width of $1/\mu$ enters into the cross section. This circumstance is due to the wave properties of the π-mesons. Actually, in the region effective for the process (in front of the nucleus) the ψ-function of the escaping meson has a shadow and the entire process is determined within the region of the penumbra.

If we set $\eta_{max} = \infty$, then

$$\sigma = (\pi/32)\, e^2 R/\mu \sim 10^{-28}\ \text{cm}^2. \qquad (10)$$

The preceding considerations were based on a specific model of the nucleus, viz., an "absolutely black" sphere of radius R. Employing a method developed in Ref. 2, one can generalize the problem to any arbitrary law of interaction between π-mesons and nucleons. The correction for semi-transparency which occurs for the case of strong absorption by heavy nuclei is of the order of $1/\mu R$ to the cross section for "black" nuclei.

The author expresses his sincere gratitute to I. Ia. Pomeranchuk for his valuable advice and continuing interest in this work.

[1] I. Ia. Pomeranchuk, Dokl. Akad. Nauk SSSR **96**, 265, 481 (1954).

[2] Iu. A. Vdovin, Dokl. Akad. Nauk SSSR **105**, 947 (1955).

[3] L. D. Landau and I. Ia. Pomeranchuk, J. Exptl. Theoret. Phys. (U.S.S.R.) **24**, 505 (1953).

Translated by A. Skumanich
155

Contribution to the Theory of Reactions Involving Polarized Particles

A. M. BALDIN AND M. I. SHIROKOV
P. N. Lebedev Physical Institute, Academy of Sciençes, USSR

(Submitted to JETP editor December 6, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 784-785 (April, 1956)

THE general theory of the correlation phenomena which depend on the law of conservation of the total angular momentum has been considerably developed in Refs. 1-3.

We have obtained the tensor momenta (see the determination below) for the two body problem in the most general case, when the incident and scattered particles and the particles of the target are in arbitrary spin states (for example, polarized).

The diagonality of the S-matrix with respect to the total impulse, total energy E, total angular momentum J and its projection M is made use of, and the transformation theory of Dirac[4] is applied systematically. The advantage of such an approach lies in the fact that it establishes a direct connection between its results and the conservation laws, allows errors and inexactitudes to be avoided more easily than in other approaches[2-5] and allows a direct generalization to the case of reactions involving more than two particles.

The reaction $a + b \rightarrow \Lambda + \theta$ was considered. Particles Λ and θ are in general different from a and b. All the particles have spins and rest masses not equal to zero (they could be either nuclei or "elementary" particles).

All the investigations are caried out in the center of mass system; hence, the indices of the total impulse and the diagonality with respect to them are not shown expressly.

The elements of the S-matrix have the form:

$$(i_\Lambda\, i_\theta s' l' \alpha' \mid S^{JE} \mid i_a\, i_b s l \alpha)\, \delta_{JJ'} \delta_{MM'} \delta(E' - E) \qquad (1)$$

where the symbols are as follows: i_Λ, i_θ are the particle spins, s and s' are the total orbital angular momenta of the two particles relative to the center of mass before and after the reaction, respectively, α and α' are the remaining variables which are not specified here (for example, variables connected with the internal state of the particles). Equation (1) is an expression of the conservation laws.

Since the magnitudes expressly shown above are not directly measured, but instead the directions of motion and the projections of the spins are measured, we must obtain the elements of an S-matrix corresponding to experiment from the elements of (1). In order to do this we must first of all go from the $s'l'JM$ representation to the $s'\nu' l'\mu'$ representation (the corresponding unitary transformation is the matrix of the Clebsch-Gordan coefficients $C^{JM}_{s'\nu' l'\mu'}$), and then with the help of the transformation

$$i^{l} Y^{*}_{l'\mu'}(\vartheta_\Lambda \varphi_\Lambda)\, C^{s'\nu'}_{i_\Lambda \lambda i_\theta n}\ {}^{1}$$

go from the $l'\mu' s'\nu'$ representation to the $\vartheta_\Lambda \varphi_\Lambda \lambda' n$ representation ($\vartheta_\Lambda \varphi_\Lambda$ are the spherical angles of the vector impulse of the particle Λ in the chosen system). The right side of (1) is transformed analogously.

The initial and final states in the problems under consideration must be assigned not to wave functions but to density matrices[6] (for example, an unpolarized beam of particles must be assigned to the density matrix $\delta_{mm'}$). The density matrices of the initial and final states are connected by means of the S-matrix

$$\rho_f = S\rho_i S^+. \tag{2}$$

The average value of the operator $\hat{A}$, which operates on the spin variables, in the form of a density matrix $(m_1|\rho|m_2)$ as just described, is given by the formula

$$\bar{A} = \mathrm{Sp}\,\rho A = \sum_{m_1 m_2} (m_1 \mid \rho \mid m_2)(m_2 \mid \hat{A} \mid m_1). \tag{3}$$

If we use the Wigner-Eckart theorem[7], Eq. (3) transforms to

$$A^{q\varkappa} = (i \,\|\, \hat{A} \,\|\, i)\,\frac{(-1)^q}{\sqrt{2q+1}}\,T^q_{\varkappa}, \tag{4}$$

where

$$T^q_{\varkappa} \tag{5}$$

$$= \sqrt{2i+1}\sum_{m_1 m_2}(-1)^{-i+m_1} C^{q\varkappa}_{i-m_1;\, im_2}(m_1 \mid \rho \mid m_2),$$

so that in order to find $A^q_{\varkappa}$ directly, the magnitude of $T^q_{\varkappa}$ rather than the density matrix is needed.

The assignment of magnitudes $T^q_{\varkappa}$ completely determines the density matrix, and conversely. Equation (5) may be considered as the transformation of the density matrix from the $m_1 m_2$ representation to the q, $\varkappa$ representation.

We call the magnitudes $T^q_{\varkappa}$ the tensor momenta (compare Refs. 1 and 3).

In this new representation the corresponding transformation of the spin operator matrices takes the form

$$\hat{A}_{JM} = \frac{1}{\sqrt{2i+1}} \tag{6}$$

$$\times \sum_{m_1 m_2}(-1)^{i+m} C^{JM}_{i-m_1;\, im_2}(m_2 \mid \hat{A} \mid m_1).$$

Using Eq. (2) and carrying out the corresponding unitary transformation, we obtain from $T^{q_a q_b}_{\varkappa_a \varkappa_b}$. The magnitudes of $T^{q_\Lambda q_\theta}_{\varkappa_\Lambda \varkappa_\theta}$ are determined in terms of

$$[(2i_\Lambda + 1)(2i_\theta + 1)]^{1/2}$$

$$\times \sum_{m_\Lambda m'_\Lambda}(-1)^{-i_\Lambda - m_\Lambda - i_\theta - m_\theta} C^{q_\Lambda \varkappa_\Lambda}_{i_\Lambda - m_\Lambda i_\Lambda m'_\Lambda}$$

$$\times C^{q_\theta \varkappa_\theta}_{i_\theta - m_\theta i_\theta m'_\theta}(\mathbf{n}_\Lambda E_\Lambda m_\Lambda m_\theta \alpha \mid \rho \mid \mathbf{n}_\Lambda E_\Lambda m'_\Lambda m'_\theta \alpha)\,],$$

the matrix elements (1), the Clebsch-Gordan coefficients and the spherical harmonics, a somewhat cumbersome expression for $T^{q_\Lambda q_\theta}_{\varkappa_\Lambda \varkappa_\theta}$.

After simplifications, the final formula takes the form:

$$T^{q_\Lambda q_\theta}_{\varkappa_\Lambda \varkappa_\theta}(\mathbf{n}_\Lambda, E_\Lambda, \alpha') = \sum_{q'\varkappa'} C^{q'\varkappa'}_{q_\Lambda \varkappa_\Lambda q_\theta \varkappa_\theta} \tag{7}$$

$$\times \sum_{s_1 s_2'}[(2q_\Lambda + 1)(2q_\theta + 1)(2s'_1 + 1)(2s'_2 + 1)]^{1/2}$$

$$\times X(i_\Lambda q_\Lambda i_\Lambda;\, s'_1 q' s'_2;\, i_\theta q_\theta i_\theta)\, T^{q'}_{\varkappa'}(\mathbf{n}_\Lambda, E_\Lambda s'_1 s'_2 \alpha'),$$

where

$$T^{q'}_{\varkappa'} = \frac{\lambda_a^2}{4}[(2i_\Lambda + 1)(2i_\theta + 1)]^{1/2}$$

$$[(2i_a + 1)(2i_b + 1)]^{-1/2}\sum(-1)^{q'+q+\varkappa'+\varkappa} \times$$

$$\times [(2s_1 + 1)(2s_2 + 1)(2s'_1 + 1)(2s'_2 + 1)]^{1/2} G_{\varkappa'}$$

$$\times (J_1 l'_1 s'_1;\, Jq;\, J_2 l'_2 s'_2)$$

$$\times G^*_{\varkappa}(J_1 l_1 s_1;\, Jq;\, J_2 l_2 s_2)(i_\Lambda i_\theta s'_1 l'_1 \alpha \mid R^{J_1}(E) \mid i_a i_b s_1 l_1 \alpha)$$

$$\times (i_\Lambda i_\theta s'_2 l'_2 \alpha' \mid R^{J_2}(E) \mid i_a l_b s_2 l_2 \alpha)\, D^J_{\varkappa\varkappa'}(\varphi_\Lambda, \vartheta_\Lambda, 0)\, T^q_{\varkappa};$$

$$T^q_{\varkappa}(\mathbf{n}_a, E_a, s_1, s_2, \alpha)$$

$$= \sum_{q_a q_b}[(2q_a + 1)(2q_b + 1)(2s_1 + 1)(2s_2 + 1)]^{1/2} \tag{8}$$

$$\times X(i_a q_a i_a;\, s_1 q s_2;\, i_b q_b i_b) \tag{9}$$

$$\times \sum_{\varkappa_a \varkappa_b} C^{q\varkappa}_{q_a \varkappa_a q_b \varkappa_b} T^{q_a q_b}_{\varkappa_a \varkappa_b}(\mathbf{n}_a, E_a, \alpha)$$

The summation Σ in Eq. (8) is to be taken over $J_1 J_2 : l'_1 l'_2 l_1 l_2 s_1 s_2 J_q$ and over $\varkappa$.

The coefficients $G_{\varkappa}$, X are determined in Refs. 3 and 8. $D^J_{\varkappa\varkappa'}(\varphi_\Lambda, \vartheta_\Lambda, 0)$ is the matrix element corresponding to the irreducible representation of the group of rotations of weight J.

The connection of our tensor momenta with the density matrix is different from the connection between the density matrix and the tensor momenta which were derived by Simon (see the corrections to Refs. 2 and 3), but when normalized they are the same. In the particular case when $q_\theta = 0$ and $q_b = 0$ our expression yields a formula which differs from Eq. (3.2) of Ref. 3 by several factors. The distinction basically depends on the difference in the definition of the tensor momenta. This distinction does not affect the final results, as compared with experiment, since a change in the definition of the tensor momenta must be accompanied by a corresponding change in the derived matrix element.

An essential factor which is needed under the summation sign Σ, as compared with Eq. (3.2) of Ref. 3, is $(-1)^{\varkappa}$.

In conclusion, we express our gratitude to Prof. M. A. Markov for his constant interest in the work. We also thank L. G. Zastavenko for his advice on a number of questions concerning the theory of representations of rotation groups.

* As has been pointed out in Ref. 5, spherical harmonics must be preceded by i^l, otherwise the result of the action of the inverse time operator will not be invariant with respect to complex angular momenta.

[1] J. M. Blatt and L. C. Biedenharn, Rev. Mod. Phys. **24**, 258 (1952)

[2] A. Simon and T. A. Welton, Phys. Rev. **90**, 1036 (1953); **93**, 1435 (1954) (errata).

[3] A. Simon, Phys. Rev. **92**, 1050 (1953); Phys. Rev. **93**, 1435 (1954) (errata).

[4] P. A. M. Dirac, *The Principles of Quantum Mechanics*, Oxford.

[5] R. Huby, Proc. Phys. Soc. (London) **A67**, 1103 (1954).

[6] D. I. Blokhintsev, *Fundamentals of Quantum Mechanics*, GTTI, 1949.

[7] G. Racah, Phys. Rev. **62**, 438 (1942).

[8] A. Arima, H. Horie and Y. Tanabe, Progr. Theoret. Phys. **11**, 143 (1954).

Translated by M. G. Gibbons
156

Scattering of Fast Neutrons by Nonspherical Nuclei. III

S. I. DROZDOV

(Submitted to JETP editor December 26, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 786-788 (April, 1956)

IN a previous communication[1] an effective cross section was calculated for the scattering of fast neutrons by a "black" nucleus having the form of an ellipse of rotation and a zero spin (even-even nucleus). In the present note these calculations are generalized to nuclei with spins different from zero (odd nuclei). In the adiabatic approximation the effective cross section is determined by the matrix element

$$f_{n'n}(\Omega) = \int d\omega \varphi^*_{n'}(\omega) f(\omega, \Omega) \varphi_n(\omega). \quad (1)$$

The amplitude for the scattering of a neutron by a stationary nucleus is:

$$f(\omega, \Omega) = i \frac{(kb)^2}{k} \xi(\mu) \frac{J_1(t)}{t}; \quad (2)$$

$$t = kb\theta [\xi^2(\mu) \cos^2(\Phi - \varphi) + \sin^2(\Phi - \varphi)]^{1/2},$$

where $\xi(\mu) = [z^2 + (1 + z^2)\mu^2]^{1/2}$, $z = a/b$. The quantity b is the radius of the largest circular cross section of the ellipsoid and $2a$ is the length of the axis of symmetry. The spherical angles ϑ and φ specify the direction of the axis of symmetry ω and the angles θ, Φ, the direction of scattering Ω. The polar axis of the external coordinate system is chosen to lie in the direction of the incident beam. We shall assume a strong coupling between the motion of the nucleons in the nucleus and the motion of the nuclear surface[2]. In this case the wave functions for the rotational states of the nucleus $\varphi_n(\omega)$ can be represented by the proper functions of the symmetric top*,

$$\varphi_n(\omega) = \sqrt{(2I + 1)/8\pi^2} D^I_{MK}(\omega), \quad (3)$$

where I is the total nuclear moment, M and K are the projections onto the external axis and axis of nuclear symmetry, repsectively, and $\omega = (\varphi_1, \theta, \varphi_2)$ represents the Eulerian angles which describe the orientation of the nucleus relative to the external coordinate system. The rotational states of the nucleus are given by