of the Auger electrons (soft) was not carried out.

The tables show that the observed lines are determined by the following transitions of the Np^{237} nucleus: 26.4; 33.3; 43.5; 59.7; 69 (?); 101 (?); 124 (?); 165; 193 (?); 208, 268; 331; 370 and 436 kev.

On the basis of the experimental results here, and of data on the investigation of the electron and α spectra of Am^{241}, a tentative scheme of the levels of the nucleus Np^{237} was constructed, and is shown in Fig. 3.

The problem of the spins and even energy levels are discussed in another work.*

We consider it our duty to thank G. N. Iakovlev for carrying out the chemical part of this research and also P.S. Samoilov who furnished assistance in the taking of the β spectrum of U^{237}

*Note added in proof. The work was published in 1955[6]. The results of the cited and present researches permit us to determine the spin of the ground state of U^{237}. It is equal to $\pm 1/2$.

[6] S. A. Baranov and K. N. Shliagin, Sessions of the Academy of Sciences, U.S.S.R., on the peaceful application of atomic energy, June 21-5, 1955, sitting OFMN, p. 251.

Translated by R. T. Beyer
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Nucleomesodynamics in Strong Coupling. I. Approximate Method. Spin-Charge Motion

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In the approximation of infinitely heavy nucleons and strong interaction with the meson field, we develop an approximate method and the associated mathematical apparatus which makes possible explicit determination of the wave functions and energy eigenvalues of the system. The mesons are assumed to be pseudoscalar, the coupling to be symmetric pseudovector (gradient) coupling, and the nucleon is assumed to be an extended source. Using the approximation method, the Hamiltonian of the system is simplified, and the spin-charge part of the system wave function is separated off and determined along with the corresponding energy.

1. INTRODUCTION

In previous papers on this same subject[1,2], it was assumed that the spin-charge and translational motion of the nucleon follow the relatively slow oscillations of the meson field adiabatically. To solve the problem, mathematical methods were used which are similar to those in the theory of polarons. Further investigations showed that in the most important case of nucleons and π-mesons the interaction is actually not as strong as is necessary to make the translational motion follow the meson field oscillations adiabatically. Quite the contrary, the vibrations of the meson field are adiabatic relative to the translational motion of the nucleon. Therefore, in zeroth approximation, we should consider the spin-charge motion and the oscillations of the meson field for the case of a fixed (infinitely heavy) nucleon. This is done in this and the succeeding paper.

Thus, in contrast to the above-mentioned earlier papers, we shall assume that only the spin-charge motion follows the meson field adiabatically, and shall, as before, use the methods of the theory of polarons[3] in treating this motion. In an earlier paper[4] it was shown, without the use of any ap-

[1] S. I. Pekar, J. Exptl. Theoret. Phys. (U.S.S.R.) 27, 398 (1954).

[2] S. I. Pekar, J. Exptl. Theoret. Phys. (U.S.S.R.) 27, 411 (1954).

[3] S. I. Pekar, *Studies in Electron Theory of Crystals*, GTI, 1951. (also available as *Untersuchungen uber die Elektronentheorie der Kristalle*, Akademische Verlag, Berlin, 1954).

[4] S. I. Pekar, J. Exptl. Theoret. Phys. (U.S.S.R.) 29, 599 (1955).

proximation methods (strong or weak coupling, quasistationary approximation, etc.), that, at least in the case where we treat the translational motion of the nucleon non-relativistically, no stationary solution of the wave equation of the system exists in the case of pseudovector coupling. We shall therefore assume the nucleon to be an extended source.

The problem treated here was first investigated by Pauli and Dancoff in 1942.[5] These authors, by cleverly introducing a new system of canonical field variables, separated off the terms in the energy operator which are proportional to the square of the total momentum of the system. The eigenvalues of the momentum are known from general arguments for any system. Thus it turns out that one knows, from general considerations, the eigenvalues of one of the terms in the Hamiltonian, the energy levels of the isobar states of the system. Unfortunately, in the paper of Pauli and Dancoff[5], the complete set of quantum numbers of the system was not given explicitely, and no systematic method for calculating matrix elements of all canonical variables was presented. Apparently it was just this point which made difficult the calculation of higher approximations of the theory and the consideration of various specific phenomena (scattering of mesons by nucleons, magnetic moment of the nucleon, nuclear forces, photoproduction of mesons, etc.). In their paper, when they considered, for example, the scattering of mesons, the isobars were not taken into account and they were unable to separate the proton and neutron states of the nucleon. Consequently, the scattering was considered only for the case of a nucleon state which is a superposition of all charge and spin states of the nucleon.

In later papers on the application of strong coupling theory with symmetric pseudovector coupling, for example in the work of Pauli[6] or Wada[7], the spin and isotopic spin of the nucleon were introduced as classical unit vectors, which is not a consistent application of the strong coupling theory developed in Ref. 5. Thus, because of the abovementioned incompleteness of the solution of the problem in Ref. 5 and the complexity of the mathematics, the theory of strong coupling with symmetric pseudovector interaction did not achieve the necessary development during the next twelve years. Since newer data speak in favor of intermediate or strong coupling and a symmetric pseudovector interaction, further attempts at a more complete solution of the problem are of interest. We here present a method for somewhat more complete solution of the problem which makes possible explicit determination of the wave function of the system, the introduction of a complete set of quantum numbers, and systematic calculation of the matrix elements of all the canonical variables, which is needed for applications of the theory.

2. INITIAL EQUATIONS AND ASSUMPTIONS

By expanding the meson field φ_α ($\alpha = 1,2,3$) and the canonically conjugate field π_α in Fourier series of the form

$$\varphi_\alpha(\mathbf{r}) = \sum_{\vec{\varkappa}} \frac{1}{\sqrt{\omega_{\vec{\varkappa}}}} q_{\alpha\vec{\varkappa}} \chi_{\vec{\varkappa}}(\mathbf{r}); \tag{1}$$

$$\pi_\alpha(\mathbf{r}) = \sum_{\vec{\varkappa}} \sqrt{\omega_{\vec{\varkappa}}}\, g_{\alpha\vec{\varkappa}} \chi_{\vec{\varkappa}}(\mathbf{r}),$$

where

$$\chi_{\vec{\varkappa}}(\mathbf{r}) = \sqrt{\frac{2}{L^3}} \begin{cases} \cos \vec{\varkappa}\mathbf{r}; & \varkappa_x \leqslant 0 \\ \sin \vec{\varkappa}\mathbf{r}; & \varkappa_x > 0 \end{cases} \tag{2}$$

(the $\chi_\varkappa(r)$ are orthonormalized in a volume L^3), we can put the Hamiltonian of the meson field in the form

$$\hat{H}_0 = \frac{1}{2} \sum_{\alpha=1}^{3} \int [\pi_\alpha^2 + (\nabla\varphi_\alpha)^2 + \mu^2\varphi_\alpha^2]\, dV \tag{3}$$

$$= \frac{1}{2} \sum_{\alpha\vec{\varkappa}} \omega_{\vec{\varkappa}} [g^2_{\alpha\vec{\varkappa}} + q^2_{\alpha\vec{\varkappa}}].$$

We shall use the natural system of units ($\hbar = c = 1$); μ is the meson mass in cm^{-1},

$$\omega_{\vec{\varkappa}} = \sqrt{\mu^2 + \vec{\varkappa}^2} \quad (\omega_{\vec{\varkappa}} > 0). \tag{4}$$

The dimensionless generalized coordinates $q_{\alpha\vec{\varkappa}}$ and momenta $g_{\alpha\vec{\varkappa}}$ of the field satisfy the following commutation relations:

$$[q_{\alpha\vec{\varkappa}},\ q_{\alpha'\vec{\varkappa}'}] = [g_{\alpha\vec{\varkappa}},\ g_{\alpha'\vec{\varkappa}'}] = 0; \tag{5}$$

$$[q_{\alpha\vec{\varkappa}},\ g_{\alpha'\vec{\varkappa}'}] = i\delta_{\alpha\alpha'}\delta_{\vec{\varkappa}\vec{\varkappa}'}.$$

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[5] W. Pauli and S. M. Dancoff, Phys. Rev. **62**, 85 (1942).

[6] W. Pauli, *Meson Theory of Nuclear Forces*, Interscience (1946).

[7] W. W. Wada, Phys. Rev. **88**, 1032 (1952).

We choose a representation in which the $q_{\alpha\vec{\varkappa}}$ are ordinary numbers; then the $g_{\alpha\varkappa}$ can be represented as differential operators $g_{\alpha\vec{\varkappa}} = -i\partial/\partial q_{\alpha\vec{\varkappa}}$. The operator for the interaction of the nucleon with the meson field is taken to be of the symmetric gradient (pseudovector) type:

$$\hat{H}' = -\frac{g}{\mu}\sqrt{4\pi}\sum_{\alpha=1}^{3}\tau_\alpha\int(\nabla\varphi_\alpha, \vec{\sigma})\, u(\mathbf{r})\, dV \qquad (6)$$

$$= \frac{g}{\mu}\sqrt{4\pi}\sum_{\alpha=1}^{3}\tau_\alpha\int\varphi_\alpha(\vec{\sigma}, \nabla u)\, dV,$$

where g is the dimensionless coupling constant, $\vec{\sigma}$ and τ_α are the familiar spin and isotopic spin matrices for the nucleon, $u(\mathbf{r})$ is the spherically symmetric form factor for the extended nucleon, normalized to unity. We shall try to find the eigenfunctions of the operator $\hat{H} = \hat{H}_0 + \hat{H}'$. Without loss of generality, we can expand these functions in terms of eigenfunctions of the operators σ_3 and τ_3, and shall denote the latter by S_1, S_2, and Q_1, Q_2, respectively. The expansion has the form:

$$\Psi = \overline{C}_1(q)\, S_1Q_1 + \overline{C}_2(q)\, S_2Q_1 + \overline{C}_3(q)\, S_1Q_2 + \overline{C}_4(q)\, S_2Q_2 \qquad (7)$$

where q stands for the collection of variables $q_{\alpha\vec{\varkappa}}$. The four basis functions $S_i \cdot Q_j$ form a complete set. Applying the operators $\vec{\sigma}$ and τ_α to the wave function (7) is equivalent to transformation of the components $\overline{C}_\nu$ by the following four-by-four matrices:

$$\sigma_1 = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}, \quad \sigma_2 = \begin{vmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{vmatrix}, \quad \sigma_3 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix},$$

$$\tau_1 = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}, \quad \tau_2 = \begin{vmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{vmatrix}, \quad \tau_3 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}. \qquad (8)$$

3. THE APPROXIMATION METHOD

The basis of our approximation is the assumption that the motion in the spin-charge degree of freedom follows the relatively slow oscillations of the meson field adiabatically. In zeroth approximation, it is assumed that for each instantaneous configuration of the meson field q, a stationary state of the system with respect to the spin-charge degree of freedom of the nucleon can be established. The mathematical formulation of this approximation consists of the following: we assume that the eigenfunction (7) can be split into two factors

$$\overline{C}_\nu(q) = C_\nu(q)\,\Phi(q); \quad \Psi = \psi(q)\,\Phi(q);$$

$$\psi(q) = C_1(q)\, S_1Q_1 + C_2(q)\, S_2Q_1 + C_3(q)\, S_1Q_2 + C_4(q)\, S_2Q_2 \equiv \begin{vmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{vmatrix}, \qquad (9)$$

chosen so that $\Phi(q)$ changes much more rapidly with $q_{\alpha\vec{\varkappa}}$ than does $\psi(q)$. This means that the second term in the expression

$$\frac{\partial\Psi}{\partial q_{\alpha\vec{\varkappa}}} = \psi\frac{\partial\Phi}{\partial q_{\alpha\vec{\varkappa}}} + \frac{\partial\psi}{\partial q_{\alpha\vec{\varkappa}}}\Phi \quad (10)$$

is much smaller than the first and can be dropped in zeroth approximation

$$\hat{H}_0\psi\Phi = {}^1/_2\sum_{\alpha\vec{\varkappa}}\omega_{\vec{\varkappa}}(q^2_{\alpha\vec{\varkappa}} - \partial^2/\partial q^2_{\alpha\vec{\varkappa}})\psi\Phi \quad (11)$$

$$\approx \psi\hat{H}_0\Phi + \ldots$$

Then the omitted terms will later be included as a small perturbation (perturbation due to non-adiabaticity).

By substituting Eq. (9) in the wave equation of the system, $\hat{H}\Psi = H\Psi$, $\hat{H} = \hat{H}_0 + \hat{H}'$, and using the approximation (11), we get two equations

$$\hat{H}'(q)\psi = H'(q)\psi, \quad (12)$$

$$[\hat{H}_0 + H'(q)]\Phi = H\Phi, \quad (13)$$

which determine $\psi(q)$ and $\Phi(q)$. In Eq. (12) the $q_{\alpha\vec{\varkappa}}$ act as parameters; the equation determines a state ψ which is stationary with respect to the spin-charge degree of freedom, for a fixed configuration q of the meson field. Equation (13) is the wave equation of oscillation of the meson field in the presence of the nucleon. Equation (13) is conveniently interpreted, using (3), as the equation of motion of a system of harmonic oscillators in the presence of a given auxiliary field $H'(q)$, which shifts the equilibrium positions of the oscillators. The potential energy of these oscillators is

$$V(q) = H'(q) + \frac{1}{2}\sum_{\alpha\vec{\varkappa}}\omega_{\vec{\varkappa}}q^2_{\alpha\vec{\varkappa}} \quad (14)$$

$$\equiv H'[\varphi_1\varphi_2\varphi_3] + \frac{1}{2}\sum_{\alpha=1}^{3}\int[(\nabla\varphi_\alpha)^2 + \mu^2\varphi_\alpha^2]\,dV.$$

If the functional

$$F[\psi\varphi_\alpha] \equiv \{\psi^*, \hat{H}'\psi\} \quad (15)$$

$$+\frac{1}{2}\sum_{\alpha=1}^{3}\int[(\nabla\varphi_\alpha)^2 + \mu^2\varphi_\alpha^2]\,dV$$

is made an extremal relative to ψ, subject to the supplementary condition that ψ be normalized, it becomes the potential energy (14) of the oscillators, since its first term, according to the variational principle of quantum mechanics, becomes $H'(q)$.

In order to determine the equilibrium positions of the oscillators $q^v_{\alpha\vec{\varkappa}}$ or the corresponding equilibrium self-consistent form of the field $\varphi^v_\alpha(\mathbf{r})$, we must find the minimum of (14) with respect to $q_{\alpha\vec{\varkappa}}$, i.e., with respect to $\varphi_\alpha(\mathbf{r})$. Thus the equilibrium self-consistent form of the field corresponds to the extremum of $F[\psi, \varphi_\alpha]$ with respect to both ψ and φ_α. This extremum is to be found by first making F an extremal relative to φ_α for fixed ψ, and then making the resultant functional an extremum relative to ψ.

If we set the variation of $F[\psi, \varphi_\alpha]$ with respect to φ_α equal to zero, we get

$$\Delta\bar{\varphi}_\alpha - \mu^2\bar{\varphi}_\alpha = -4\pi\eta_\alpha \quad (16)$$

($\bar{\varphi}_\alpha$ is the extremal value of φ_α), where the function $\eta_\alpha(\mathbf{r})$ acts as a source density for the meson field $\bar{\varphi}_\alpha$; it is equal to

$$\eta_\alpha[\mathbf{r}, \psi] = -\frac{g}{\mu}(4\pi)^{-1/2}\sum_{\beta=1}^{3}\overline{\tau_\alpha\sigma_\beta}\frac{\partial u}{\partial x_\beta}, \quad (17)$$

$$\overline{\tau_\alpha\sigma_\beta} \equiv \{\psi^*, \tau_\alpha\sigma_\beta\psi\}.$$

The solution of equation (16) has the form

$$\bar{\varphi}_\alpha(\mathbf{r}) = \int\frac{\eta_\alpha[\mathbf{r}'\psi]\exp\{-\mu|\mathbf{r}-\mathbf{r}'|\}}{|\mathbf{r}-\mathbf{r}'|}\,dV'. \quad (18)$$

Upon substituting $\bar{\varphi}_\alpha(\mathbf{r})$ in the functional (15), we get a new functional which depends only on ψ:

$$J[\psi] \equiv F[\psi, \bar{\varphi}_\alpha] = -2\pi\sum_{\alpha=1}^{3}\int\bar{\varphi}_\alpha\eta_\alpha\,dV. \quad (19)$$

We must now make this functional an extremum by varying ψ, subject to the supplementary condition $\{\psi^*, \psi\} = 1$. We shall denote the extremal of the functional $J[\psi]$ by ψ^v. When this ψ^v is substituted in formula (18), we get a meson field which we denote by $\varphi^v(\mathbf{r})$. It is self-consistent in the following sense: the Euler equation determining the extremum of $J[\psi]$ is Eq. (12) with the q's replaced by the equilibrium q^v's. Thus the ψ^v which makes $J[\psi]$ an extremum is a solution of the equation

$$\hat{H}'(q^v)\psi^v \equiv \hat{H}'[\varphi^v_\alpha]\psi^v = H'(q^v)\psi^v, \quad (20)$$

in which the parameters of the Hamiltonian, $q^v_{\alpha\vec{\varkappa}}$ themselves depend on ψ^v, since the φ^v_α are expressed in terms of ψ^v by formula (18):

$$\varphi_\alpha^v(\mathbf{r}) = \int \frac{\eta_\alpha[\mathbf{r}', \psi^v] \exp\{-\mu|\mathbf{r}-\mathbf{r}'|\}}{|\mathbf{r}-\mathbf{r}'|} dV'; \quad (21)$$

φ_α^v and ψ^v mutually determine one another through formulas (20) and (21). Such problems are customarily called self-consistent.

By expressing $J[\psi^v]$ and $H'(q^v)$ in terms of ψ^v, it is easy to get

$$J[\psi^v] = \frac{1}{2} H'(q^v) = -\frac{1}{3} G \sum_{\alpha\beta} (\overline{\tau_\alpha\sigma_\beta}^{\psi^v})^2, \quad (22)$$

where

$$\overline{\tau_\alpha\sigma_\beta}^{\psi^v} = \{\psi^{v*}, \tau_\alpha\sigma_\beta\psi^v\};$$
$$G = \frac{g^2}{2\mu^2}\iint (\nabla u(\mathbf{r}), \nabla' u(\mathbf{r}')) \frac{\exp\{-\mu|\mathbf{r}-\mathbf{r}'|\}}{|\mathbf{r}-\mathbf{r}'|} dV dV'. \quad (23)$$

In a semiclassical treatment of the system, where the spin-charge motion is treated quantum-mechanically while the meson field oscillations are treated classically, the functions φ_α^v, ψ^v would be an exact solution of the problem, and would describe a state in which the static meson field φ_α^v corresponds to the minimum of the potential energy (14), the oscillators are at rest in their equilibrium positions, and the spin-charge state ψ^v is determined by the exact quantum equation (20).

However, in the investigation of most phenomena, the meson field must be treated quantum-mechanically. In this case, the oscillators of the meson field cannot be at rest, but will go through zero-point vibrations in the neighborhood of the equilibrium positions $q^v_{\alpha\vec{\varkappa}}$. These oscillations are described by Eqs. (12) and (13). We shall solve them approximately, under the assumption that the oscillators stay for the most part in the neighborhood of their equilibrium positions. In other words, it is assumed that the meson field $\varphi_\alpha(\mathbf{r})$ varies only slightly around $\varphi_\alpha^v(\mathbf{r})$ during the course of its oscillation.

4. DETERMINATION OF THE SPIN-CHARGE PART OF THE WAVE FUNCTION

We find ψ^v as the extremal of the functional (19). The latter is expressed explicitly in terms of ψ as follows:

$$J[\psi] \equiv -\frac{1}{3} G \sum_{\alpha,\beta=1}^{3} (\overline{\tau_\alpha\sigma_\beta})^2, \quad (24)$$

where

$$\begin{aligned}
&\overline{\tau_1\sigma_1} = C_1^*C_4 + C_2^*C_3 + C_3^*C_2 + C_4^*C_1, \quad \overline{\tau_2\sigma_3} = -iC_1^*C_3 + iC_2^*C_4 + iC_3^*C_1 - iC_4^*C_2,\\
&\overline{\tau_1\sigma_2} = -iC_1^*C_4 + iC_2^*C_3 - iC_3^*C_2 + iC_4^*C_1, \quad \overline{\tau_3\sigma_1} = C_1^*C_2 + C_2^*C_1 - C_3^*C_4 - C_4^*C_3,\\
&\overline{\tau_1\sigma_3} = C_1^*C_3 - C_2^*C_4 + C_3^*C_1 - C_4^*C_2, \quad \overline{\tau_3\sigma_2} = -iC_1^*C_2 + iC_2^*C_1 + iC_3^*C_4 - iC_4^*C_3,\\
&\overline{\tau_2\sigma_1} = -iC_1^*C_4 - iC_2^*C_3 + iC_3^*C_2 + iC_4^*C_1, \quad \overline{\tau_3\sigma_3} = C_1^*C_1 - C_2^*C_2 - C_3^*C_3 + C_4^*C_4,\\
&\overline{\tau_2\sigma_2} = -C_1^*C_4 + C_2^*C_3 + C_3^*C_2 - C_4^*C_1.
\end{aligned} \quad (24')$$

We set $C_k = \sqrt{u_k}e^{i\varepsilon_k}$ ($k = 1, 2, 3, 4$; $u_k \geqslant 0$, ε_k is real), and get

$$Z \equiv \sum_{\alpha\beta} (\overline{\tau_\alpha\sigma_\beta})^2 = 8|\sqrt{u_1u_4}\, e^{i\varepsilon} - \sqrt{u_2u_3}|^2 + (u_1 + u_2 + u_3 + u_4)^2, \quad (25)$$

where $\varepsilon = \varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4$. When we set the derivatives of $Z - \lambda(u_1 + u_2 + u_3 + u_4)$, with respect to ϵ and u_k equal to zero (λ is a Lagrange multiplier), we arrive at the following system of equations:

$$(u_1u_2u_3u_4)^{1/2} \sin\varepsilon = 0; \quad (26)$$
$$u_1 = u_4; \quad u_2 = u_3; \quad (u_1 - u_2)(1 + \cos\varepsilon) = 0;$$
$$u_1 + u_2 = 1/2.$$

This system can be satisfied only if $\sin\epsilon = 0$, i.e., $\epsilon = \pi\nu$ where ν is an integer. For odd ν, the solution is any set of numbers u_k satisfying the conditions

$$u_1 = u_4, \quad u_2 = u_3, \quad u_1 + u_2 = {}^1/_2. \tag{27}$$

This gives $Z = 3$, For even ν, the solution of Eq. (26) has the form

$$u_1 = u_2 = u_3 = u_4 = {}^1/_4; \quad Z = 1. \tag{28}$$

Clearly, the stable state of the nucleon is the state with the lowest energy, i.e., the case of (27). In this state [cf. Eq. (24), $J[\psi^\nu] = -G$. From now on we shall treat only this state.

Without loss of generality, we can multiply all the components C_k of any solution by a constant of modulus unity. This means that all the phases ϵ_k can be increased simultaneously by an arbitraty real number. We shall choose this number so that $\epsilon_1 + \epsilon_4 = 0$, whereupon $\epsilon_2 + \epsilon_3 = -\pi\nu$, with ν odd. Then the solution (27) takes the form:

$$C_4 = C_1^*, \quad C_3 = -C_2^*, \tag{29}$$

$$|C_1|^2 + |C_2|^2 = {}^1/_2.$$

Rotation of the coordinate system, first through angle φ about the Oz axis, and then through angle θ about the Oy axis results in the following coordinate transformation:

$$r' = \|a\| r, \quad \|a\| = \begin{vmatrix} \cos\vartheta\cos\varphi & \cos\vartheta\sin\varphi & -\sin\vartheta \\ -\sin\varphi & \cos\varphi & 0 \\ \sin\vartheta\cos\varphi & \sin\vartheta\sin\varphi & \cos\vartheta \end{vmatrix}. \tag{30}$$

The spinor ψ is transformed in the following way as a result of such a rotation:

$$\psi' = \|S\|\psi, \quad \|S\| = \begin{vmatrix} \cos\frac{\vartheta}{2}e^{i\varphi/2} & \sin\frac{\vartheta}{2}e^{-i\varphi/2} & 0 & 0 \\ -\sin\frac{\vartheta}{2}e^{i\varphi/2} & \cos\frac{\vartheta}{2}e^{-i\varphi/2} & 0 & 0 \\ 0 & 0 & \cos\frac{\vartheta}{2}e^{i\varphi/2} & \sin\frac{\vartheta}{2}e^{-i\varphi/2} \\ 0 & 0 & -\sin\frac{\vartheta}{2}e^{i\varphi/2} & \cos\frac{\vartheta}{2}e^{-i\varphi/2} \end{vmatrix}. \tag{31}$$

(31)

The quantities $\overline{\tau_\alpha\sigma_1}$, $\overline{\tau_\alpha\sigma_2}$, $\overline{\tau_\alpha\sigma_3}$ transform like the components of a three-dimensional vector. Therefore the functional (24) is invariant under the transformation (31). Consequently, if ψ^0 is an extremal of $J[\psi]$, then $\|S\|\psi^0$ will also be an extremal. We can similarly rotate the coordinate system in the isotopic spin space. The matrix $\|b\|$ of the rotation will have a form similar to (30) except that we introduce in place of ϑ and φ the corresponding angles β and γ. Under such a rotation, ψ again transforms like a spinor:

$$\psi'' = \|T\|\psi', \quad \|T\| = \begin{vmatrix} \cos\frac{\beta}{2}e^{i\gamma/2} & 0 & \sin\frac{\beta}{2}e^{-i\gamma/2} & 0 \\ 0 & \cos\frac{\beta}{2}e^{i\gamma/2} & 0 & \sin\frac{\beta}{2}e^{-i\gamma/2} \\ -\sin\frac{\beta}{2}e^{i\gamma/2} & 0 & \cos\frac{\beta}{2}e^{-i\gamma/2} & 0 \\ 0 & -\sin\frac{\beta}{2}e^{i\gamma/2} & 0 & \cos\frac{\beta}{2}e^{-i\gamma/2} \end{vmatrix}. \tag{32}$$

Under such a rotation, the quantities $\overline{\tau_1\sigma_\beta}$ $\overline{\tau_2\sigma_\beta}\overline{\tau_3\sigma_\beta}$ transform like the components of a three-dimensional vector in the isotopic spin space, while the functioanl (24) remains invariant. Consequently,

$$\psi^\nu = \|T\|\cdot\|S\|\psi^0 \tag{33}$$

is also an extremal of $J[\psi]$. If we choose as our "initial" solution ψ^0 the special case of (29):

$$C_1^0 = C_4^0 = 2^{-1/2}, \quad C_2^0 = C_3^0 = 0, \tag{34}$$

then we can express the "rotated" solution (33) in the following form:

$$\begin{aligned} C_1^v &= \frac{1}{\sqrt{2}}\left(\cos\frac{\vartheta}{2}\cos\frac{\beta}{2}e^{i\delta/2} + \sin\frac{\vartheta}{2}\sin\frac{\beta}{2}e^{-i\delta/2}\right); \\ C_2^v &= \frac{1}{\sqrt{2}}\left(-\sin\frac{\vartheta}{2}\cos\frac{\beta}{2}e^{i\delta/2} + \cos\frac{\vartheta}{2}\sin\frac{\beta}{2}e^{-i\delta/2}\right); \\ C_3^v &= \frac{1}{\sqrt{2}}\left(-\cos\frac{\vartheta}{2}\sin\frac{\beta}{2}e^{i\delta/2} + \sin\frac{\vartheta}{2}\cos\frac{\beta}{2}e^{-i\delta/2}\right); \\ C_4^v &= \frac{1}{\sqrt{2}}\left(\sin\frac{\vartheta}{2}\sin\frac{\beta}{2}e^{i\delta/2} + \cos\frac{\vartheta}{2}\cos\frac{\beta}{2}e^{-i\delta/2}\right), \end{aligned} \tag{35}$$

where $\delta = \varphi + \gamma$. It can be shown that every extremal $J[\psi]$ of the form (29) is given by the expression (35) with the appropriate choice of angles ϑ, β and δ. If the angles are limited to the intervals $0 \leqslant \delta \leqslant \pi$, $0 \leqslant \vartheta \leqslant 2\pi$, $0 \leqslant \beta \leqslant 2\pi$, then only one set of angles will correspond to each ψ^v.

Even though four independent angles ϑ, φ, β, γ appear in the rotation matrices $||S||$ and $||T||$, the "rotated" solution (35) is determined by only three independent angles ϑ, β and $\delta = \varphi + \gamma$.

Corresponding to the initial ψ^0 and the "rotated" ψ^v we should introduce densities $\eta_\alpha^0(\mathbf{r})$ and $\eta_\alpha^v(\mathbf{r})$ as well as the self-consistent meson fields $\varphi_\alpha^0(\mathbf{r})$ and $\varphi_\alpha^v(\mathbf{r})$. The relation between them is given by formulas (17) and (21). The quantities η_α and φ_α are three-dimensional vectors in the isotopic spin space. Therefore

$$\begin{aligned} \eta_\alpha^v(\mathbf{r}) &= \sum_{\alpha_1=1}^{3} b_{\alpha\alpha_1}\eta_{\alpha_1}^0(\|a^{-1}\|\mathbf{r}) \\ &= -\frac{g}{\sqrt{4\pi}\,\mu}\sum_{\alpha_1=1}^{3} b_{\alpha\alpha_1}(\|a\|\overline{\tau_{\alpha_1}\sigma}^{\psi^0}, \mathbf{r})\frac{1}{r}\frac{\partial u}{\partial r} \\ &= -\frac{g}{\sqrt{4\pi}\,\mu}\sum_{\alpha_1\beta_1\beta} b_{\alpha\alpha_1}a_{\beta\beta_1}\overline{\tau_{\alpha_1}\sigma_{\beta_1}}^{\psi^0}\frac{x_\beta}{r}\frac{\partial u}{\partial r} \\ &= -\frac{g}{\sqrt{4\pi}\,\mu}\sum_{\beta=1}^{3}\overline{\tau_\alpha\sigma_\beta}^{\psi^v}\frac{\partial u}{\partial x_\beta}, \end{aligned} \tag{36}$$

where

$$\overline{\tau_\alpha\sigma_\beta}^{\psi^v} = \sum_{\alpha_1,\beta_1=1}^{3} b_{\alpha\alpha_1}a_{\beta\beta_1}\overline{\tau_{\alpha_1}\sigma_{\beta_1}}^{\psi^0}. \tag{37}$$

Using formulas (34), (24), and (37), we get

$$\begin{aligned} &\overline{\tau_1\sigma_1}^{\psi^v} = \cos\vartheta\cos\beta\cos\delta + \sin\vartheta\sin\beta, \quad \overline{\tau_2\sigma_3}^{\psi^v} = -\sin\vartheta\sin\delta, \\ &\overline{\tau_1\sigma_2}^{\psi^v} = -\cos\beta\sin\delta, \quad \overline{\tau_3\sigma_1}^{\psi^v} = \cos\vartheta\sin\beta\cos\delta - \sin\vartheta\cos\beta, \\ &\overline{\tau_1\sigma_3}^{\psi^v} = \sin\vartheta\cos\beta\cos\delta - \cos\vartheta\sin\beta, \\ &\overline{\tau_2\sigma_1}^{\psi^v} = -\cos\vartheta\sin\delta, \; \overline{\tau_3\sigma_2}^{\psi^v} = -\sin\beta\sin\delta, \\ &\overline{\tau_2\sigma_2}^{\psi^v} = -\cos\delta, \overline{\tau_3\sigma_3}^{\psi^v} = \sin\vartheta\sin\beta\cos\delta + \cos\vartheta\cos\beta. \end{aligned} \tag{38}$$

5. CONSIDERATION OF SMALL OSCILLATIONS OF THE MESON FIELD IN THE NEIGHBORHOOD OF THE EQUILIBRIUM FORM

The self consistent solutions $\varphi_\alpha^v(\mathbf{r})$ which we found in the preceding section correspond to the equilibrium positions of the meson field oscillators. We now go on to consider the small vibrations of $\varphi_\alpha(\mathbf{r})$ in the neighborhood of the equilibrium form $\varphi_\alpha^v(\mathbf{r})$. We shall assume that the oscillators of the meson field vibrate mainly close to the configuration in which their potential energy is a minimum. This means $\varphi_\alpha(\mathbf{r})$ is almost always close to one of the functions $\varphi_\alpha^v(\mathbf{r})$. Thus if we set

$$\varphi_\alpha(\mathbf{r}) = \varphi_\alpha^v(\mathbf{r}) + \varphi_\alpha'(\mathbf{r}), \tag{39}$$

and select the appropriate angles ϑ, β, and δ for each $\varphi_\alpha(\mathbf{r})$, we can regard $\varphi'_\alpha(\mathbf{r})$ as a small perturbation in that portion of the space in which $\varphi_\alpha^v(\mathbf{r})$

is essentially different from zero. The term $\varphi'_\alpha(\mathbf{r})$ drops out in zeroth approximation, so that equation (12) becomes (cf. Eqs. 6, 21, 17, 23):

$$\hat{H}'(q^v)\psi_s^v \equiv -\frac{2}{3}G\sum_{\alpha\beta}\overline{\tau_\alpha\sigma_\beta}^{\psi^v}\tau_\alpha\sigma_\beta\psi_s^v = H'_s(q^v)\psi_s^v. \quad (40)$$

This solution simplifies considerably if we go over to the wave function ψ_s^0 by means of the substitution (33). Then Eq. (40) becomes:

$$-\frac{2}{3}G\sum_{\alpha\beta}\overline{\tau_\alpha\sigma_\beta}^{\psi^0}\tau_\alpha\sigma_\beta\psi_s^0 \equiv -\frac{2}{3}G\begin{vmatrix}1 & 0 & 0 & 2\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 2 & 0 & 0 & 1\end{vmatrix}\begin{vmatrix}C_{1s}^0\\ C_{2s}^0\\ C_{3s}^0\\ C_{4s}^0\end{vmatrix} = H'_s(q^v)\psi_s^0. \quad (41)$$

The four eigenfunctions and eigenvalues of this equation have the form:

$$1)\ H'_1 = -2G,\quad 2)\ H'_2 = {}^2/_3G,\quad 3)\ H'_3 = {}^2/_3G.\quad 4)\ H'_4 = {}^2/_3G; \quad (42)$$

$$\psi^0 \equiv \psi_1^0 = \begin{vmatrix}\frac{1}{\sqrt{2}}\\ 0\\ 0\\ \frac{1}{\sqrt{2}}\end{vmatrix},\quad \psi_2^0 = \begin{vmatrix}0\\ 1\\ 0\\ 0\end{vmatrix},\quad \psi_3^0 = \begin{vmatrix}0\\ 0\\ 1\\ 0\end{vmatrix},\quad \psi_4^0 = \begin{vmatrix}\frac{1}{\sqrt{2}}\\ 0\\ 0\\ -\frac{1}{\sqrt{2}}\end{vmatrix}.$$

From this equation we see that the ground state of the unperturbed problem is non-degenerate, so that the energy of the spin-charge motion in first approximation can, as usual, be expressed in terms of the exact energy operator (6) and the zeroth order wave function:

$$H'(q) = \{\psi^{v*}, \hat{H}'\psi^v\} \quad (43)$$

$$= \frac{g\sqrt{4\pi}}{\mu}\sum_{\alpha\beta}\overline{\tau_\alpha\sigma_\beta}^{\psi^v}\int\varphi_\alpha\frac{\partial u}{\partial x_\beta}dV =$$

$$= -4\pi\sum_\alpha\int\varphi_\alpha(\mathbf{r})\,\eta_\alpha^v(\mathbf{r})\,dV.$$

We now proceed to Eq. (13) which describes the oscillations of the meson field. Upon substituting $H'(q)$ in the form (43) in the operator on the left side of the equation and using Eqs. (3), (39), (16), and (17), we get

$$\hat{H}_0 + H'(q) = H'(q^v) \quad (44)$$

$$+\frac{1}{2}\sum_\alpha\int[(\nabla\varphi_\alpha^v)^2 + \mu^2\varphi_\alpha^{v2}]\,dV$$

$$+\frac{1}{2}\sum_\alpha\int[\pi_\alpha^2 + (\nabla\varphi'_\alpha)^2 + \mu^2\varphi_\alpha'^2]\,dV$$

$$= J[\psi^v] + \frac{1}{2}\sum_{\alpha\vec{\varkappa}}\omega_{\vec{\varkappa}}\left[(q_{\alpha\vec{\varkappa}} - q_{\alpha\vec{\varkappa}}^v)^2 - \frac{\partial^2}{\partial q_{\alpha\vec{\varkappa}}^2}\right].$$

A question arises as to how the parameters

$$\vartheta \equiv v_1,\quad \beta \equiv v_2,\quad \delta \equiv v_3, \quad (45)$$

should be chosen so that the functions $\varphi_\alpha^v(\mathbf{r})$ give the best approximation to $\varphi_\alpha(\mathbf{r})$. Obviously the values of v_i must depend on $\varphi_\alpha(\mathbf{r})$, since the parameters of an approximation always depend on the function to be approximated. We should regard as the best choice of the v_i that set for which the energy $H(q)$ is most accurately given by formula (43). According to the variational principle of quantum mechanics, the highest accuracy in calculating $H(q)$ is achieved for that choice of the v_i which makes the energy (43) a minimum. The minimum condition has the following form:

$$\sum_{\alpha=1}^{3}\int \varphi_\alpha \frac{\partial \varphi_\alpha^v}{\partial v_i} dV = 0, \quad i = 1, 2, 3. \qquad (46)$$

The v_i are determined from these three equations; they are functionals of $\varphi_\alpha(\mathbf{r})$, i.e., functions of $q_{\alpha\vec{\varkappa}}$. If we introduce the operator $\hat{\omega} = \sqrt{\mu^2 - \Delta}$ and use Eq. (16), Eq. (46) can be rewritten as

$$\sum_\alpha \int \varphi_\alpha \frac{\partial \hat{\omega}^2 \varphi_\alpha^v}{\partial v_i} dV = 0, \quad i = 1, 2, 3. \qquad (47)$$

There are also other methods for determining the parameters v_i. For example, we can approximate φ_α by functions φ_α^v using the method of least squares, and choose the v_i so that the integral

$$\sum_\alpha \int [\varphi_\alpha - \varphi_\alpha^v]^2 dV \qquad (48)$$

is a minimum. Upon expanding the square of the difference and noting that the square of the first term and the integral of the square of the second term do not depend on v_i, we can express the minimum condition in the form

$$\sum_\alpha \int \varphi_\alpha \frac{\partial \varphi_\alpha^v}{\partial v_i} dV = 0, \quad i = 1, 2, 3. \qquad (49)$$

These equations can be used in place of (47) for determining the v_i; the results thus obtained are very close to those from (47) so long as the φ_α actually differ little from the φ_α^v. We could also use a system of equations intermediate in type between (47) and (49):

$$\sum_\alpha \int \varphi_\alpha \frac{\partial \hat{\omega} \varphi_\alpha^v}{\partial v_i} dV = 0, \quad i = 1, 2, 3. \qquad (50)$$

The operator (44) is the required Hamiltonian, determining the motion of the meson field in the presence of the nucleon. Its eigenvalues give the energy H of the total system, as shown by Eq. (13). The eigenfunctions of this operator, $\Phi(q)$, when multiplied by ψ^v give the wave functions of the system (cf. Eq. 9). The operator (44) is essentially different from the energy operator of a system of independent harmonic oscillators, since the quantities v_i, and consequently also the $q^v_{\alpha\vec{\varkappa}}$, are complicated functions of the $q_{\alpha\vec{\varkappa}}$.

It is appropriate here to compare our method with that of Geilikman[8], who represents the meson field in the form

$$\varphi_\alpha(\mathbf{r}) = f_\alpha(\mathbf{r}) + \varphi'_\alpha(\mathbf{r}), \quad \varphi'_\alpha \ll f_\alpha \qquad (51)$$

and treats $f_\alpha(\mathbf{r})$ as a classical quantity, commuting with π_α. He determines the quantity $f_\alpha(\mathbf{r})$ as the minimum of our expression (14), i.e., as the equilibrium form of the meson field when it is treated classically.

As we pointed out earlier, there are an infinity of mimima of Eq. (14), determined by the parameters $\vartheta, \beta, \delta, \varphi_\alpha^v(\mathbf{r})$, so that f_α depends essentially on ϑ, β, δ. In formula (51), $\varphi'_\alpha(\mathbf{r})$ can be treated as a small quantity only if, for each $\varphi_\alpha(\mathbf{r})$, the parameters v_i are determined in an appropriate way [for example, as done above, through Eq. (47), (49), or (50).] Consequently the v_i are functions of the $q_{\alpha\vec{\varkappa}}$ and therefore $f_\alpha(\mathbf{r}, v_i)$ does not commute with π_α, in contradiction to the assumption of Ref. 8.

No explicit form of the wave function for the spin-charge motion was found in Ref. 8, so that the author did not discover the presence of parameters v_i (or an equivalent set). As a result, the quantities $q^v_{\alpha\vec{\varkappa}}$, in the operator analogous to (44), were treated as constants, and the operator itself was regarded to be the energy operator of a system of independent harmonic oscillators. As a result Geilikman's results[8,9] differ essentially from ours.

6. SOME PROPERTIES OF THE FUNCTION $v_i(\ldots q_{\alpha\vec{\varkappa}} \ldots)$, AND SOME RELATIONS WHICH ARE NEEDED LATER

We shall obtain various results without determining the explicit form of the function $v_i(\ldots q_{\alpha\vec{\varkappa}} \ldots)$, by using some of the properties of these functions which are already apparent from the implicit definition of v_i given by formulas (47), (49) or (50). We write these formulas in the form

$$\sum_\alpha \int \varphi_\alpha \frac{\partial \hat{\omega}^n \varphi_\alpha^v}{\partial v_i} dV = 0, \qquad (52)$$

$$i = 1, 2, 3, \quad n = 0, \text{ or } 1, \text{ or } 2.$$

If we expand φ_α and φ_α^v in Fourier series of the form (1), Eq. (52) is replaced by

$$\sum_{\alpha\vec{\varkappa}} q_{\alpha\vec{\varkappa}} \omega_{\vec{\varkappa}}^{n-1} \frac{\partial q^v_{\alpha\vec{\varkappa}}}{\partial v_i} = 0. \qquad (53)$$

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[8] B. T. Geilikman, Dokl. Akad. Nauk SSSR **90**, 359 (1953).

[9] B. T. Geilikman, Dokl. Akad. Nauk SSSR **90**, 991 (1953); **91**, 39, 225 (1953).

It is obvious that if we multiply all the $q_{\alpha\vec{\varkappa}}$ by an arbitrary common factor t, the roots v_i of equation (53) remain unchanged. Consequently,

$$v_i(\cdots tq_{\alpha\vec{\varkappa}}\cdots) = v_i(\cdots q_{\alpha\vec{\varkappa}}\cdots), \tag{54}$$

i.e., the v_i are homogeneous functions of degree zero in the $q_{\alpha\vec{\varkappa}}$, so that

$$\sum_{\alpha\vec{\varkappa}} \frac{\partial v_i}{\partial q_{\alpha\vec{\varkappa}}} q_{\alpha\vec{\varkappa}} = 0. \tag{55}$$

We can also show that

$$\int \sum_{\alpha} [\hat{\omega}^{n_1}\varphi_\alpha^v(\mathbf{r})][\hat{\omega}^{n_2}\varphi_\alpha^v(\mathbf{r})]\, dV \tag{56}$$

$$= \int \sum_{\alpha} \varphi_\alpha^0(\mathbf{r})\, \hat{\omega}^{n_1+n_2}\varphi_\alpha^0(\mathbf{r})\, dV$$

does not depend on v_i. The proof can be given by making use of the invariance of the function in the integrand with respect to rotations in both ordinary and isotopic space, and by changing variables in the integral. If we set $n_1 = n_2 = n/2$ in (56) and differentiate with respect to v_i and v_j, we get

$$\sum_{\alpha} \int \frac{\partial \varphi_\alpha^v}{\partial v_i} \hat{\omega}^n \varphi_\alpha^v dV = 0, \tag{57}$$

$$\sum_{\alpha} \int \frac{\partial \varphi_\alpha^v}{\partial v_i} \frac{\partial \hat{\omega}^n \varphi_\alpha^v}{\partial v_j} dV = -\sum_{\alpha} \int \frac{\partial^2 \varphi_\alpha^v}{\partial v_i \partial v_j} \hat{\omega}^n \varphi_\alpha^v dV. \tag{58}$$

Later on we shall repeatedly deal with the quantities

$$R_{ij}^{(n)} = -\sum_{\alpha} \int \frac{\partial^2 \varphi_\alpha^v}{\partial v_i \partial v_j} \hat{\omega}^n \varphi_\alpha^v dV \tag{59}$$

$$= \sum_{\alpha\vec{\varkappa}} \omega_{\vec{\varkappa}}^{n-1} \frac{\partial q_{\alpha\vec{\varkappa}}^v}{\partial v_i} \frac{\partial q_{\alpha\vec{\varkappa}}^v}{\partial v_j},$$

which we now calculate. The field φ_α^v, defined by formula (21), can be expressed, using formula (36), as

$$\varphi_\alpha^v(r) = -\frac{g}{\sqrt{4\pi}\,\mu} \sum_{\beta=1}^{3} \overline{\tau_\alpha \sigma_\beta}^{\psi^v} \int \frac{\partial u(\mathbf{r}')}{\partial x_\beta'} \frac{\exp\{-\mu|\mathbf{r}-\mathbf{r}'|\}}{|\mathbf{r}-\mathbf{r}'|} dV' = -(\overline{\tau_\alpha \sigma}^{\psi^v}, \nabla W) \tag{60}$$

$$= -\frac{1}{r}\frac{dW}{dr}(\overline{\tau_\alpha \sigma}^{\psi^v}, \boldsymbol{r}),$$

where

$$W(r) = \frac{g}{\mu}\frac{1}{\sqrt{4\pi}} \int u(\mathbf{r}') \frac{\exp\{-\mu|\mathbf{r}-\mathbf{r}'|\}}{|\mathbf{r}-\mathbf{r}'|} dV'. \tag{61}$$

If we substitute (60) in (59), we get

$$R_{ij}^{(n)} = -\frac{1}{2} I^{(n)} \sum_{\alpha\beta} \overline{\tau_\alpha \sigma_\beta}^{\psi^v} \frac{\partial^2 \overline{\tau_\alpha \sigma_\beta}^{\psi^v}}{\partial v_i \partial v_j}. \tag{62}$$

Here we have introduced the notation

$$2\int \frac{\partial W}{\partial x_\beta} \hat{\omega}^n \frac{\partial W}{\partial x_{\beta'}} dV = \delta_{\beta\beta'} I^{(n)}. \tag{63}$$

From formulas (62) and (38) we get

$$R_{11}^{(n)} = R_{22}^{(n)} = R_{33}^{(n)} = I^{(n)}, \tag{64}$$

$$R_{12}^{(n)} = R_{21}^{(n)} = -I^{(n)} \cos\delta,$$

$$R_{13} = R_{31}^{(n)} = R_{23}^{(n)} = R_{32}^{(n)} = 0.$$

Now we calculate $\partial v_i / \partial q_{\alpha\vec{\varkappa}}$. For this purpose we differentiate (52) or (53) with respect to $q_{\alpha\vec{\varkappa}}$:

$$\sum_{\alpha_1, j} \int \varphi_{\alpha_1} \frac{\partial^2 \hat{\omega}^n \varphi_{\alpha_1}^v}{\partial v_i \partial v_j} \frac{\partial v_j}{\partial q_{\alpha\varkappa}} + \frac{\partial q_{\alpha\vec{\varkappa}}^v}{\partial v_i} \omega_{\vec{\varkappa}}^{n-1} = 0. \tag{65}$$

Since, according to our assumption, φ_{α_1} is close to $\varphi_{\alpha_1}^v$, we replace φ_{α_1} in the integrand by the approximation $\varphi_{\alpha_1}^v$. Then by using (59), we can write equation (65) in the form

$$\sum_{j=1}^{3} R_{ij}^{(n)} \frac{\partial v_j}{\partial q_{\alpha\vec{\varkappa}}} = \frac{\partial q_{\alpha\vec{\varkappa}}^v}{\partial v_i} \omega_{\vec{\varkappa}}^{n-1}, \quad i = 1, 2, 3. \tag{66}$$

The quantities $\partial v_j / \partial q_{\alpha\vec{\varkappa}}$ are determined from these three linear equations as follows:

$$\frac{\partial \vartheta}{\partial q_{\alpha\vec{\varkappa}}} = \frac{\omega_{\vec{\varkappa}}^{n-1}}{I^{(n)} \sin^2\delta} \left(\frac{\partial q_{\alpha\vec{\varkappa}}^v}{\partial \vartheta} + \cos\delta \frac{\partial q_{\alpha\vec{\varkappa}}^v}{\partial \beta} \right), \tag{67}$$

$$\frac{\partial \beta}{\partial q_{\alpha\vec{\varkappa}}} = \frac{\omega_{\vec{\varkappa}}^{n-1}}{I^{(n)} \sin^2\delta} \left(\frac{\partial q_{\alpha\vec{\varkappa}}^v}{\partial \beta} + \cos\delta \frac{\partial q_{\alpha\vec{\varkappa}}^v}{\partial \vartheta} \right), \tag{68}$$

$$\frac{\partial \delta}{\partial q_{\alpha \vec{\varkappa}}} = \frac{\omega_{\vec{\varkappa}}^{n-1}}{I^{(n)}} \frac{\partial q^{v}_{\alpha \vec{\varkappa}}}{\partial \delta}. \tag{69}$$

The approximate method which we have developed enabled us to reduce the initial problem of finding the eigenvalues and eigenfunctions of the operator $\hat{H}_0 + \hat{H}'$ (cf. Eqs. 3 and 6) to the solution of Eq. (13) with the operator (44). The latter describes the oscillations of the meson field in the presence of the nucleon; the spin-charge degrees of freedom of the system no longer appear in it. The solution of this equation is given in a succeeding paper, which appears in this same issue.

The following approximations have been made in this paper:

1) The approximation of "infinitely heavy nucleons"; the nucleon was assumed to be fixed.

2) The adiabatic approximation: we dropped the second term on the right hand side of (10), which led to the approximate formula (11). 3) It

3) It was assumed that the meson field $\varphi_{\alpha}(\mathbf{r})$ deviates only slightly from the self-consistent field $\varphi^{v}_{\alpha}(\mathbf{r})$, if the parameters ϑ, β, δ are suitably chosen. This assumption (39) is used for the calculation of the energy $H'(q)$. by the method of small perturbations [formula (43)] and for the transition from formula (65) to formula (66).

The omitted small quantities which are enumerated above will be taken into account as small perturbations in one of the later papers. Including these terms will enable us to estimate the accuracy of the theory and to derive criteria for its applicability. It will be shown that assumptions 2 and 3 are based on one and the same inequality: $g / \mu a \gg 1$. Here a is the effective radius of the nucleon, which is defined to be

$$a^{-1} = \iint u(\mathbf{r}) \frac{1}{|\mathbf{r} - \mathbf{r}'|} u(\mathbf{r}')\, dV\, dV'.$$

The inequality we have given defines the case of strong coupling.

Translated by M. Hamermesh
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