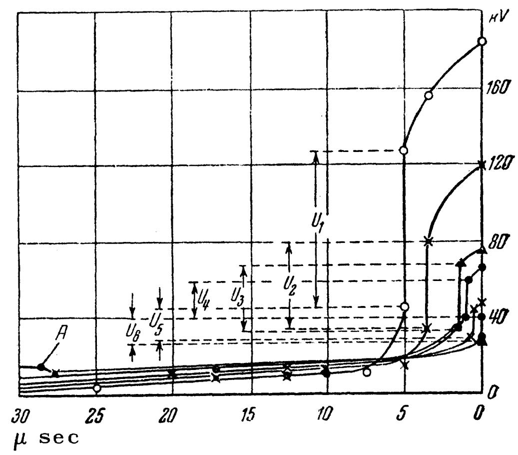

FIG. 4. Curves showing the decay of the voltage in the formation of a discharge in distilled water (positive point and plane); $U_1 = 84$; $U_2 = 44$; $U_3 = 34$; $U_4 = 22$; $U_5 = 16$; $U_6 = 12$ kV

tion and thermal ionization. During this stage the temperature of the column increases and gas is produced; this process is terminated with an "explosion" and the ejection of liquid[11]. At this point molecules of non-ionized liquid penetrate the column, the current diminishes, and there is a corresponding increase in the voltage (fourth stage; *A* of Fig. 4 and *C* of Oscillograph IV, Fig. 3).

It is evident from Oscillographs V and VI of Fig. 3 that even in the case of a very large overvoltage (3.5 times) a relatively long time (about 5 μsec) is reguired for the formation of the positive column which short-circuits the electrodes. During this period, as may be seen from the nicks in the front of the pulses in Oscillographs VI, small streamers are formed; these do not start breakdown but cause a certain amount of ionization in the gap. The development of the filamentary positive column from the positive point also depends on random factors. This is seen in Oscillographs IV of Fig .3 in which breakdown occurs after a lag of 184 μsec in one case and 52 μsec in the other although the amplitude of the applied pulse was the same in both cases.

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Energy Spectrum of the Penetrating Particles of Extensive Cosmic Ray Showers

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The depth-intensity curve of penetrating shower particles absorbed in lead and rock of water equivalent of 3.7, 5.9, 27, 65.5 and 148 meters has been measured. The studies are carried out at an elevation of 400 meters above sea level. On the basis of the resulting path lengths, the integral energy spectrum of penetrating shower particles is found in the form of a power function. This function has the form $E^{-\gamma}$, where $\gamma = 0.62 \pm 0.05$ (for the energy interval from 5×10^8 to 3×10^{10} ev).

INVESTIGATION of the characteristics of penetrating shower particles is essential in connection with the mechanism of generation of extensive cosmic ray showers that is proposed by a group of Soviet students working under the guidance of Skobel'tsyn[1-4]. According to their idea, the formation of an extensive atmospheric shower appears to be the result of a nuclear-cascade process in which the main role is played by nuclear-active and penetrating components. The latter carry the major part of the energy of the whole shower, and determine the significant characteristics of the shower, its spatial structure, and the energy balance among its different components.

To this day, studies of the nature of extensive cosmic ray showers furnish the only key to understanding the interactions of high energy (of the order of 10^{15} - 10^{18} ev) particles with matter, and those physical processes which accompany these interactions. From this point of view, investiga-

[1] D. V. Skobel'tsyn, G. T. Zatsepin and V. V. Miller, Phys. Rev. 71, 315 (1947).

[2] D. V. Skobel'tsyn, Dokl. Akad. Nauk SSSR 67, 45 (1949).

[3] D. V. Skobel'tsyn, Dokl. Akad. Nauk SSSR 67, 225 (1949).

[4] G. T. Zatsepin, Dokl. Akad. Nauk SSSR 67, 993 (1949).

tion of the nuclear active and penetrating components of extensive showers holds the most interest. Nevertheless, these components of extensive cosmic ray showers remain relatively little studied. In particular, the mass spectrum of these components has not been investigated, nor has the spatial distribution of high energy penetrating particles along the radius of the shower. The goal of the present work is the investigation of the energy spectrum of penetrating particles of extensive cosmic ray showers.

1. EXPERIMENTAL ARRANGEMENT

In order to investigate the integral energy spectrum of penetrating shower particles, the attenuation of the intensity of these particles was studied by means of absorbers of various thicknesses. Lead and iron filters of a total thickness of 3.7 meters water equivalent were used in one series of experiments. Absorbing layers composed mainly of rocky soil of thicknesses 5.9, 27, 65.5 and 148 meters water equivalent were used in another series.

The measurements were carried out at an elevation of 400 meters above sea level. The intensity versus depth curve is dependent upon the energy spectrum of the penetrating components[5]. To obtain the one relationship from the other, it is necessary to use the relation between the energy and path length of penetrating particles. Let us deduce this relationship for μ-mesons, since at present it is generally accepted that the major part of the penetrating component of cosmic radiation underground consists of μ-mesons.[6,7] The loss of energy by fast μ-mesons can be represented, according to Wilson[7], by

$$dE/dx = a + bE + c\ln(E\cdot 10^{-3}). \qquad (1)$$

where E is expressed in mev. The first term on the right of Eq. (1) represents the magnitude of the energy lost by the fast μ-mesons in ionizing processes, the second term comprises radiation loss and the loss through nuclear interactions, and the third term gives the energy lost by the μ-mesons through Cerenkov radiation.

TABLE 1

Values of dE/dx (mev g/cm^2)

Energy in bev	a		bE		$c\ln(E\cdot 10^{-3})$	
	earth	lead	earth	lead	earth	lead
1	2.2	1.3	0.005	0.019	0	0
10	2.2	1.3	0.05	0.19	0.2	0.1
100	2.2	1.3	0.5	1.9	0.45	0.2
1000	2.2	1.3	5	19	0.7	0.3

From Table 1 it can be seen that at about 100 bev the term which is proportional to E becomes comparable in magnitude to the term representing the ionization losses.

For smaller energies, the ionization losses far exceed the other losses. In particular, in our case, where the maximum measurable energy of the particles does not exceed 3×10^{10} ev one needs to consider only the ionization losses. In this case, the energy loss versus path length relationship takes the form:

$$dE/dx = 2.2 \text{ mev g/cm}^2 \text{ for rock;}$$

$$dE/dx = 1.3 \text{ mev g/cm}^2 \text{ for lead.} \qquad (2)$$

Knowing the relationship between the energy and the path length of particles, and using the dependence of the intensity of the penetrating shower particles on the absorber thickness, one can obtain the integral energy spectrum by means of a well-known relationship[5].

2. MEASUREMENT TECHNIQUE

The measurement of the intensity of the penetrating shower particles under absorbers was carried out by means of Geiger-Mueller counters set up in an arrangement providing for two, three, and fourfold coincidences. The counters were connected

[5] L. Janossy, *Cosmic Rays*, p. 214 Sec. 11L 285 (Russian translation) 1949.

[6] C. A. Randall, Phys. Rev. **87**, 241 (1952).

[7] G. Wilson, *Physics of Cosmic Rays*, p. 331 IIL, Moscow, Leningrad, (Russian translation) (1954).

in parallel, and were joined in rows in such a manner that the total effective area of each row equalled one square meter. In all, there were five such rows, three of which were arranged on top of the ground at the vertices of a right-angle triangle, with legs equal to 30 meters. The other two rows were placed one atop the other underground. Figure 1 shows the schematic arrangement of the rows.

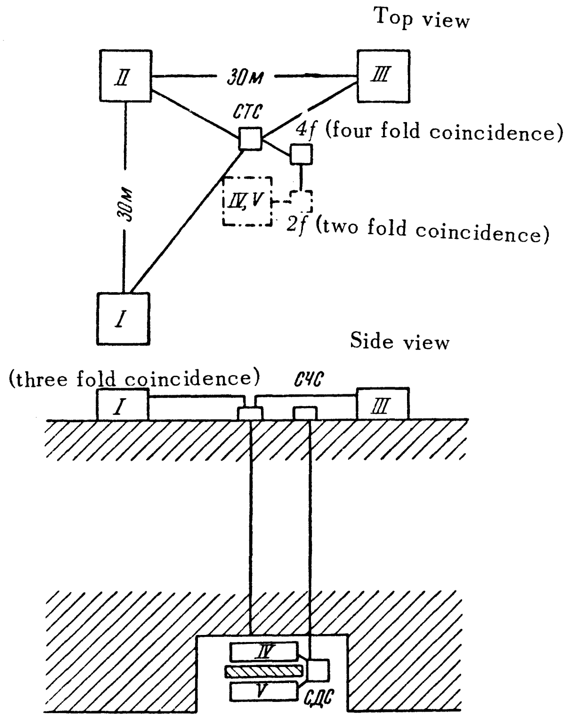

FIG. 1. *I, II, III, IV* and *V* rows of counters; *3f, 4f, 2f* scheme of 3, 4 and 2 fold coincidence.

The arrangement of rows I, II and III was not changed during the course of the entire experiment. Rows IV and V were moved in depth after each series of measurements. The impulses from counter rows I, II and III were channeled for three-fold coincidences. Each functioning of the three-fold coincidence system was caused by the passage of an extensive cosmic ray shower through the rows of counters. By this means, a three-fold coincidence of the impulses from the upper rows registered an extensive cosmic ray shower. The penetrating component of this shower was registered with the aid of the underground row of counters, whose impulse was channeled for coincidence with the master signal received from the three-fold coincidence of the upper counters. The underground part of the arrangement consisted of two rows. These two rows were of identical construction, were placed one above the other, and were connected for two-fold coincidence. This duplication of the rows registering underground particles was used to eliminate accidental coincidences between impulses from the upper and lower portions of the arrangement, which were not caused by shower particles. In several series of measurements eight centimeters of lead were placed between the two underground rows of counters.

The resolving time of the three and four-fold coincidence systems was 3.5×10^{-6} sec. For the two-fold coincidence systems it was 1.5×10^{-6} sec. Such a system of registering shower particles completely eliminated all possible false coincidences arising in separate parts of the arrangement. By means of this arrangement, the frequency of three-fold coincidences in the upper rows and the frequency of four-fold coincidences between impulses from the upper and lower rows was measured during the entire series of experiments. From the experimental values of three and four-fold coincidences per unit time with different absorber thicknesses, there were determined the intensities of penetrating shower particles at these depths (see Sec. 4 below).

3. RESULTS OF MEASUREMENTS

In Table 2 are given the pertinent properties of the absorbers which were used in the course of the work. In the first three cases, 8 cm of lead was placed between the underground counter rows IV and V (Fig. 1). In addition, measurements were made at a depth of 27 meters water equivalent of the increase in the number of coincidences upon removal of the lead absorber from between rows IV and V. During this trial no change was observed in the number of four-fold coincidences recorded. The remaining two series of measurements were carried out without any lead absorber between rows IV and V.

The resulting measurements of the frequency of three and four-fold coincidences are reported in Table 3.

4. COMPUTATION AND DISCUSSION OF RESULTS

The experimental results given in Table 3 furnish the means for establishing the relationship governing the decrease of the intensity of penetrating particles with increasing absorber thickness. We shall first ascertain the relationship between the intensity of penetrating shower particles at a given depth and the frequency of four-fold coincidences at that depth. The frequency of four-fold coincidences, produced by the passage of shower particles through four rows of counters, is equal to

TABLE 2

Absorbing material	Thickness	Density in gm/cm^3	Thickness in meters water equivalent.
Pb, Fe, Al	18 см Pb+ +8 см Fe+ +10 см Al+ +8 см Pb	11 7,3 2,7	3,7
Brick Masonry	2,7 м+ +8 см Pb	1,85	5,9
Rock	11,0 м	2,55	27,0
Rock	28,5 м	2,3	65,5
Rock	55,0 м	2,7	148,0

TABLE 3

Depth in meters water equivalent	Duration of measurements in hours	No. of 3-fold coinc.	No. of 4-fold coinc.	3-fold coinc. per hr.	4-fold coinc. per hr.	100 α
3,7	97	1360	107	13,9±0,7	1,1 ±0,1	3
5,9	118	1615	108	13,5±0,6	0,85±0,07	2,4
27	416	5720	120	13,6±0,4	0,29±0,03	0,75
65,5	554	7625	105	13,7±0,3	0,19±0,02	0,46
148	852	11 180	106	13,9±0,2	0,13±0,01	0,3

$$C_4 = A\int_0^\infty \rho^{-(\varkappa+1)}\prod_{i=1}^{4}(1-e^{-\rho_i\sigma})\,d\rho. \qquad (3)$$

Here $A\rho^{-(\varkappa+1)}$ is the differential spectrum of the density of extensive cosmic ray showers, ρ_i is the density of shower particles falling on the ith row of counters, σ is the effective area of each row of counters.

To evaluate the integral (3), we must determine ρ_i with the aid of the spatial distribution function of the shower particles, taking into account the geometry of the experimental arrangement. However, considering the unwieldiness of the resulting expression and the fact that the spatial distribution function of the shower particles is not known with sufficient accuracy at large distances, such an evaluation cannot be justified. Instead, for a first approximation, we can assume that for recorded showers the densities ρ_i of particles falling on rows I, II and III respectively of the above-ground portion of the arrangement ($i = 1, 2, 3$), are identical and are equal to ρ. Furthermore, the density of penetrating particles ρ_4 will be equal to $\alpha\rho$, where α is the fraction of particles that consists of penetrating particles. Thus integral (3) takes the form

$$C_4 = A\int_0^\infty \rho^{-(\varkappa+1)}(1-e^{-\rho})^3(1-e^{-\alpha\rho})\,d\rho. \qquad (4)$$

This takes into account that for our arrangement $\sigma = 1$. Similar integrals are investigated in detail in the works of Daudin[8] and Zatsepin[9], where the method of solution can be found.

Integral (4) can be broken up into two integrals: $C_4 = I_1 - I_2$ where

$$I_1 = A\int_0^\infty (1-e^{-\rho})^3\rho^{-(\varkappa+1)}\,d\rho; \qquad (5)$$

$$I_2 = A\int_0^\infty (1-e^{-\rho})^3 e^{-\alpha\rho}\rho^{-(\varkappa+1)}\,d\rho.$$

Using the method shown in the latter reference[8] we obtain

$$I_1 = \frac{A\Gamma(3-\varkappa)}{\varkappa(\varkappa-1)(\varkappa-2)}[3-3\cdot 2^\varkappa+3^\varkappa], \qquad (6)$$

$$I_2 = \frac{A\Gamma(3-\varkappa)}{\varkappa(\varkappa-1)(\varkappa-2)} \qquad (7)$$

$$\times\ [\alpha^\varkappa - 3(1+\alpha)^\varkappa + 3(2+\alpha)^\varkappa - (3+\alpha)^\varkappa].$$

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[8] J. Daudin, J. Phys. Radium 8, 301 (1947).

[9] G. T. Zatsepin, Dissertation, FIAN, (1950).

Inserting the numerical value $x = 1.4$, we finally obtain

$$C_4 = \frac{A\Gamma(1.6)}{1.4 \cdot 0.4 \cdot 0.6}[0.262 + \alpha^{1.4} - 3(1+\alpha)^{1.4} + 3(2+\alpha)^{1.4} - (3+\alpha)^{1.4}]. \quad (8)$$

Knowing the number of four-fold coincidences per unit time C_4 at a given depth, it is possible to determine the fraction of penetrating particles α at this depth. To do this it would be necessary to insert a numerical value for A in equation (8). However, it is more convenient to compute α from the relationship between the frequency of four-fold coincidences and the frequency of three-fold coincidences. The frequency of the three-fold coincidence is:

$$C_3 = A\int_0^\infty (1 - e^{-\rho})^3 \rho^{-(x+1)}\, d\rho. \quad (9)$$

Equation (9) coincides with (6). For the ratio C_4/C_3 we obtain

$$\frac{C_4}{C_3} = 1 - \frac{\alpha^{1.4}}{0{,}262} - \frac{3}{0.262}(1+\alpha)^{1.4} + \frac{3}{0.262}(2+\alpha)^{1.4} - \frac{1}{0.262}(3+\alpha)^{1.4}. \quad (10)$$

The third, fourth and fifth terms of right side of equation (8) can be expanded in powers of α. Furthermore ,bearing in mind the small magnitude of α (a few percent), it is possible to disregard the terms containing α^2. Therefore

$$C_4 / C_3 = 3\ 2\alpha\,(1 - 1.2\,\alpha^{0.4}). \quad (11)$$

Substituting experimental values of C_4 and C_3 into the left side of this expression from the fifth and sixth columns of Table 3, we obtain values for α which are given in the last column of Table 3.

Figure 2 shows the dependence of intensity on depth according to the data given in the first and last columns of Table 3. The ordinate is the logarithm of the intensity, and the abscissa is the logarithm of the effective depth expressed in meters of water. Within the limits of error, all experimental points lie on a straight line. Therefore the dependence of the intensity of penetrating particles on the depth can be expressed as a power function of the form

$$I = c_1 x^{-\gamma}. \quad (12)$$

The value of the exponent, as determined from the experimental results, is 0.62 ± 0.05.

Given the above, it is possible to construct the integral energy spectrum of the penetrating par-

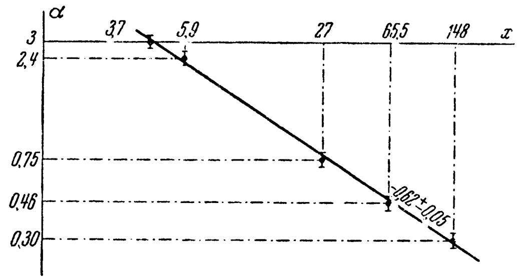

FIG. 2

ticles from the change of the intensity with depth. For this purpose we substitute the value of x obtained from expression (2) into (12); as a result we obtain:

$$N(>E) = c_2 E^{-\gamma}. \quad (13)$$

This is the expression for the energy spectrum of penetrating particles of extensive cosmic ray showers. Here N is the number of penetrating particles which have an energy greater than E. The constant C_2 represents the total number of penetrating particles in the entire shower at an elevation of 400 meters above sea level. The differential energy spectrum has the form:

$$NdN = c_3 E^{-(\gamma+1)}\, dE, \quad (14)$$

where NdN is the number of penetrating shower particles with energy lying between E and $E + dE$. With the aid of this spectrum, it is possible to find the lower limit of the mean energy of penetrating shower particles. For the limits of integration employed in finding the mean value, one must use the limiting values of the measurable energy, namely, 5×10^8 and 3×10^{10} ev. By carrying out the integration we obtain the value $\bar{E}_{min} = 1.7 \times 10^9$ ev for the lower limit of the mean energy of penetrating particles.

Comparing the energy spectrum of the penetrating particles of the vertical component of a cosmic shower at sea level[10] with the energy spectrum of the penetr ating particles of extensive atmospheric showers, we come to the conclusion that shower particles are absorbed significantly more slowly than are the particles of the vertical component. This fact confirms the work of George[11]. George, comparing his results for measurements of extensive shower underground with the results of Greisen, was able to advance the hypothesis that the depth dependence of the intensity of penetrating shower particles must be a power relationship, with an ex-

[10] J. G. Wilson, Nature **158**, 414 (1946).

[11] E. P. George, J. W. Mac Anuff and W. Sturgess, Proc. Phys. Soc **A66**, 346 (1953).

ponent equal to 0.66, down to depths of 1600 meters water equivalent.

The results of our direct measurements definitely confirm George's hypothesis. However, in spite of George's assertion, one must expect the curve of the energy spectrum to start falling sharply at certain depths significantly less than 1600 meters water equivalent in the contrary case, where the mean energy of penetrating shower particles becomes more like that reported by Eidus and co-workers[12].

[12] L. Kh. Eidus, M. P. Adamovich, I. L. Ivanovskaia, V. S. Nikolaev and M. S. Tuliankina, J. Exptl. Theoret. Phys. (U.S.S.R.) 22, 440 (1942).

For this reason, further investigation of the intensity of the penetrating components of extensive cosmic ray showers at significantly greater depths is of definite interest.

In conclusion, I wish to express my thanks to Professor E. L. Andronikashvili for proposing the problem and guiding the work; to Professors C. H. Vernov and G. T. Zatsepin for valuable discussions, and to M. F. Bibilashvili, G. N. Muskhelishvili, G. E. Chikovan and G. R. Khutsishvili for assistance rendered.

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The Electromagnetic Field of a Linear Emitter Located inside an Ideally Conducting Parabolic Screen

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On the basis of a critical examination of researches previously published, the inadequate basis of the results obtained in them is pointed out. A rigorous solution of the problem is given. It is also shown that the solution takes the form corresponding to the approximation of geometrical optics in the limiting case of very high frequencies.

AT first glance, the problem under consideration appears to be almost trivial, since the variables are separated in the basic equation of the problem. However, the desired choice of partial solutions, which can serve for the construction of the solution of the problem which satisfies all requirements, included the appropriate radiation principle at infinity, and the correct behavior at the focus of the cylinder, and the actual obtaining of such a solution is not so simple, as could be shown. In particular, nowhere, to our knowledge, in the series of researches[1-4] on the wave problem for a parabolic region, is any fundamental solution of the problem under consideration given in the desired form.* Such a problem is taken up in the present work. It is shown further that the solution, in the limiting case of very high frequencies, takes the form which corresponds to the ray approximation of optics.

*For more detail, see Sec. 1 of Ref. 5.

[1]Encyklopädie Math. Wissensch. 5, 3, 511.

[2]P. S. Epstein, Dissertation, Munich, 1914.

[3]W. Magnus, Jahresber. deut. Math. Verein. 50, 140 (1940).

[4]W. Magnus, Z. Physik 118, 343 (1941).

[5]G. A. Grinberg, N. N. Lebedev, I. P. Skal'skaia and Ia. S. Ufliand, Dokl. Akad. Nauk SSSR 95, 961 (1954).

1. STATEMENT OF THE PROBLEM

We consider the problem of the reflection of electromagnetic waves from a conducting screen which has the form of a parabolic cylinder. We assume a linear vibration source placed inside the cylinder along its focal line.** The current strength of the source is $I = I_0 e^{i\omega t}$, where I_0 = const = amplitude of the current, ω=angular frequency.

We set up a cartesian coordinate system (x,y,z) so that the axis Ox lies in the plane of symmetry of the parabolic cylinder, and the axis Oz coincides with the exciting current. In addition, we introduce the parabolic coordinates (α, β) with the help of the relations

*The case of arbitrary disposition of the source is considered in Sec. 8.