between ψ and $\bar{\psi}$ are evidently Hermitian conjugates.

The Fourier components of $G(x, x')$ in the region $p^2 \sim m^2$ are determined by the matrix element $\langle 0 | \psi(x) | p \rangle$, where $p^2 \sim m^2$. In other words, in the Fourier expansion of the operator $\psi(x)$, it is sufficient to determine only the part of the spectrum of $\psi(p)$ where $p^2 \sim m^2$. In the state in which $p^2 \sim m^2$, there is a single particle interacting with the electric field, and the magnitude $\Delta = |p^2 - m^2| m^{-2}$ is a measure of the energy of the photons which this particle may emit or absorb. We make the assumption, justified below, that for the effect under consideration only a connection with the low frequency part of the electromagnetic field is necessary. Choosing a system of reference in which the motion of the particle is nonrelativistic, we may write the nonrelativistic Schrödinger equation

$$i\,\frac{\partial\psi(x)}{\partial t} = \left\{m + eA_0(x) + \frac{1}{2m}\,(\hat{\mathbf{p}} - e\mathbf{A}(x))^2\right\}\psi(x), \tag{2}$$

where $\hat{p} = -i\nabla$. This equation is correct for the description of the interaction of the "free" part of the Fourier expansion of $\psi(x)$ ($p^2 \sim m^2$) with the low frequency part of the electromagnetic field. Neglecting terms quadratic in the field, since, as may be shown, they give rise to a higher order contribution in the final result, we substitute into Eq. (2)

$$\psi(\mathbf{x}, t) = P_\tau \exp\left\{-i\int^{t} j_\mu A_\mu(\mathbf{x}, \tau)\,d\tau\right\}\psi_0(\mathbf{x}, t), \tag{3}$$

where P_τ is the time-ordering operator and $j_\mu = \{e, e\mathbf{v}\}^*$. The operator $\psi_0(\mathbf{x}, t)$ satisfies the free particle equation

$$i\partial\psi_0/\partial t = (m + \mathbf{p}^2/2m)\,\psi_0.$$

In the approximation $e^2 \ll 1$ we may consider $A_\mu(\mathbf{x}, \tau)$ in Eq. (3) as the free-field operator; hence, ψ_0 and A_μ commute. For the Green's function $G(x, x')$ (more exactly, for the "free" part $p^2 \sim m^2$ in the Fourier expansion) we obtain

$$G(x, x') = G_0(x, x')\,\langle P_\tau P_{\tau'}\Big(\exp\left\{-i\int^{t} j_\mu A_\mu(\mathbf{x}, \tau)\,d\tau\right\} \times \exp\left\{i\int^{t} j_\nu A_\nu(\mathbf{x}', \tau')\,d\tau'\right\}\Big)_+\rangle_0,$$

where $G_0(x, x')$ is the free particle Green's function.

Using the formula

$$\exp(A + B) = \exp(A)\cdot\exp(B)\cdot\exp\left(-\frac{1}{2}[A, B]\right),$$

which is correct when there exists a number $[A, B]$, and taking an average over the photonic vacuum, we obtain the following simple transformations:

$$G(x, x') = G_0(x, x') \tag{4}$$

$$\times\exp\Big\{-\frac{1}{2}\Big[\int^{t}\!\!\int^{t} \langle (A_\mu(\mathbf{x}, \tau), A_\nu(\mathbf{x}, \tau'))_+\rangle_0\, j_\mu j_\nu\, d\tau\, d\tau' + \int^{t'}\!\!\int^{t'} \langle (A_\mu(\mathbf{x}', \tau), A_\nu(\mathbf{x}', \tau'))_+\rangle_0\, j_\mu j_\nu\, d\tau\, d\tau'\Big] + \int^{t}\!\!\int^{t'} \langle (A_\mu(\mathbf{x}, \tau), A_\nu(\mathbf{x}', \tau'))_+\rangle_0\, j_\mu j_\nu\, d\tau\, d\tau' + I(t, t')\Big\},$$

where

$$I(t, t') = \begin{cases} \displaystyle\int^{t}\!\!\int^{t'} \theta(\tau' - \tau)\,[A_\mu(\mathbf{x}, \tau), A_\nu(\mathbf{x}', \tau')]\, j_\nu j_\mu\, d\tau\, d\tau'; & t > t', \\ \displaystyle -\int^{t}\!\!\int^{t'} \theta(\tau - \tau')\,[A_\mu(\mathbf{x}, \tau), A_\nu(\mathbf{x}', \tau')]\, j_\nu j_\mu\, d\tau\, d\tau'; & t < t'. \end{cases} \tag{5}$$

$$[A_\mu(x), A_\nu(x')] = -4\pi i\delta_{\mu\nu} D(x - x'); \qquad \langle (A_\mu(x), A_\nu(x'))_+\rangle_0 = 4\pi D_{F\mu\nu}(x - x').$$

The assumption of adiabatic cessation of the interaction

$$e = e(t) = \begin{cases} e, & t > T_0 \\ e \exp \alpha t, & t < T_0 \end{cases} \quad (T_0 \to -\infty)$$

allows us to neglect the values of all the integrals taken at the lower limit. In particular, Eq. (5) depends only on the lower limit, and hence is equal to zero. Carrying out the integration with respect to $d\tau$ and $d\tau'$ in Eq. (4) in the indicated manner, and varying Eq. (4) with respect to δe^2, we take the Fourier component of the resulting relation in the momentum region $|p^2 - m^2| \ll m^2$ which is of interest to us. We find

$$\delta G(p) = \frac{i\delta e^2}{\pi} \left\{ \int [G(p) - G(p-k)] \left[\left(v^2 - \frac{(vk)^2}{k^2} \right) + \frac{(vk)^2}{k^2} d_l(k^2) \right] \frac{d^4k}{\omega^2 (k^2 + i\varepsilon)} \right\}. \quad (6)^*$$

The calculation is most simply carried out in the system of reference in which the velocity of the particle $\mathbf{v} = \mathbf{p}/m$ is equal to zero. Then

$$\delta G(p) = \frac{i\delta e^2}{\pi} \left\{ \int [G(p) - G(p-k)] \left[\left(1 - \frac{\omega^2}{k^2} \right) + \frac{\omega^2}{k^2} d_l(k^2) \right] \frac{d^4k}{\omega^2 (k^2 + i\varepsilon)} \right\}.$$

Carrying out the integration with respect to $d\omega$ according to the usual rules of contouring[3], we find that the terms sought are obtained by taking a calculation at the point $k^2 = 0$:

$$\delta G(p) = \frac{\delta e^2}{2\pi} \left\{ \int [G(p) - G(p-k)] \frac{d|\mathbf{k}|}{|\mathbf{k}|} \right\} [3 - d_l(0)]; \quad (\omega = +|k|). \quad (7)$$

The essential logarithmic region of interaction in this integral is $|p^2 - m^2|/m \ll k \ll m$. The region of high frequencies leads to renormalization effects, which, naturally, cannot be correctly taken into account in this technique. Carrying out the integration in (7) for low frequencies, we obtain

$$\delta G(p) = G(p) \left(\frac{\delta e^2}{2\pi} \right) [3 - d_l(0)] \ln \left(\frac{m^2}{p^2 - m^2} \right)$$

or

$$G(p) = G_0(p) \left(\frac{m^2}{p^2 - m^2} \right)^{(e^2/2\pi)[3 - d_l(0)]}$$

where $G_0(p)$ differs from the Green's function of a free particle by the renormalization factors. Thus, the appearance of the additional singularity (1) in the Green's function of a particle interacting with an electromagnetic field is connected only with the classical properties of the electric current being produced by the particle in its uniform motion.

In conclusion, I wish to express my deep gratitude to A. A. Abrikosov and I. M. Khalatnikov for discussions of this work.

* Designations are those used in Ref. 3.

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3 R. Feynman, Phys. Rev. 76, 749, 769 (1949).

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Transformation of Positive Helium Ions Colliding with Inert Gas Atoms into Negative Ions

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In studying the processes which take place during the passage of He^+ ions through rarefied gases, we noted that negative ions He^- occurred in the beam after it had passed through the gas.

The experiments were carried out in the double mass spectrometer arrangement described in Ref. 1. A beam of He^+ ions of given energy was separated by the magnetic mass-monochromator, after which it entered the gas-filled collision chamber. For a gas pressure of $\sim 3 \times 10^{-4}$ mm Hg in the chamber, we can keep the pressure in the remaining parts of the apparatus at a level $< 1 \times 10^{-5}$ mm Hg. The composition of the beam which has passed through the collision chamber is investigated by means of a magnetic mass analyzer.

If we admit a beam of He^+ ions into the collision chamber and select a suitable intensity of magnetic field in the mass analyzer, we can pass through it He^+ ions which will retain their charge and their velocity after the passage. On reversing the direction of the magnetic field in the mass