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* The corresponding expression for σ_d in the article of Berestetskii[1] contains several misprints.

[1] V. B. Berestetskii and I. Ia. Pomeranchuk, Dokl. Akad. Nauk SSSR **81**, 1019 (1951).

[2] Rochester conference, 1955.

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On the Theory of Galvanomagnetic Effects in Metals

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IN this communication we develop the theory of galvanomagnetic effects in metals, without making any special assumptions on the law of dispersion of conduction electrons or on the form of the collision integral.

1. An electron in a metal is here taken to be a quasiparticle carrying a charge e, with energy ϵ and a quasi-momentum p; ϵ is a periodic function of p, with the period of the reciprocal lattice. With an electron moving in a constant magnetic field $\mathbf{H}(H_x = H_y = 0;\ H_z = H)$, ϵ and p_z remain unaltered. Thus, in the momentum zone an electron moves along the curve

$$\varepsilon(\mathbf{p}) = \text{const}; \quad p_z = \text{const}. \tag{1}$$

From the equations of motion, $d\mathbf{p}/dt = (e/c)[\mathbf{v}, \mathbf{H}]$ $(\mathbf{v} = \nabla_{\mathbf{p}}\epsilon)$ we obtain:

$$t = -(c/eH)\int dl/v_{\perp}. \tag{2}$$

Here dl represents an element of arc of curve (1), taken in the direction of motion, while $v_{\perp}$ represents a projection of velocity vector upon the plane (x, y). The character of an electron's motion along a trajectory within the momentum zone actually depends on whether curve (1) is closed (i.e., whether it consists of a series of closed curves each of which is disposed within the limits of a single space in the reciprocal lattice)*, or whether it is open (i.e., whether it passes uninterruptedly through the entire reciprocal lattice). If curve (1) is closed, then the transition along this curve is

$$T = -\frac{c}{eH}\oint\frac{dl}{v_{\perp}} = -\frac{2\pi m^* c}{eH}; \quad m^* = \frac{1}{2\pi}\frac{\partial S}{\partial\varepsilon}; \tag{3}$$

where $S = S(\varepsilon, p_z)$ corresponds to the area of intersection of surface $\epsilon(p) = \text{const}$ with plane $p_z = \text{const}$.

The quantity m^* can actually be called the effective mass of an electron in a magnetic field. The sign of this effective mass (and of the period as well) depends on whether the energy within the surface $\epsilon(p) = \epsilon$ is smaller than $\epsilon (m^* > 0)$ or larger than $\epsilon (m^* < 0)$; this criterion does not hold true for the intersecting curves. It should be noted here that the concept of effective mass cannot be introduced for open trajectories.

2. While describing the state of an electron in a magnetic field it is proper to use variables ϵ, p_z and a dimensionless variable $\tau = t/T_0$, indicating the location of an electron in the trajectory (1) within the momentum zone ($T_0 = 2\pi c m_0/eH$, with m_0 representing the mass of a free electron). T_0 was introduced for the sake of convenience, so as to free τ from its dependence on H. The kinetic equation for the distribution function of f in the variable chosen by us is

$$\frac{\partial f}{\partial\tau}\dot{\tau} + \frac{\partial f}{\partial p_z}\dot{p}_z + \frac{\partial f}{\partial\varepsilon}\dot{\varepsilon} + \left(\frac{\partial f}{\partial t}\right)_{\text{cr}} = 0. \tag{4}$$

(In studying a stationary case, the values of $\dot{\tau}$, $\dot{p}_z$, $\dot{\varepsilon}$ are obtained from the equations of motion). Assuming that $f = f_0 - et_0\psi_i E_i$, we align (4) along the electric field E. Noting that $\dot{\varepsilon} = e\mathbf{v}\mathbf{E}$; $\dot{p}_z = eE_z$, $\dot{\tau} = 1/T_0$, we obtain

$$\begin{aligned} &\partial\psi_i/\partial\tau + \gamma\hat{W}\psi_i = \gamma f_0'(\varepsilon)\, v_i; \\ &\hat{W}\psi_i = t_0(\partial\psi_i/\partial t)_{\text{cr}};\ \gamma = H_0/H; \end{aligned} \tag{5}$$

where f_0 represents the equilibrium Fermi function, t_0 represents the characteristic time of relaxation, and $T_0(H_0) = t_0$. The limiting condition for Eq. (5) is represented, for closed curves, by the condition of periodicty (with a period of T/T_0) of the function ψ_i and, for open trajectories, by the boundedness of the function ψ_i.

As the mean value of Eq. (5) we obtain

$$\overline{\hat{W}\psi_i} = f_0'(\varepsilon)\,\overline{v}_i. \tag{6}$$

The prime in the above equation indicates that the mean value was obtained with respect to τ. For the closed curves, assuming that $2\pi m_0 v_x = -\partial p_y/\partial\tau$, $2\pi m_0 v_y = \partial p_x/\partial\tau$, we obtain $\overline{v}_\alpha = 0$ $(\alpha = x, y)$.

3. We will find the solution for Eq. (5) under conditions (6) for large fields ($\gamma \ll 1$), in the form of a series* with interval γ. Computations show that for closed curves (1)

$$\psi_x = \gamma\,[-(f_0'/2\pi m_0)\,p_y + C_x] + \gamma^2\psi_x^{(2)} + \ldots,$$
$$\psi_y = \gamma\,[(f_0'/2\pi m_0)\,p_x + C_y] + \gamma^2\psi_y^{(2)} + \ldots, \qquad (7)$$
$$\psi_z = C_z + \gamma\psi_z^{(1)} + \ldots,$$

C_i represents solely the functions of ϵ, p_z. For the open curves (1): $\psi_i = C_i^{(0)} + \gamma\psi_i^{(1)} + \ldots$. The functions $\psi_i^{(k)}$, C_i are determined by means of Eqs. (5) and (6), and are dependent on the collision integral. The fact that the zero member is absent in the expansion of ψ_α for the case of closed curves is related to the condition $v_\alpha = 0$. Here we assume that the interval $\delta\varepsilon \sim \Theta$ of the smearing out of the Fermian function f_0 (near ϵ_0) does not contain open trajectories (ϵ_0 = energy limit at absolute zero; θ = temperature).

4. Making use of Eq. (7), : s possible to represent the conductivity tensor σ_{ik}, in the case when interval $\delta\varepsilon \sim \Theta$ of the spreading out of the Fermi function f_0 contains only closed trajectories, in the form

$$\sigma_{ik}(H) = \begin{pmatrix} \gamma^2 a_{xx} & \gamma a_{xy} & \gamma a_{xz} \\ \gamma a_{yx} & \gamma^2 a_{yy} & \gamma a_{yz} \\ \gamma a_{zx} & \gamma a_{zy} & a_{zz} \end{pmatrix}, \qquad (8)$$

in which the subdivision of the factors of matrix a_{ik} begins with the zero value of member γ.

The factor a_{xy} is not dependent on the collision integral when γ approaches zero. Making use of expression (7), we obtain:

$$a_{xy}^{(0)} \sim \int f_0' d\varepsilon \int dp_z \int p_y \partial p_x/\partial\tau \cdot d\tau \qquad (9)$$
$$= \int f_0' d\varepsilon \int dp_z \oint p_y dp_x .$$

Assuming that $f_0'(\varepsilon) = -\delta(\varepsilon - \varepsilon_0)$, we find $a_{xy}^{(0)} \sim V_1 - V_2 \sim n_1 - n_2$. Here $V_1 - V_2$ are the volumes bounded by the closed surfaces $\varepsilon(p) = \varepsilon_0$, within which $\epsilon < \epsilon_0$ ($m^* > 0$) and conversely, $\varepsilon > \varepsilon_0$ ($m^* < 0$); here n_1 represents the number of bound electronic states with a positive mass m^*, and n_2 represents the number of free electronic states with a negative effective mass. In this way the magnitudes of n_1 and n_2 in the following theory may be properly considered as the number of electrons and "holes". It must be emphasized here that the introduction of these concepts is possible only in the case where the energy interval $\delta\epsilon$ contains only closed nonintersecting surfaces of constant energy. In this case we obtain asymptotically (for large fields) $\sigma_{xy} = ec\,(n_1 - n_2)/H$; at $n_1 = n_2$ $a_{xy} \sim \gamma$, that is, $\sigma_{xy} \sim H^{-2}$. In the case of closed trajectories on open surfaces, the integral (9) has no such simple meaning and cannot be expressed in terms of a number of particles.

In the case when integral $\delta\varepsilon \sim \Theta$ contains a layer of open trajectories, the quantities σ_{ik} approach a finite limit as $H \to \infty$. This is related to the fact that the expansion of all the ψ_1, in this case begins with the zero term in γ. Consequently, experimental determination of the asymptoticity of $\sigma_{ik}(H)$ in strong magnetic fields allows us, in principle, to explain the topology of isoenergetic surfaces near the limiting value of Fermia energy.

5. The specific resistance of a metal in a magnetic field is usually measured by making use of the Hall effect. Assuming that magnetic field is perpendicular to the direction of current (an assumption commonly made for this experiment), and choosing the direction of current for the x-axis, we obtain $\rho(H) = (\sigma^{-1})_{xx}$. If there are no open surfaces within the interval ϵ, then, making use of expression (2) we find that

$$\rho = [a_{yy}^{(0)} a_{zz}^{(0)} + (a_{yz}^{(0)})^2] / (a_{xy}^{(0)})^2\, a_{zz}^{(0)}$$

approaches saturation when the number of "holes" is not equal to the number of electrons ($n_1 \neq n_2$). When the number of "holes" is equal to the number of electrons ($n_1 \neq n_2$) and $a_{xy}^{(0)}$ becomes zero, it becomes necessary to consider the subsequent terms in the expansion of σ_{ik} in respect to γ. This introduces a quadratic increase of resistance with a magnetic field. Consequently, the existence of a group of metals with an indefinitely increasing resistance in a magnetic field, which is known from experiment[1], can be explained by the most general assumptions of the electronic theory of metals. A similar result has been previously arrived at in the simplest common cases[2].

Apparently the law of Kapitza[3] (linear increase of resistance with magnetic field) represents a transition zone between the quadratic relation of resistance in small fields[4] and quadratic relation with a different coefficient in strong fields.

Consideration of the departure of the temperature from zero leads to the saturation of resistance even in the case when $n_1 = n_2$. However, this saturation is reached in very strong fields $H \sim H_0\, e^{\Delta\varepsilon/\Theta}$, where $\Delta\epsilon$ represents the distance between the final Fermi surface and the nearest open surface.

In the case when energy interval $\delta\epsilon$ contains open surfaces (or when the Fermi surface itself is open) then, as can be seen from the above, the absolute resistance ρ approaches saturation at $H \gg H_0$. The entire matter actually depends on the temperature. The point of greatest interest here lies in measuring of asymptoticity of the Hall coefficient $R = \rho_{xy} H$ in large fields. For the closed surfaces, R does not depend on the direction and is equal to $R = 10ec(n_1 - n_2)(n_1 \mp n_2)$. In the case of open surfaces, the magnitude and even the sign of R depends on the field.

The authors are indebted to L. D. Landau and E. C. Borovik for their valuable discussions.

* It should be noted that if plane (x, y) does not coincide with any of the crystallographic planes, then the curves are not identical.

* If we assume, as is commonly done, that $W \equiv 1$, then Eqs. (5) and (6) can be easily solved for any values of γ. In this way we obtain

$$\psi_i = \gamma \int_0^\infty e^{-\gamma\tau'} v_i(\tau - \tau')\, d\tau' \cdot f_0'(\varepsilon),$$

and for conductivity σ_{ik} we obtain the expression

$$\sigma_{ik} = \frac{2e^2 \tau_0}{h^3} \int \varphi_{ik}(\varepsilon, p_z)\, f_0'(\varepsilon)\, (dp);$$

$$\varphi_{ik} = \gamma \int_0^\infty e^{-\gamma\xi} \overline{v_i(\tau)\, v_k(\tau + \xi)}\, d\xi.$$

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ANNOUNCEMENT

Beginning with the present (August, 1956), Soviet Physics JETP will appear monthly. Volume 3 (August, 1956-January, 1957) will contain translations of all articles appearing in Volume 30 of the Journal of Experimental and Theoretical Physics of the USSR (January-June, 1956). Volume 4 (February-July, 1957) will be a translation of Volume 31 (July-December, 1956) of the Soviet Journal.

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2	591	2	7	$A = \frac{e^2 H_0^2 \delta_{00}}{mc^2}$	$A = \frac{e^2 H_{00}^2 \delta_{00}^2}{mc^2}$
2	754	1	3 ff.	————	[14] B. B. Kinsey and G. A. Bartholomew, Phys. Rev. **82**, 380 (1951). [15] B. B. Kinsey and G. A. Bartholomew, Phys. Rev. **83**, 234 (1951).
2	771	1	10	Intermediate State	Intermediate State of Tin
	771	1	19	sphere of lead	sphere of tin
3	145	1	1	$R = 10\ ec$	$R = 1/ec$