sphere, r_0 its radius and ω the angular velocity of rotation. Lense and Thirring[8] have shown that the field of Eq. (3) results in an additional precession of the perihelion of the satellite(planet) by the angle (in angular seconds per century)

$$\Psi_R = -\frac{\pi^2 r_0^2 Y}{9c^2 \tau T^2 (1-e^2)^{3/2}}, \tag{4}$$

$$\Delta = \frac{|\Psi_R|}{\Psi} = \frac{8}{15}\left(\frac{r_0}{a}\right)^2 \frac{T}{\tau(1-e^2)^{1/2}},$$

where τ (in days) is the period of rotation of the central sphere producing the field. In Eq. (4) it is assumed, for simplicity, that the plane of the orbit coincides with the equator of the rotating sphere and that the rotation of both the satellite and the sphere take place in the same direction. In a general case[8] a multiplicative factor $(1-3\sin^2(i/2))$, appears in Eq. (4), where i is the angle between the equatorial plane and the plane of the orbit. The angle of rotation of the nodes is smaller by a factor of 2 than the angle of Eq. (4) and has an opposite sign. To obtain the total effect it is only necessary to add algebraically the precession of the perihelion of Eq. (4) with the precession of Eq. (2).

In the case of Mercury ($a = 5.8 \times 10^{12}$, $T = 88$ days, $r_0 = r_\odot = 6{,}96\cdot 10^{10}$ and $\tau = \tau_\odot \simeq 28$ days) $\Delta \approx 2.5 \times 10^{-4}$ and $\Psi_r = 0.01''$. At the present time the accuracy of measurement of the precession of the perihelion has reached the order of $1''$. For a nearby satellite of earth the picture is quite different. For h=400 km, $T \approx 1.54$ hours: $\Delta \approx 3 \times 10^{-2}$ and $\Psi_r \approx -43''$ per century. Thus the relativistic effect connected with the rotation of the earth is of the same magnitude as the total relativistic effect for Mercury. Thus it appears desirable to give attention to the possibility of a measurement of this relativistic "rotation effect".

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Translated by M. J. Stevenson
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Regularized Theory of Field System

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(Submitted to JETP editor October 15, 1955)
J. Exper. Theoret. Phys. USSR 30, 214-216
(January, 1956)

IN the papers[1,2], it is shown that the meaning of the relativistically invariant regularization removing the divergences in the current field theory consists in replacing the usual field equations by equations with higher order derivatives. However, because of the known difficulties related to negative energies[3], the problem of interpretation of the field theory with higher order derivatives is not solved.

For sake of simplicity let us consider a neutral scalar field, subject to the equation:

$$\prod_{i=0}^{n} (\Box - \varkappa_i^2)\, \Phi(x) = -e'\rho(x), \tag{1}$$

where $\varkappa_0 < \varkappa_1 < \ldots \varkappa_n$, and $\rho(x)$ is the density of the field sources. Equation (1) is equivalent to the system of equations

$$(\Box - \varkappa_i^2)\, \Phi_i(x) = -e' C_i \rho(x), \quad i = 0, 1, 2, \ldots, n, \tag{2}$$

where the constants C_i are

$$C_i = \left[\prod_{i \neq j=0}^{n} (\varkappa_j^2 - \varkappa_i^2)\right]^{-1} \tag{3}$$

and satisfy the Pauli-Villars regularization conditions. In this case the solution of Eq. (1) has the form

$$\Phi(x) = \sum_{i=0}^{n} \Phi_i(x). \tag{4}$$

In the absence of sources, the solution of (2) can be written in the form of a Fourier expansion:

$$\Phi_i(x) = \left(\frac{\hbar c^2 |C_i|}{2L^3}\right)^{1/2} \sum_k \omega_i^{-1/2} \{a_i(k) \exp(-i\omega_i t + i\mathbf{kr}) + a_i^{+}(k) \exp(i\omega_i t - i\mathbf{kr}), \tag{5}$$

where $\omega_i = (k^2 + \varkappa_i^2)^{1/2}$. The expression for the entire energy of a free field takes the form:

$$H=\sum_{i=0}^{n} H_i = {}^1/_2 \sum_{i=0}^{n} \sum_{k} \hbar\omega_i (a_i^+ a_i + a_i a_i^+)(-1)^i. \quad (6)$$

The canonical quantization gives the following relations for the operators a_i, a_i^+.

$$[a_i(k), a_j^+(k')] = (-1)^i \delta_{ij}\delta_{kk'}. \quad (7)$$

It follows from Eq. (6) that to the "usual" fields there correspond even i's, and to the "unusual" fields, odd i's. From Eq. (7) it follows that, for usual fields, the particle number operator is $N_i = a_i^+ a_i$, and, for unusual fields, it is $N_i = a_i a_i^+$. Expression (6) takes then the form

$$H=\sum_{i=0}^{n} H_i = \sum_{i=0}^{n} \sum_{k} \hbar\omega_i (N_i + {}^1/_2)(-1)^i. \quad (8)$$

An attempt to interpret this theory in the sense that it describes a system of fields with rest masses $m_i = \hbar\varkappa_i/C$, and coupling constants $e_i = e'C_i$ encounters some difficulties in what concerns negative energies of the free unusual field [as it can be seen from Eq. (8)]. To eliminate these difficulties and to conserve the advantages of the theory (convergence), let us do the following: let us write the Heisenberg S– matrix corresponding to the interaction of the considered field (described by equations with derivatives of order $2(2n + 1)$ of the type (1) with the field of the sources, and let us average this S– matrix over the vacuum of the n unusual fields

$$S = \Big\langle \sum_{r=0}^{\infty} \left(-\frac{i}{\hbar c}\right)^r \frac{1}{r!} \quad (9)$$

$$\times \int_{-\infty}^{\infty} dx_1 \ldots dx_r P[H(x_1), \ldots H(x_r)] \Big\rangle_{\text{vac. unusual Fields}}$$

$$H(x) = -e'\rho(x)\Phi(x) = -e'\sum_{i=0}^{n} \Phi_i(x)\rho(x). \quad (10)$$

The S-matrix (9) does not depend on the changes of the unusual fields: we take it as the fundamental law describing the interaction between the field of the sources and the $(n + 1)$ usual fields, characterized by rest-masses m_i $(i = 0, 2, \ldots 2n)$ and by coupling constants e_i which are expressed in terms of rest-masses of the usual and unusual fields. It is clear that, inversly, to any given system of $(n + 1)$ fields with identical spin, characterized by rest-masses and coupling constants, we can associate a relativistically invariant, finite S-matrix (9); the unknown constants e' and the rest-masses m_i (i odd) of unusual fields are determined in terms of rest masses m_i $(i = 0, 2, \ldots 2n)$ of the usual (real) fields, and of their constants e_i of coupling with the sources, by the equation

$$e'C_i = e_i,\ i = 0, 2, 4 \ldots, 2n. \quad (11)$$

It is easy to see that the S-matrix (9) corresponds to a quantum system of fields described with higher derivatives (1) in the Heisenberg representation, and obeying a particular conservation law: for any real transition of the system the vacuum of the unusual fields has to be conserved. Quanta of the unusual fields may appear only in intermediate states. One has to point out, however, that this theory is not unique. The system of $(n + 1)$ fields can be described by equations with derivatives of order either $2(2n + 1)$ or $2(2n + 2)$ – which correspond to the existence of n or $n + 1$ unusual fields.

A complete uniqueness of the formalism is obtained by the restriction to equations containing derivatives of the lowest order. According to Eqs. (11) and (3) this restriction corresponds to the physical requirement that no other experimental constants occur in the theory except the rest masses and the constants of coupling of these fields with the sources; this is the same requirement as in ordinary electrodynamics. However, in this case, the theory with a single field is not regularized. To regularize the S-matrix it is necessary and sufficient that the sources interact with at least two fields of identical spins.

If one removes the restriction of lowest order derivatives, the S-matrix is always regularized – but in addition to the rest masses and the coupling constants of the usual field, an additional arbitrary constant e' enters the theory*. It can be determined by comparing the theory with experimental data.

The system of equations (11) does not depend on the tensor dimensionality of the field operator $\phi(x)$. The theory can be thus applied to any system of integral spin fields, for example to a system of mesons interacting with nucleons.

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* Equation (11) is then a system of $n + 1$ equations for $(n + 2)$ unknowns.

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Electron Plasma Oscillations in an External Electric Field

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J. Exper. Theoret. Phys. USSR **30**, 216-218
(January, 1956)

THIS letter is concerned with the determination of the frequency of oscillations of electron plasma placed in a constant and uniform electric field E_0*.

We shall denote by $F(\mathbf{r}, \mathbf{v}, t)$ a distribution function of the plasma electrons. This function satisfies the kinetic equation

$$\frac{\partial F}{\partial t} + \mathbf{v}\,\mathrm{grad}\,F + \frac{e}{m}(\mathbf{E}_0 + \mathbf{E})\frac{\partial F}{\partial \mathbf{v}} + J[F] = 0,$$

where $J(F)$ is the collision integral and $\mathbf{E}$ is the electric field specified by the plasma oscillations and satisfying the relation

$$\mathrm{div}\,\mathbf{E} = 4\pi e\int F d\mathbf{v} - 4\pi e n_0$$

where n_0 is the equilibrium density of the ions. The equilibrium distribution function F_0 of the electrons in the absence of oscillations has the form[2]

$$F_0 = f_0(v^2) + \mathbf{E}_0\mathbf{v} f_1(v^2),$$

$$f_0(v^2) = C\exp\left\{-w + \int_0^w \left[1 + \frac{w}{\xi}\right]^{-1} dw\right\},$$

$$f_1(v^2) = -\frac{el}{m}\frac{\partial f_0}{\partial v},$$

where $w = mv^2/2T$, $\xi = (M/6m)(eE_0 l/T)^2$, m and M are the masses of electrons and ions respectively, l is the mean free path of electrons and T the temperature. If we assume that the distribution function F differs only slightly from F_0, we obtain the following equations for $f = F - F_0$ and the field $\mathbf{E}$:

$$\frac{\partial f}{\partial t} + \mathbf{v}\,\mathrm{grad}\,f + \frac{e\mathbf{E}_0}{m}\frac{\partial f}{\partial \mathbf{v}} + \frac{e\mathbf{E}}{m}\frac{\partial F_0}{\partial \mathbf{v}} + \frac{1}{\tau}f = 0, \quad (1)$$

$$\mathrm{div}\,\mathbf{E} = 4\pi e\int f d\mathbf{v},$$

where the term f/τ phenomenologically takes into account the presence of collisions (τ is the average time between collisions).

We seek a solution for the set of equations (1) according to Landau[3] in the form

$$f(\mathbf{r}, \mathbf{v}, t) = \int f_{\mathbf{k}}(\mathbf{v}, t)\, e^{i\mathbf{k}\mathbf{r}} d\mathbf{k}, \; f_{\mathbf{k}}(\mathbf{v}, t) \quad (2)$$

$$= \frac{1}{2\pi i}\int_{-i\infty+\sigma}^{i\infty+\sigma} f_p(\mathbf{v})\, e^{pt} dp,$$

$$\mathbf{E}(\mathbf{r}, t) = -\mathrm{grad}\,\varphi = -i\int \mathbf{k}\varphi_{\mathbf{k}}(t)\, e^{i\mathbf{k}\mathbf{r}} d\mathbf{k},$$

$$\varphi_{\mathbf{k}}(t) = \frac{1}{2\pi i}\int_{-i\infty+\sigma}^{i\infty+\sigma} \varphi_p e^{pt} dp, \; \sigma > 0,$$

and we obtain

$$\varphi_p = \frac{\frac{4\pi m}{E_0}\int \exp\left\{-\frac{m}{eE_0}\int_0^{v_x}\left(p + i\mathbf{k}\mathbf{v} + \frac{1}{\tau}\right)dv_x\right\}\left\{\int_{-\infty}^{v_x} g(\mathbf{v})\exp\left\{\frac{m}{eE_0}\int_0^{v_x}\left(p + i\mathbf{k}\mathbf{v} + \frac{1}{\tau}\right)dv_x\right\}dv_x\right\}d\mathbf{v}}{k^2 - \frac{4\pi e}{E_0}\int \exp\left\{\frac{-m}{eE_0}\int_0^{v_x}\left(p + i\mathbf{k}\mathbf{v} + \frac{1}{\tau}\right)dv_x\right\}\left\{\int_{-\infty}^{v_x} i\mathbf{k}\frac{\partial F_0}{\partial \mathbf{v}}\exp\left[\frac{m}{eE_0}\int_0^{v_x}\left(p + i\mathbf{k}\mathbf{v} + \frac{1}{\tau}\right)dv_x\right]dv_x\right\}d\mathbf{v}}, \quad (3)$$