Ionization Spectrum of the Cosmic Ray Soft Component at Sea Level

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A new method for the investigation of ionization spectra of cosmic ray particles is worked out. Ionization spectra are taken for particles with penetration in the intervals 2-3, 3-5, 5-9, and 9-15 cm of lead. Conclusions are reached about the distribution of the proton component at sea level among these penetration intervals.

1. INTRODUCTION

BY investigating the ionization spectra of cosmic ray particles of known penetration, it is possible to determine the composition of cosmic rays.[1] The basic difficulty of this method is the poor separation of various mass particles. The insufficient resolution is determined by the nature of the ionization energy loss fluctuations, which for most cosmic ray particles, obey the Landau distribution.[2] The distinguishing characteristic of this distribution is the high probability of fluctuations in the direction of large energy loss. However, it is particularly in the region of large ionization that the ionization spectra of hyperons, protons, or K-mesons of the soft cosmic ray component are located. Since the percentage of these particles in the composition of cosmic rays at sea level or at mountain altitudes is relatively small, it turns out that the ionization spectra of these particles can not be successfully separated from those parts of the ionization spectra of cosmic ray μ-mesons and electrons which are caused by fluctuations in the direction of large energy loss. In the method proposed below, this difficulty is overcome to a large extent.

2. THE BASIC IDEA

Let each particle of the analyzable particle flux pass successively through n layers of matter, in each of which it loses some energy by ionization. The basic idea of our method is to measure the smallest value of the totality of ionization losses $\Delta_1, \Delta_2, \ldots \Delta_n$. A suitable arrangement of equipment automatically selects this value for measurement, and rejects all others. We assume here that the material of all layers is the same, and that all layers have the same thickness.*

*The selection of the smallest value of ionization as an experimental method using two gas proportional counters was proposed by L. D. Landau and first carried out by Nikitin.[1]

Let us examine, in such a measuring set up, the probability of finding a particle in the interval of ionization loss Δ, $\Delta + d\Delta$. If the ionization loss in each of the n identical layers obeys a Landau distribution, the desired probability is

$$n\frac{1}{\xi}\varphi\left(\frac{\Delta-\Delta_0}{\xi}\right)\left[\psi\left(\frac{\Delta-\Delta_0}{\xi}\right)\right]^{n-1}d\Delta = \frac{1}{\xi}\Phi_n\left(\frac{\Delta-\Delta_0}{\xi}\right)d\Delta, \tag{1}$$

where we introduce the general function

$$\Phi_n(\lambda) = n\varphi(\lambda)\left[\psi(\lambda)\right]^{n-1}, \tag{2}$$

in which φ and ψ are Landau's general functions[2] (see Ref. 2 for remaining definitions). Thus, instead of an ionization spectrum which can be described by Landau's distribution function

$$f(x, \Delta) = \frac{1}{\xi}\varphi\left(\frac{\Delta-\Delta_0}{\xi}\right), \tag{3}$$

the above method of measurement will yield a spectrum which obeys the distribution

$$f_n(x, \Delta) = \frac{1}{\xi}\Phi_n\left(\frac{\Delta-\Delta_0}{\xi}\right). \tag{4}$$

Let us now evaluate the probability of ionization loss larger than a particular value, i.e., we calculate the integral

$$S(\Delta_a) = \int_{\Delta_a}^{\infty} f_n(x, \Delta)\, d\Delta. \tag{5}$$

Since we are interested in large values of Δ_a, we shall use the asymptotic expressions for the general functions φ and ψ, as developed in Landau's paper. We then easily obtain, for large values of Δ_a,

$$S(\Delta_a) = 1/\omega_a^n, \tag{6}$$

where ω_a is connected to Δ_a by the equation

$$(\Delta_a - \Delta_0)/\xi = \omega_a + \ln \omega_a - 0.37. \quad (7)$$

It follows from Eq. 6 that the number of particles with ionization higher than the given value Δ_a is $\omega_a^{\,n-1}$ times smaller with the present method of measurement (i.e., selection of the smallest value) than with the ordinary method where the particle passes through only a single layer of matter (a single proportional counter). For the majority of cosmic ray particles and the counters used in the work described here, $\omega_a \approx 15$ if $\Delta_a = 2\,\Delta_0$. This means that if ionization spectra are taken with three counters ($n = 3$) the number of μ-mesons or electrons with more than twice the expected value of ionization is some 200 times smaller than for ionization spectra taken with a single counter. Consequently, with a large enough number of counters, it is possible to measure the ionization spectra of hyperons or protons of the soft cosmic ray component under conditions almost completely free of background caused by parts of meson and electron ionization spectra.

3. DESCRIPTION OF THE EQUIPMENT

We have chosen to use scintillation counters as a method of measuring ionization loss. The possibility of using scintillation counters for this purpose, and their superiority to gas proportional counters has been proved earlier by Meshkovskii and Shebanov[3] and other workers.[4,5]

The physical arrangements are shown in Fig. 1. Here 1 – 5 are five layers of scintillator material. The scintillators, especially prepared for this experiment, were polystyrene plastic activated with 1, 1, 4, 4 – tetraphenylbutadine – 1, 3 in a concentration of 1.8 %. The polymerization of the doubly distilled styrene was carried out at 200° C in the course of 12 – 15 hours. The tetraphenylbutadine was prepared by the method of Valeur.[6] Scintillater size was 1 × 3.5 × 7.5 cm. After machining, the scintillater plates were carefully polished, mounted in reflecting housings and glued to the photocathodes of type FEU – 19 photomultipliers for better optical contact.

For this experiment, exceptional photomultipliers were selected in order to give the least instrumental line broadening. A scintillation γ spectrometer based on the Compton effect was set up, and photomultipliers selected for least line width from a Co^{60} source.

The numbers 6 – 10 on Fig. 1 are lead filters. The thickness of filter 6 was chosen such that the penetration of particles passing through it was greater than 2 cm of lead. The roman numerals denote arrays of Geiger-Muller counters. For the measurement of ionization spectra of particles with 2 – 3 cm penetration, a gating pulse due to the coincidence of I + II + III – IV – IVa – V was used; for particles with penetration 3 – 5 cm — the coincidence I + II + III + IV – IVa – V – Va – VI ; for penetration 5 – 9 cm. — I + II + III +V – VI – VIa – VII; for penetration 9 – 15 cm.— I + II + III + VI – VII; for penetration greater than 9 cm. — I + II + III + VI.

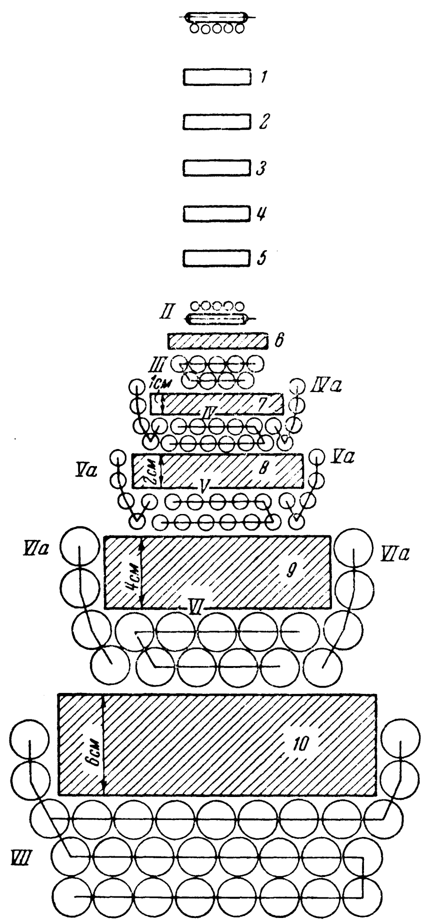

FIG. 1. Diagram of the apparatus

In the selected solid angle, path length of particles through the scintillaters varied by ± 5%. Material over the apparatus was kept to a minimum (0.5 cm plywood and roofing iron). The apparatus was placed in a thermostatically controlled enclosure where the temperature was held constant to ± 0.2°. The temperature controlled enclosure was found necessary in order to keep the photomultiplier gains constant. Voltages on the photomultipliers was held constant to ± 0.1 %.

Photomultiplier gains were checked once in 24 hours by the following method. Simultaneously with the measurements described above, integral ionization spectra of the cosmic ray hard component were taken for all five scintillation counters. Using the integral spectra taken in the course of twenty four hours, the maxima of the differential spectra were found by a method described in reference 3. In the event of a shift in the position of the maximum for any of the counters, the high voltage on the photomultiplier was adjusted. Over long periods of operation the equipment showed no sharp fluctuations of photomultiplier gain as long as the temperature and voltage were held constant within the stated limits.

A block diagram of the electronics is shown in Fig. 2. Pulses from scintillation counters 1 – 5 were put through linear amplifiers with negative

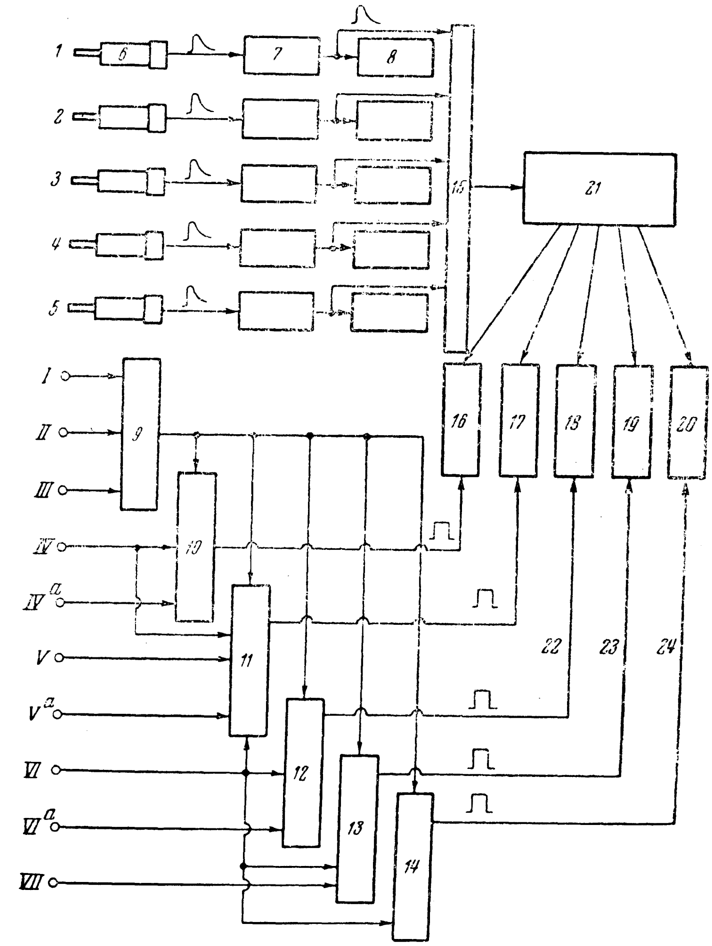

FIG. 2. Block diagram of the electronics: *1 – 5* — scintillationcounters, *6* – FEU – 19 photomultiplier, *7* – amplifier, *8* – integral analyser, *9 – 14* – coincidence circuits, *10 – 13* anticoincidence circuits, *15* – circuit to select the smallest pulse, *16 – 20* – groups of 25 registers, *21* – 25 channel pulse height analyser, *22 – 24* – gating pulses, *I – VII* –arrays of Geiger-Muller counters.

feedback. Subsequently, pulses were fed into integral registers, as well as a circuit which selected the smallest pulse. The latter consisted of a coincidence circuit using crystal diodes. The linearity of this minimum pulse selection circuit was checked with generator in the amplitude interval 2 – 40 V.

The selected smallest pulse was measured by a 25 channel differential pulse height analyser, which made use of two type LP – 1 electronic switches.* Each channel of the pulse height

*The differential analyzer was built by E. V. Kuznetsov, to be published.

analyser was connected to five mechanical registers. Gating pulses, corresponding to particular intervals of particle penetration were also fed to the registers. The output of the pulse height analyser was registered only if the gating pulse was present. Thus a single 25 channel analyser fed 125 mechanical registers, and made it possible to take 5 simultaneous ionization spectra corresponding to various intervals of particle penetration.

In the interpretation of heavy particle ionization spectra it was important to know that events recorded in the region of high ionization were not the result of the passage of several particles. To eliminate multiple events from the results, we constructed a special system.

Sets I and II of 5 mm diameter Geiger-Muller tubes each consisted of two layers, the tube axes of one layer being perpendicular to those of the other. Thus sets I and II defined a coordinate system. A circuit with a neon lamp (type MTX – 90) was connected to each counter tube in sets I and II in such a way as to light when a charged particle passed through the counter tube. A similar circuit was connected to each of the sets of counters III, IV, IVa, V, VI, VIa, and VII. Neon lamps were also connected to each channel of the pulse height analyser. All lamps were displayed on a single panel.

During passage of a particle through the system (coincidence I + II + III), the corresponding neon lamps lit, and the panel was photographed by a motion picture camera. In this way it was possible to differentiate between the passage of a single particle and the passage of several and at the same time measure the amount of ionization caused by either type of event. A system to register the multiple events could work simultaneously with the equipment used to make the basic measurements.

4. EXPERIMENTAL RESULTS

Measurements were carried out during 1300 hours of equipment operation. During 900 of these hours, i.e., about 70% of the total time, multiple events were registered in addition to the basic measurements. During the 900 hours, in the soft component there was only one case of simultaneous passage of two particles through the equipment (and 5000 single events). The one multiple event was found in the penetration interval 2– 3 cm lead and showed twice minimum ionization. The measurement of multiple passage events shows that this type of background was essentially absent in our ionization spectra of the soft component of cosmic rays.

The ionization spectra are shown in Figs. 3–6.

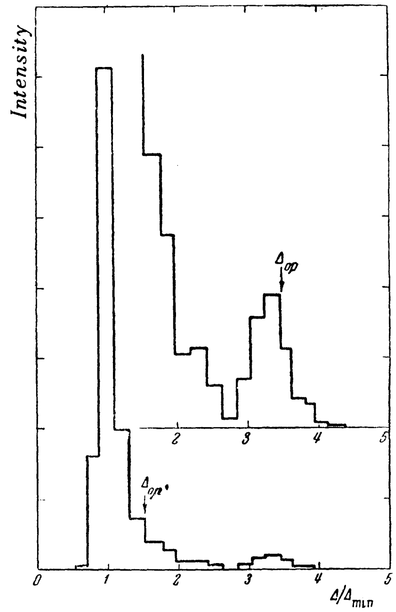

FIG. 3. Ionization spectrum for particles with penetration of 2–3 cm Pb.

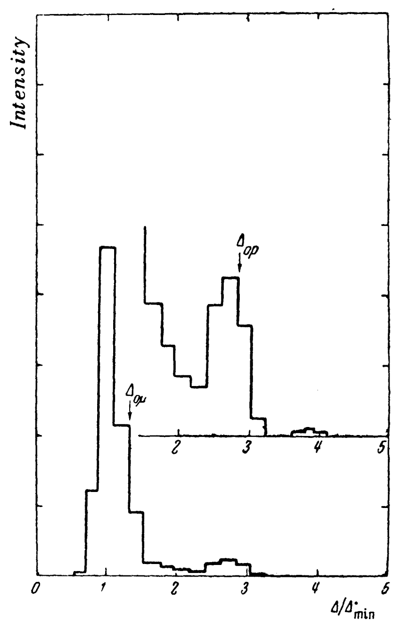

FIG. 4. Ionization spectrum for particles with penetration of 3–5 cm Pb

The abscissa is ionization in units of minimum ionization, the ordinate is the relative intensity in a particular differential interval of ionization. In the upper portions of Figs. 3 – 6 the right portion of each spectrum is shown in a larger scale. We note that two groups are clearly seen on each drawing. As will be shown in Sec. 5, the right group is the ionization spectrum of the proton component of cosmic rays. The left group corresponds to mesons and electrons of the soft component.

As has been described above, in our method of measuring ionization an electronic circuit automatically selects the smallest pulse from an array of five scintillation counters for measurement. This method of measurement can lead to serious errors if there are background pulses in a single counter which are small compared to the pulse magnitude at the maximum of the ionization spectrum, which, according to Landau's theory, decreases very rapidly in the direction of small energy losses. It is clear that the appearance of such small pulses which have nothing to do with the ionization spectrum can strongly distort the results of our method of measurement. To investigate this possibility we performed a separate experiment in which the ionization spectrum of the hard component was measured with a single scintillation counter, and the position of the maximum was found in the vicinity of the proton maximum(see Fig. 3). The results, taken with good statistical accuracy, showed the absence of pulses to the left of the ionization curve of a single scintillator.

Another control experiment evaluated the background due to possible inefficiencies of the anti-coincidence schemes. Results showed that the anti-coincidence arrays of our apparatus were at least 99.9% efficient, i.e., the number of particles which are falsely registered as having been absorbed in a particular filter do not exceed 0.1 % of the number of particles passing through the filter. In the proton spectrum region this anti-coincidence inefficiency can give no visible background since a majority of the particles passing through a given filter of our apparatus are mesons whose background in the region of high ionization is already eliminated by the selection of the smallest pulse. An insignificant background (order 2 – 3%) due to this inefficiency applies to the left portion of the spectra (Figs. 3 – 6).

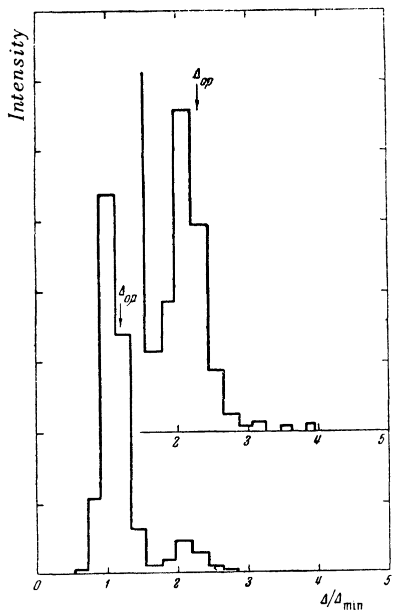

FIG. 5. Ionization spectrum for particles with penetration of 5–9 cm Pb.

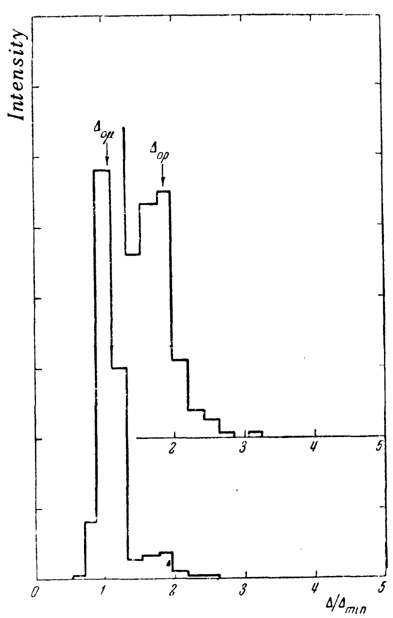

FIG. 6. Ionization spectrum for particles with penetration of 9 – 15 cm Pb.

5. DISCUSSION OF THE RESULTS

For interpretation of the results the abscissa of figures 3 – 6 were given in units of minimum ionization $\Delta_{0\,min}$. Simultaneously with measurements of the soft component spectra, measurements were made of the hard component (particles with penetration greater than 9 cm of lead), and the position of the ionization maximum Δ_0 calculated by averaging the values of Δ_0 for mesons from a momentum spectrum of mesons at sea level.

The magnitude of the most probable value of ionization Δ_0 was calculated from the equation:

$$\Delta_0 = \xi [\ln \xi + 2 \ln (p/\mu) + \ln (2m/I^2) - \beta^2 + 0.37 - \delta] \quad (8)$$

where δ is a correction for density (Eq. 8 without the quantity δ has been given by Landau in Ref. 2).

We computed the density correction from data of Sternheimer.[7] As in that reference, the ionization potential I for styrene was taken as 60 ev.

For the calculation of Δ_0 we used the known meson spectrum as given by Rossi,[8] taking into account more recent work.[9-20] The results of these workers, together with Rossi's spectrum are plotted in Fig. 7. The results of references 14

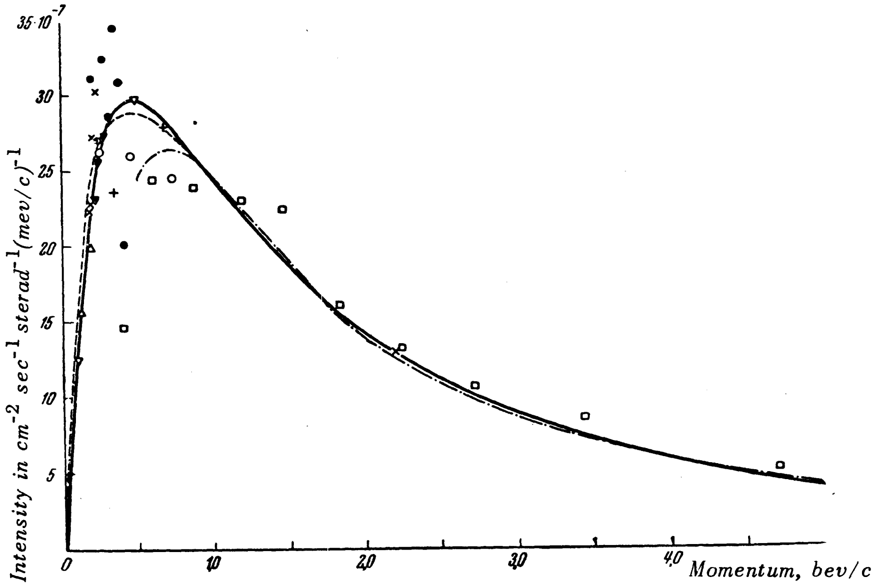

FIG. 7. Meson spectrum at sea level. —Rossi[8], —Owen and Wilson[20], Δ— Ref. 9, ▼— Ref. 10, O—Ref. 12, ∇— Ref. 13, +— Ref. 16, □— Ref. 17, × — Ref. 18, ●—Ref. 19.

and 15 are not shown because they differed too much from all others; results of Glaser et al[11] are not shown because their spectrum is very similar to Rossi's. We see that notwithstanding the large number of papers which came out after Rossi's review, no important modifications need be made. We feel, however, that some changes should be made in the regions of high and low momenta . For momenta $p > 10000$ mev/ c we used the exponential law $S(p) \sim p^{-2.44}$, as found by Owen and Wilson ref 20. For momenta below 1 000 mev/ c we adopted the spectrum given by a dotted line on Fig. 7. For the region below 500 mev/c the curve is determined by taking $S(R)$ = const, if S is measured in gm^{-1} sec^{-1} $sterad^{-1}$ and R in gm / cm^2 of air. The existence of a plateau in this momentum region is shown in several papers.[12, 18, 19]

The result of averaging the values of Δ_0 on the meson spectrum, for the hard component, was $\overline{\Delta}_0 = 1.01\ \Delta_{0min}$. Thus the position of the maximum in the hard component spectra was calibrated in units of minimum ionization. The closeness of the magnitude of Δ_0 to that of minimum ionization is explained by our use of scintillation counters made of polystyrene, for which, as for some other organic scintillators,[3-5] theory predicts almost no increase of ionization with increase of particle energy due to the density effect, as expressed in Eq. 8.

In the calculation of Δ_0 for μ-mesons of the soft component we must take into account that in the described method of measuring ionization loss, i.e., by selection of the least loss, the maximum of the ionization spectrum are shifted to smaller values of ionization compared to the maxima obtained with a single counter. This circumstance follows from the fact that with selection of the least pulse the spectrum does not obey differential distribution of a single counter, but is the result of multiplying the differential curve by the ($n-1$) power of the integral curve, as has been mentioned in Sec. 2. It follows that the new position of the maximum depends on the shape of the distribution for a single counter. If the single counter obeys the Landau distribution, it is easy to derive an expression for the position of the ionization curve maximum Δ_{0n} which would be obtained in a system of n counters:

$$\Delta_{0n} = \Delta_0 (1 + \lambda_n \xi / \Delta_0), \tag{9}$$

where λ_n is the number for which the general function $\varphi_n(\lambda)$, introduced in Sec. 2, has a maximum. For example, in our case of five counters $\lambda_5 \approx -1.0$.

It follows from equation 9 that the shift of the maximum, i.e., the ratio Δ_{0n} / Δ_0, varies slowly with the speed of the particle since the magnitude of ξ / Δ_0 depends only little on speed, as seen in Eq. 8. Numerical calculation shows that, within the limits of Landau's theory, the maximum spread of values of Δ_{0n} / Δ_0 for $n = 5$ is not greater than 0.5%. Thus the apparatus described above can be considered to a sufficient degree of accuracy as a linear device with respect to the various groups of particles whose ionization losses obey the Landau distribution.

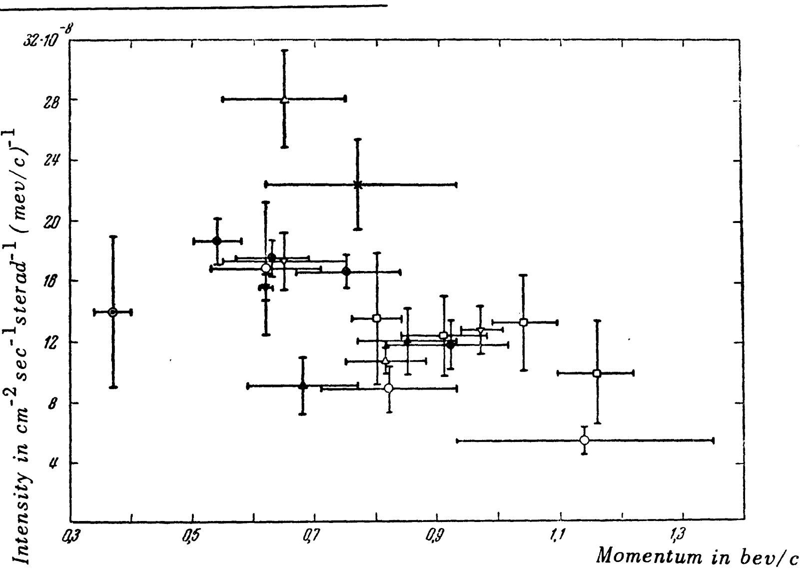

FIG. 8. Proton spectrum of cosmic rays at sea level: ⊙—Ref. 22, O— Ref. 24, Δ — Ref. 18, ∇— Ref. 25, ▼ — Ref. 26, ▲— Ref. 27, ◘— Ref. 28, ●—data of this paper.

Proceeding from this it is possible to calculate the value of Δ_0 for μ-mesons in the penetration intervals we investigated without the use of Eq. 8. These values are shown in Figs. 3–6 and are labeled with the letter $\Delta_{0\mu}$. The absence of a well defined μ-meson maximum in Figs. 3–6 is not surprising, since according to the literature[19] there are few μ-mesons in the soft component of cosmic rays. The left ends of the spectra in these figures are chiefly caused by electrons. In particular these particles are in the penetration interval 2–3 cm of lead, in which the number of μ-mesons is about 15% of the number of electrons.

It can be seen in Fig. 3 that the maximum of the electrons spectrum coincides with the value of minimum ionization, as would be expected from calculation with Eq. 8.

In calculating the magnitude of Δ_0 for protons in the measured penetration intervals we can not use Eq. 8, which comes from Landau's theory, since for protons of this penetration and for such counters as we used the basic condition for applicability of the theory is not fulfilled:

$$G = (\xi / E_{\max}) \ll 1 \tag{10}$$

($E_{\max}$ is the maximum transferable energy). Therefore we made use of Symon's results, as described in Rossi's book[21], where calculations are made for more general cases, up to $G = 10$.

Using Symon's results we also calculated the shift of the maximum Δ_{0n} / Δ_0 for particles with various large values of G. It turned out that the value of Δ_{0n} / Δ_0 for such particles is quite close to that of Δ_{0n} / Δ_0 for particles obeying a

Landau distribution. Variations in this quantity of more than 0.5% begin only for $G \gtrsim 1.5$. Since the protons in the measured penetration interval have $G < 1.4$, we may conclude that our equipment is linear for the position of the proton spectra maxima.

Penetration interval in cm of lead	Observed number of protons: Data of preliminary measurements	Observed number of protons: Data from which the spectra of Figs. 3–6 were obtained	The number of protons in percent: of all particles in the penetration interval	The number of protons in percent: of all particles with penetration greater than 9 cm lead.
2 — 3	47	95	4.34 ± 0.45	0.193 ± 0.017
3 — 5	70	109	7.33 ± 0.68	0.247 ± 0.019
5 — 9	75	166	8.62 ± 0.70	0.350 ± 0.023
9 — 15	—	130	7.90 ± 1.26	0.295 ± 0.041

Results of the calculation of Δ_0 according to Symon are indicated in Figs. 3–6 by the letter Δ_{0p}. We see that the positions of the right spectrum maxima are quite close to the calculated value Δ_{0p}, although still different. In particular the experimentally observed position of the maxima is smaller than the theoretical value by 4–5%. We have not been able to formulate a sufficiently convincing hypothesis to explain this discrepancy.

From the above discussion it follows that the right portion of the curves in Figs. 3–6 are the ionization spectra of the proton component of cosmic radiation. It is then possible to draw conclusions about the number of protons in cosmic rays at sea level in the measured penetration intervals. The results are given in a table, and presented as a momentum spectrum of protons (Figure 8.) where the results of previous work are also given[18, 22-28]. In drawing our points on Fig. 8 and in calculating the statistical errors we took into account the data of preliminary experiments which are not included in the spectra of Figs. 3–6, although indicated in the table. With regard to the number of protons the results of the preliminary experiments are as accurate as the results which were used to construct the spectra of Figs. 3–6. The absolute intensity of the vertical proton flux was evaluated relative to the mesons of more than 9 cm lead penetration, whose flux was taken as 0.81×10^{-2} cm^{-2} sec^{-1} sterad^{-1}, in accordance with the spectrum of figure 7 and the exponential law $p^{-2.44}$ for $p > 10000$ mev/c.

In conclusion we express deep appreciation to V. P. Rumiantsev for aid in the work, and to Iu. F. Orlov for fruitful discussions of the results.

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Generalized Method for Calculating Damping in Relativistic Quantum Field Theory

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An infinite system of coupled equations is constructed, of which each describes a process involving the emission and absorption of a certain number of particles. This system is shown to be equivalent to the Tomonaga-Schwinger equation. The solution, which is derived by a process of successive substitutions, leadsto results for the S-matrix which generalize the results ofthe theory of radiation damping.

IT has often been pointed out that the solution ofthe problems ofthe quantum theory of fields by means of perturbation theory leads to difficulties in a number of cases. In order to eliminate these difficulties, and to study the limits of applicability of the results obtained by means of perturbation theory, one has to develop a more consistent method of solving the equations of field theory. The present paper represents an attempt to develop a method of solution which guarantees that the normalization remains correct, and from which one obtains as an approximation the results of damping theory, and, as a zero-order approximation, perturbation theory.

We start from the usual equation for the scattering matrix:

$$i\hbar c\delta U[\sigma]/\delta\sigma(x) = \mathcal{H}(x)\,U[\sigma], \qquad (1)$$

where $\mathcal{H}(x)$ as usual, is the operator of the interaction Hamiltonian, which, if we limit the discussion to the case ofthe electron-positron and the electromagnetic fields (we consider these fields for definiteness, although the result can be generalized directly) takes the form

$$\mathcal{H}(x) = -j_\mu A_\mu \qquad (2)$$

$$= -\frac{e}{2}\,[\bar{\psi}(x)\gamma_\mu\psi(x) - \psi(x)\bar{\psi}(x)\gamma_\mu]\,A_\mu(x).$$

The operators $\bar{\psi}$, ψ, A_μ satisfy the equations of free fields and therefore, if we write

$$\bar{\psi}(x) = \bar{u}(x) + v(x);\ \ \psi(x) = u(x) + \bar{v}(x); \qquad (3)$$

$$A_\mu(x) = A_\mu^{(+)}(x) + A_\mu^{(-)}(x),$$

we may interpret $A_\mu^{(+)}$, $\bar{u}$ and v as free-particle creation operators (creating photons, electrons and positrons, respectively) and $A_\mu^{(-)}$, u and $\bar{v}$ as absorption operators forthe same free particles.

This basic equation is quite general, i.e., it describes any arbitrary process involving the creation and annihilation of particles in arbitrary states due to their mutual interaction. Moreover, as has already been pointed out,[1] Eq. (1) does not describe each ofthese processes separately, but contains them all simultaneously. In the process of solving the equation by perturbation theory, this connection between different processes is lost sight of in practice, and each is considered in isolation. We shall try to put forward a method of solution which is free from this disadavantage, and which uses this connection between different processes as a starting-point.

The physical picture behind this change is the following: As a result of the interactions between the particles the probability of the initial state decreases in the course of time, while simultaneously the probabilities of the mutually com-