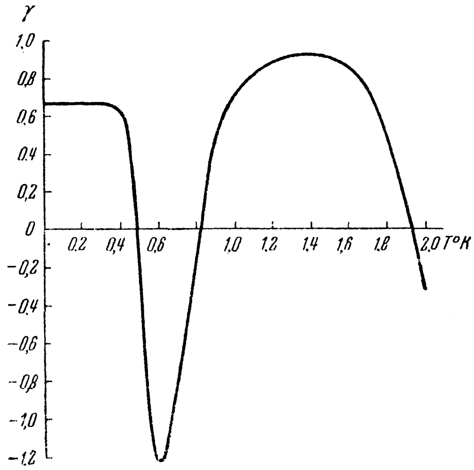

effect upon the velocity of second sound in a standing wave is of the order of a fraction of a percent:

In conclusion, I thank L. D. Landau for his consideration of these results.

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Remarks on One Variant of the Equations of a Nonlocal Field

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IN a nonlocal field theory, where the wave-function of a particle $U(x_\mu, \xi_\mu)$ depends on "internal" variables ξ_μ as well as the ordinary space-time variables x_μ ($\mu = 1, 2, 3, 4$) it is natural to interpret the orbital angular momentum of "internal" motion as the intrinsic angular momentum (spin) of the particle. We will assume the function U to be scalar; then the equation of Markov[1] for a free particle in the momentum representation has the form:

$$\left\{\omega^2 k_\mu^2 - \frac{\partial^2}{\partial r_\mu^2} + r_\mu^2 + \frac{2}{k_\mu^2}\left[\left(k_\mu \frac{\partial}{\partial r_\mu}\right)^2 - (k_\mu r_\mu)^2\right]\right\} \chi(k_\mu, r_\mu) = 0, \quad (1)$$

where

$$\chi(k_\mu, r_\mu) = \int \exp\{-ik_\mu x_\mu\}\, U(x_\mu, r_\mu)\, d^4x,$$

and λ and ω are constants with the dimensions of lengths. We introduce the notation $f = -\omega^2 k^2_\mu$; this is the square of the mass measured in units of $\hbar/\omega$. In a rest system Eq. (1) assumes the form

$$\left(-\frac{\partial^2}{\partial r_i^2} + r_i^2 - \frac{\partial^2}{\partial r_0^2} + r_0^2\right)\chi = f\chi, \quad (2)$$

where $i = 1, 2, 3$ and r_0 is the real variable $r_0 = -ir_4$. The solutions of (2) will be sought in the form $X(r_\mu) = g(r_i)\,\Phi(r_0)$, separating the dependence on space and time variables. On the function X we impose the requirement of boundedness in all of the 4-space of internal coordinates.

For the functions $g(r_i)$ and $\Phi(r_0)$ we get the equations

$$(-(\partial^2/\partial r_i^2) + r_i^2)\, g = kg, \quad (3)$$

$$(-(\partial^2/\partial r_0^2) + r_0^2)\, \Phi = (f - k)\, \Phi, \quad (4)$$

where k is a constant of separation of variables. The solution of Eq. (3) in spherical coordinates $r = |\mathbf{r}|, \theta, \varphi$, as is well known, has the form

$$g_{klm}(r, \theta, \varphi) = Y_{lm}(\theta, \varphi)\, r^l e^{-r^2/2} L^{l+1/2}_{(k-2l-3)/4}(r^2),$$

where Y_{lm} is the spherical function and L is the associated Laguerre polynomial. Here the quantity k assumes the values

$$k = 4n + 2l + 3, \quad (5)$$

where $l = 0, 1, 2, \ldots$; $n = 0, 1, 2 \ldots$

Thus for given k the internal angular momentum l can assume the values $0, 2, \ldots, (k-3)/2$ or $k, 3, \ldots, (k-3)/2$ depending on whether k is odd or even. The projection of the internal angular momentum m assumes the values $|m| \leqslant l$.

Equation (4) has bounded solutions only for

$$f - k = 2n_0 + 1, \quad n_0 = 0, 1, 2, \ldots \quad (6)$$

Its solution then has the form

$$\Phi_{n_0}(r_0) = H_{n_0}(r_0)\, e^{-r_0^2/2},$$

where H_n is the Hermite polynomial. From condition (6) we obtain that for given f the quantity k can assume the values $3, 5, \ldots, f - 1$. However, from conditions (5) and (6) it is evident that f can assume the values $2\beta + 4$

where $\beta = 0,1,2, \ldots$.

For given f the desired solution of (2) has the form

$$\chi_{fklm}(r, \theta, \varphi, r_0) = g_{klm}(r, \theta, \varphi)\, \Phi_{(f-k-1)/2}(r_0)$$

and is degenerate with multiplicity $(\beta+1)(\beta+2)(\beta+3)/6$. The obtained solutions make it possible to classify all states according to their mass, internal angular momentum and "internal time number" k . Thus, for $f = 4$ we have the lowest nondegenerate state, in which $k = 3$ (corresponding to $n_0 = 0$) and the angular momentum $l = 0$. For $f = 6$, we have two states with different spins; in the first $k = 3$, $l = 0$ and in the second $k = 5$, $l = 1$. For $f = 8$, for example, we have in addition to states $k = 3$, $l = 0$ and $k = 5$, $l = 1$ also the states for $k = 7$ with angular momenta $l = 0$ and $l = 2$.

The considerations presented have, finally, an illustrative character and lie in the direction of attempts[2,3] to introduce spin in a natural way in a theory with a mass spectrum.

In conclusion we express our gratitude for his guidance to Prof. M. A. Markov.

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Further Discussion on the Quantum Theory of the Radiating Electron

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NELIP[1] has made a series of critical remarks in his reply to our work on the quantum theory of the radiating electron. However, these critical remarks are based on a modification of our Eq. 18[2] which is given by Nelip[1] in the form

$$\varepsilon = (1 - \beta^2 \sin^2 \vartheta)\left(1 + \frac{\nu}{2n}\right) + O\left(\frac{\nu^2}{n^2}, \frac{\nu^3}{n^3} \cdots\right), \qquad \text{(A)}$$

(cf. Eq. A, p. 423 of Ref. 1). As a matter of fact, this formula has the form

$$\varepsilon = (1 - \beta^2 \sin^2 \vartheta)(1 + \nu/2n). \qquad \text{(B)}$$

(cf. Eq. (18) of Ref. 2).

In connection with this formula we made an observation that the higher order terms of $(\frac{\nu}{n})^2$ of the expansion should also have the small multiplier $(1 - \beta^2 \sin^2 \vartheta)$. Therefore the expression (A) has the form

$$\varepsilon = (1 - \beta^2 \sin^2 \vartheta)\left(1 + \frac{\nu}{2n}\right) + \left(1 - \beta^2 \sin^2 \vartheta\right) O\left(\frac{\nu^2}{n^2}, \frac{\nu^3}{n^3} \cdots\right), \qquad \text{(C)}$$

which was indeed used by us in our previous work[2] and also in our following articles on the quantum theory of the radiating electron (cf., for example, Eq. (43) of Ref. 3).

Furthermore, Nelip[1] ascribes to us still a second inaccurate formula (B) (p. 423) which does not follow in any way out of our Eqs. (16) and (18) of Ref. 2. Therefore the critical remarks that our method developed in Ref. 2 has a small region of applicability by its limitation to the magnitudes $\nu^3/n^2 \ll 1$ appear to be a misunderstanding as they are based on the modification pointed out above. From our original Eq. (18)[2] it follows that the terms discarded by us are of the order $(\nu/n)^2$.

Therefore we cannot accept the criticism made by Nelip and our previous observations should remain valid.[4,5]

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[3]A. A. Sokolov and I. M. Ternov, J. Exptl. Theoret. Phys. (U.S.S.R.) 25, 698 (1953).

[4]A. A. Sokolov, J. Exptl. Theoret. Phys. (U.S.S.R.) 24, 488 (1953).

[5]Sokolov, Matveev and Ternov, Dokl. Akad. Nauk SSSR 102, 65 (1955).

Translated by M. J. Stevenson
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