The connection of our tensor momenta with the density matrix is different from the connection between the density matrix and the tensor momenta which were derived by Simon (see the corrections to Refs. 2 and 3), but when normalized they are the same. In the particular case when $q_\theta = 0$ and $q_b = 0$ our expression yields a formula which differs from Eq. (3.2) of Ref. 3 by several factors. The distinction basically depends on the difference in the definition of the tensor momenta. This distinction does not affect the final results, as compared with experiment, since a change in the definition of the tensor momenta must be accompanied by a corresponding change in the derived matrix element.

An essential factor which is needed under the summation sign Σ, as compared with Eq. (3.2) of Ref. 3, is $(-1)^{\varkappa}$.

In conclusion, we express our gratitude to Prof. M. A. Markov for his constant interest in the work. We also thank L. G. Zastavenko for his advice on a number of questions concerning the theory of representations of rotation groups.

* As has been pointed out in Ref. 5, spherical harmonics must be preceded by i^l, otherwise the result of the action of the inverse time operator will not be invariant with respect to complex angular momenta.

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Scattering of Fast Neutrons by Nonspherical Nuclei. III

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In a previous communication[1] an effective cross section was calculated for the scattering of fast neutrons by a "black" nucleus having the form of an ellipse of rotation and a zero spin (even-even nucleus). In the present note these calculations are generalized to nuclei with spins different from zero (odd nuclei). In the adiabatic approximation the effective cross section is determined by the matrix element

$$f_{n'n}(\Omega) = \int d\omega \varphi^*_{n'}(\omega) f(\omega, \Omega) \varphi_n(\omega). \quad (1)$$

The amplitude for the scattering of a neutron by a stationary nucleus is:

$$f(\omega, \Omega) = i \frac{(kb)^2}{k} \xi(\mu) \frac{J_1(t)}{t}; \quad (2)$$

$$t = kb\theta [\xi^2(\mu) \cos^2(\Phi - \varphi) + \sin^2(\Phi - \varphi)]^{1/2},$$

where $\xi(\mu) = [z^2 + (1 + z^2)\mu^2]^{1/2}$, $z = a/b$. The quantity b is the radius of the largest circular cross section of the ellipsoid and $2a$ is the length of the axis of symmetry. The spherical angles ϑ and φ specify the direction of the axis of symmetry ω and the angles θ, Φ, the direction of scattering Ω. The polar axis of the external coordinate system is chosen to lie in the direction of the incident beam. We shall assume a strong coupling between the motion of the nucleons in the nucleus and the motion of the nuclear surface[2]. In this case the wave functions for the rotational states of the nucleus $\varphi_n(\omega)$ can be represented by the proper functions of the symmetric top*,

$$\varphi_n(\omega) = \sqrt{(2I + 1)/8\pi^2} D^I_{MK}(\omega), \quad (3)$$

where I is the total nuclear moment, M and K are the projections onto the external axis and axis of nuclear symmetry, repsectively, and $\omega = (\varphi_1, \theta, \varphi_2)$ represents the Eulerian angles which describe the orientation of the nucleus relative to the external coordinate system. The rotational states of the nucleus are given by

$$\varepsilon_{IK} = (\hbar^2/2J)\{I(I+1) - K(K+1)\}; \qquad (4)$$

$$I = K, K+1, \ldots$$

To determine the cross section for the excitation of different rotational states n' one must compute the corresponding matrix element (1). Employing Eq. (1) through (3) along with the known relation for $D^J_{MK}(\omega)$ we obtain

$$f_{n'n}(\Omega) \equiv f^{I'M'K}_{IMK}(\Omega)$$

$$= i(-1)^{M-K} e^{i(M-M')\Phi} \frac{(kb)^2}{4\pi k} \sqrt{(2I'+1)(2I+1)}$$

$$\times \sum_{\varkappa} C^{\varkappa M-M'}_{I'-M'M} C^{\varkappa 0}_{I'-KIK} \sqrt{\frac{(\varkappa - M' + M)!}{(\varkappa + M' - M)!}}$$

$$\times \int_{-1}^{1} d\mu\, \xi_{\lambda}(\mu)\, P^{M'-M}_{\varkappa}(\mu)$$

$$\times \int_{0}^{2\pi} d\varphi \cos(M'-M)\varphi \frac{J_1(t)}{t};$$

$$t = kb\theta\sqrt{\xi^2(\mu)\cos^2\varphi + \sin^2\varphi}. \qquad (5)$$

Here $C^{JM}_{jmj'm'}$ is the Clebsch-Gordan coefficient and $P^m_{\varkappa}(\mu)$ is the associated Legendre polynomial. The external axis of quantization is chosen to be in the direction of the incident beam.

The differential cross section for the excitation of the rotational state ϵ_{IK} is given by

$$\sigma^{I'K}_{IK}(\theta) = \frac{1}{2I+1} \sum_{M,M'} |f^{I'M'K'}_{IMK}(\Omega)|^2.$$

Hence, inserting Eq. (5) into this expression and employing the known relations for the Clebsch-Gordan coefficients we find

$$\sigma^{I'K}_{IK}(\theta) = \frac{4(kb)^4}{\pi^2 k^2} \sum_{\varkappa,m} \frac{2I'+1}{2\varkappa+1} (C^{\varkappa 0}_{I'KI-K})^2 I^2_{\varkappa m}(\theta); \qquad (6)$$

$$I_{\varkappa m}(\theta) = \frac{1}{8} \sqrt{(2\varkappa+1)\frac{(\varkappa-m)!}{(\varkappa+m)!}}$$

$$\times \int_{-1}^{1} d\mu\, \xi(\mu)\, P^m_{\varkappa}(\mu) \int_{0}^{2\pi} d\varphi \cos m\varphi \frac{J_1(t)}{t}.$$

The functions $I_{\varkappa m}(\theta)$ were investigated in Ref. 1 in the study of scattering for even-even nuclei. Because of the symmetry of the nucleus they are different from zero only for even $\varkappa$, m in which case, $I_{\varkappa m} = -I_{\varkappa,-m}$. The functions $I_{\varkappa m}(\theta)$ oscillate, roughly speaking, like the function $J_1(kR\theta)/(kR\theta)$ wherein the value of θ for which $I_{\varkappa m}(\theta) = 0$ depends upon $\varkappa$ and m.

For $I' = I$ Eq. (6) becomes the differential cross section for scattering without an energy exchange, viz.,

$$\sigma^{IK}_{IK}(\theta) = \frac{4(kb)^4}{\pi^2 k^2} \sum_{\varkappa,m} \frac{2I+1}{2\varkappa+1} (C^{\varkappa 0}_{IKI-K})^2 I^2_{\varkappa m}(\theta). \qquad (7)$$

We see that this differential elastic cross section for neutron scattering by nonspherical nuclei with spin different from zero ($K \neq 0$) does not pass through zero for scattering angles $\theta < 1$. This is understandable since the cross section given by Eq. (7) is represented by a sum of cross sections which correspond to transitions between states with different projections of I, i.e., M before, M' after scattering. On the other hand, there is the exceptional case of an even-even nucleus with $I = K = 0$ where the angular distribution for elastic scattering has a zero similar to the cross section for elastic scattering by the diffraction of fast neutrons by a spherical nucleus. Actually, for $I = K = 0$ Eq. (6) shows that the cross section for exciting the state $\epsilon_{I'}$ of an even-even nucleus is

$$\sigma_{I'}(\theta) = \frac{4(kb)^4}{\pi^2 k^2} \sum_{m=-I'}^{I'} I^2_{I'm}(\theta),$$

in agreement with the result of Ref. 1. From this equation we see that the cross section for elastic scattering is

$$\sigma_0(\theta) = \frac{4(kb)^4}{\pi^2 k^2} I^2_{00}(\theta).$$

The differential cross section for scattering into the direction Ω with the excitation of different nuclear rotational states is given by

$$\sigma^{IK}_s(\theta) = \frac{1}{2I+1} \sum_{I',M',K} |f^{I'M'K}_{IMK}(\Omega)|^2.$$

It is convenient to represent this cross section in another form (see Ref. 1):

$$\sigma^{IK}_s(\theta) = \frac{1}{2I+1} \sum_{M} \int d\omega\, |f(\omega,\Omega)\, \varphi_n(\omega)|^2.$$

Employing this equation with Eqs. (2) and (3) we find

$$\sigma^{IK}_s(\theta)$$

$$= \frac{1}{8\pi^2} \sum_{\varkappa,M} (-)^{M-K} C^{\varkappa 0}_{I-MIM} C^{\varkappa 0}_{I-KIK} \int d\omega D^{\varkappa}_{00}(\omega)\, |f(\omega,\Omega)|^2.$$

Furthermore, it is easily seen that,

$$\sum_{\varkappa, M} (-)^{M-K} C^{\varkappa 0}_{I-MIM} C^{\varkappa 0}_{I-KIK} D^{\varkappa}_{00}(\omega) = 1, \qquad (8)$$

hence,

$$\sigma_s^{IK}(\theta) = \frac{2}{\pi}\frac{(kb)^4}{k^2}\int_0^1 d\mu \xi^2(\mu) \int_0^{\frac{\pi}{2}} d\varphi \left[\frac{J_1(t)}{t}\right]^2 = \sigma_s(\theta). \qquad (9)$$

Thus, the differential scattering cross section $\sigma_s(\theta)$ does not depend on the initial state of the nucleus, i.e., on the index I, K. Graphs of the function $\sigma_s(\theta)$ for different degrees of nuclear deformation and different neutron energies have been published[1].

The integrated cross section $\sigma_s = \int d\Omega \sigma_s(\theta)$, does not depend on the energy (in agreement with the results of Ref. 1) and has the form

$$\sigma_s = \pi b^2 \int_0^1 d\mu \xi(\mu).$$

The dependence of σ_s on the degree of nuclear deformation was considered in Ref. 1.

The total cross section for all scattering processes is specified by the imaginary part of the amplitude for elastic scattering evaluated at $\theta=0$, i.e.,

$$\sigma_t^{IK} = \frac{4\pi}{K}\frac{1}{2I+1}\sum_M \operatorname{Im} f^{IMK}_{IMK}(\Omega)\Big|_{\theta=0}.$$

This reduces with the use of Eq. (5) to the total cross section $\sigma_t^{IK} = \sigma_t = 2\pi b^2 \int_0^1 d\mu \xi(\mu) = 2\sigma_s$. The cross section for capture is $\sigma_c = \sigma_t = \sigma_s = \dot{\sigma}_s$. Thus, the total neutron scattering cross section and absorption cross section do not depend on the initial state of the nucleus either.

It is easy to see that for the case of spherical nuclei the above formulas reduce to the formulas given by the diffraction of neutrons by "black" spherical nuclei.

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* Because of the reflection symmetry of the nucleus, the wave function (3) has to be symmetrized correspondingly[2]. However, for our purposes we can use the nonsymmetric expression.

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Concerning the Radiative Correction to the μ-Meson Magnetic Moment

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ANALYSIS of the structure of the present quantum theory of fields indicates its inapplicability to distances of the order of $\hbar/\lambda_0 \sim 10^{-13} - 10^{-14}$ cm[1]. Consequently, for those quantum-electrodynamic processes where momenta (real or virtual) of the order of λ_0 play a role, deviations from the usual formulas are to be expected. For the evaluation of these deviations in the integrals which appear in the determination of radiative corrections (integrals over virtual momenta) one can restrict the upper limit of integration to λ_0. Since these integrals converge for momenta $\sim m$, then the deviations should be of the order of m^2/λ_0^2, where m is the mass of the charged particle. This is why the deviations for the magnetic moment of the electron appear only in the third order radiative correction, viz., $\alpha^3 (\alpha = e^2/\hbar c)$ [2]. In the case of the heavier μ-meson, the finiteness of λ_0 affects the first radiation correction ($\sim \alpha$) and one expects a different value than predicted by Schwinger's formula. If one assumes that the μ-meson is devoid of any specific interactions which are greater than the electromagnetic one, then the problem can be treated as one in pure electrodynamics.

For the determination of the magnitude of the change in the radiative correction to the magnetic moment of the μ-meson (a change which is dependent upon the finiteness of λ_0) we shall consider, as is customary[3,4], the vertex portion of the scattering matrix of the third order Λ_i. The