with quantum numbers 1, 0, 0; l', m' and with momentum k of the emerging electron (atomic units are used here). In the radial integral which enters into the matrix element, the important region is evidently $r_1, r_2 \lesssim 1/Z$. In this region the wave function of the initial state coincides with the Coulomb function with accuracy to a normalization factor A_{nl}. To find A_{nl}, we make use of the fact that the initial and Coulomb functions are quasi-classical for $r \gg 1/Z$. If we write out the quantization rule and the normalization condition for them, we get, after some simple transformations,

$$A_{nl} = B_{nl} \sqrt{(n^3/Z^2)(\partial E_{nl}/\partial n)},$$

where B_{nl} is the normalization coefficient of the Coulomb function, E_{nl} is the energy of the corresponding level. In this case, it is assumed in the calculations that for $n = 1, 2$, $A_{nl} = B_{nl}$.

For large n_2 and for $n_1 = 2$,

$$1/\tau = A_{l_1 l_2} \partial E_{n_2 l_2} / \partial n_2 ,$$

where $A_{l_1 l_2}$ is almost independent of n_2. Therefore, replacing the sum over n_2, beginning with $n_2 = 3$, by an integral, we can write the total number of Auger-transitions per unit time in the form:

$$(1/\tau)_\Sigma = (1/\tau)_{L-L} + \sum_{l_1=0}^{1} \sum_{l_2=0}^{2} A_{l_1 l_2} \int_{n_2=3}^{\infty} (\partial E_{n_2 l_2}/\partial n_2)\, dn_2 + \sum_{l_1=0}^{1} A_{l_1 3} \int_{n_2=4}^{\infty} (\partial E_{n_2 3}/\partial n_2)\, dn_2 + \ldots$$
$$= (1/\tau)_{L-L} - \sum_{l_1=0}^{1} \sum_{l_2=0}^{2} A_{l_1 l_2} E_{3 l_2} - \sum_{l_1=0}^{1} A_{l_1 3} E_{43} - \ldots ,$$

where small terms of the type

$$A_{l_1 l_2} (\partial E_{n_2 l_2}/\partial n_2)(\partial E_{n_1 l_1}/\partial n_1) \qquad (n_1 > 2).$$

are discarded.

In the first approximation,

$$(1/\tau)_\Sigma = (1/\tau)_{L-L} - \sum_{l_1=0}^{1} \sum_{l_2=0}^{2} A_{l_1 l_2} E_{3 l_2} .$$

We can put $E_{3 l_2}$ in the form:[3]

$E_{3l} = -(Z - s_l)^2/18$, where s_l is the screening constant

For $(1/\tau)_{L-L}$, making use of the well-known results of reference 2, we obtain for $Z = 47$, after some computation,

$$(1/\tau)_\Sigma = 45{,}9 \text{ atomic units} \qquad (1)$$

A quantity defined from experiment is the coefficient

$$\alpha_K = Z^4 (1/\tau)_\Sigma / (1/\tau)_{\text{rad}},$$

where $(1/\tau)_{\text{rad}}$ is the number of radiative transitions per unit time. For $Z = 47$: $(1/\tau)_{\text{rad}} = 0.197$ atomic units.[4] Making use of Eq. (1), we obtain $\alpha_K = 1.14 \times 10^6$. The experimental value is[4] $\alpha_K = 1.14 \times 10^6$. Calculation with the help of the Coulomb functions gives $\alpha_K = 1{,}65 \cdot 10^6$.

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Scattering of Fast Neutrons by Semitransparent Nonspherical Nuclei

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THE scattering of fast neutrons by an opaque nonspherical nucleus with spin zero has been studied by Drozdov[1,2]. The scattering of fast neutrons by an even-even semitransparent nonspherical nucleus is considered in the present work.

According to Bohr and Mottelson[3,4], the even-even nuclei in their rotational states have the form of an ellipsoid of revolution and the wave function of such a state is a spherical harmonic $Y_{lm}(\omega)^*$, where l, m are the spin of the nucleus and the projection of that spin, ω represents the angles ϑ, φ which characterize the direction of the axes of symmetry of the ellipsoid. The rotational levels are determined by the formula $E_l = (\hbar^2/2I)\, l(l+1)$, $l = 0, 2, 4, \ldots$, where I is the effective moment of inertia of the nucleus. If we introduce the eccentricity of the nucleus $\epsilon = 1 - (a/b)^2$ (b is the radius of the largest circular cross section of the ellipsoid), then $I = 1/4 I_0 \epsilon^2$, where I_0 is the ordinary moment of inertia of a spherical nucleus of equal volume. For most nuclei, $|\epsilon| \approx 0.2$-0.3. For fast neutrons, we can use the adiabatic approximation and consider the scattering of the neutron on a fixed nucleus. We can then write the wave function of the system in the form $\Psi(\mathbf{r}, \omega) \approx u_{\mathbf{k}}(\mathbf{r}, \omega) \times Y_{l_0 m_0}(\omega)$, where $u_{\mathbf{k}}$ is the wave function of the neutron scattered by a fixed nucleus whose orientation is given by the angles ω. In what follows we shall consider that the nucleus is in the ground state before scattering, i.e., $l_0 = 0$.

To calculate the amplitude of the scattering, we make use of the formula of Francis and Watson[5]:

$$f(\Omega, \omega) = -\frac{\mu U}{2\pi h^2} \int \exp\{i(\mathbf{k}-\mathbf{k}')\mathbf{x} + i(n-1)kD(\mathbf{x})\}\, d\mathbf{x}, \tag{2}$$

Here k, k′ are the propagation vectors of the incident and scattered neutron, respectively, Ω represents the angles which define the direction of k′; $D(\mathbf{x})$ is the distance traversed by the incident neutron (in the nucleus) which is found at the point x; integration is carried out over the volume of the ellipsoid, whose orientation is defined by the angles ω; the complex potential U is connected with the complex index of refraction n by the relation $U = -(h^2k^2/2\mu)(n^2-1)$. The region of applicability of Eq. (2) is evidently limited by the condition $|U|/E \ll 1$, where E is the energy of the neutron. The cross section for scattering in the direction Ω with excitation of the rotational level l, m is determined by the formula

$$\sigma_{lm}(\Omega) = \frac{1}{4\pi} \left| \int d\omega Y_{lm}(\omega) f(\omega, \Omega) \right|^2. \tag{3}$$

The total cross section of all processes and the cross section of capture are given by the relations

$$\sigma_t = \frac{1}{k} \operatorname{Im} \int d\omega f(\Omega, \omega) \Big|_{\theta=0},$$

$$\sigma_c = \frac{1}{4\pi\lambda} \int d\omega\, d\mathbf{x} e^{-D(\mathbf{x})/\lambda},$$

where $\lambda = 1/(2k\operatorname{Im} n)$, θ is the scattering angle. If we take the direction of the vector k as the axis of quantization, then the symmetry properties of Eq. (2) lead to the relations $\sigma_{l,m}(\Omega) = 0$ if l is odd, $\sigma_{l,m}(\Omega) = \sigma_{l,-m}(\Omega)$; $\sigma_{l,m}(\Omega) = 0$ for $\theta = 0$ if $m \neq 0$.

With the help of a coordinate transformation which transforms the ellipsoid into a sphere, with a change to a cylindrical system of coordinates, the integral (2) becomes

$$f(\Omega, \omega) = ab^2k^2 \frac{n^2-1}{q} \times \int_0^1 J_0(\rho\varkappa) \sin(q\sqrt{1-\rho^2}) \exp\{ip\sqrt{1-\rho^2}\} \rho\, d\rho, \tag{4}$$

where

$$\varkappa = 2kb \sin\frac{\theta}{2} \left[1 - \varepsilon\cos^2\gamma - \left(a\sin\frac{\theta}{2} \Big/ b\xi(\vartheta)\right)^2\right]^{1/2},$$

$$p = ka\frac{n-1}{\xi(\vartheta)},$$

$$q = p\left(1 + \frac{2\sin^2(\theta/2)}{n-1}\right), \quad \xi(\vartheta) = \sqrt{1-\varepsilon\sin^2\vartheta},$$

γ is the angle between the vector $\mathbf{k}' - \mathbf{k}$ and the axis of symmetry of the ellipsoid. With the use of Eqs. (3) and (4), it can be shown that the excitation cross section of the lth level is proportional to ϵ^l, i.e., excitation is practicable only with $l = 2$.

The integral (4) is computed for small angles $kb\theta < 1$. For angles which satisfy the condition $2kb\sin\theta/2 \gg 1$ for (4), we get the asymptotic expression

$$f(\Omega, \omega) \approx ab^2k^2 \frac{1-n^2}{4q} \left[\frac{(q+p)\exp(i\sqrt{\varkappa^2+(q-p)^2})}{\varkappa^2+(q+p)^2} + \frac{(q-p)\exp(i\sqrt{\varkappa^2+(q+p)^2})}{\varkappa^2+(q-p)^2}\right]. \tag{5}$$

Calculation of the cross section (3) is carried out by means of a decomposition of the integral (4) (or its asymptotic expression) in a series in the small parameter ϵ.

If we start out in the calculation of the amplitude of the scattering, not from Eq. (2), but from the optical diffraction formula:[6]

$$f(\Omega, \omega) = \frac{ik}{2\pi} \int_S [1 - e^{2i(n-1)ks}]\, e^{i(\mathbf{k}-\mathbf{k}')\mathbf{x}} df_n, \tag{6}$$

(here $2s$ is the penetration of the neutron in the nucleus; integration is carried out over the area of the projection of the ellipsoid on a plane perpendicular to the vector k) then the following expression is obtained:

$$f(\Omega, \omega) = ikb^2\xi(\vartheta) \int_0^1 [1 - \exp(2ip\sqrt{1-\rho^2})] J_0(\varkappa_0\rho) \rho\, d\rho, \tag{7}$$

where

$$\varkappa_0 = kb \sin\theta \left[\cos^2\varphi + \xi^2(\vartheta)\sin^2\varphi\right]^{1/2},$$

φ is the azimuthal angle of the axis of symmetry, measured from the plane determined by the vectors k, k′. It is easy to see that Eq. (4) transforms into Eq. (7) for small angles of scattering θ, with accuracy to $(n+1)/2 \approx 1$. Evidently the optical approach of Eq. (6) is valid only for very small scattering angles.

In the Born approximation, which is valid for $kR|U|/E \ll 1$ (R is the radius of the nucleus), we can set $n-1=0$ everywhere in Eq. (4) except in the multiplicative factor n^2-1. We then obtain

$$f(\Omega,\omega) = \sqrt{2\pi}\hbar^{-2}\mu ab^2 U y^{-3/2} J_{3/2}(y), \tag{8}$$

where $y = 2kb\sin(\theta/2)\sqrt{1-\varepsilon\cos^2\gamma}$. Here the dependence of the amplitude on the angles ω is determined only by the angle γ. Therefore, if we choose as the angle of quantization the direction of the vector k′ − k only such states will be excited for which the projection of the momentum is equal to zero. The cross section of excitation of the lth level can be calculated, not by Eq. (3), but by the simpler formula

$$\sigma_l(\Omega) = (2l+1)\left|\int_0^\pi P_l(\cos\gamma)\, f(\gamma,\Omega)\, d\cos\gamma\right|^2; \tag{9}$$

the cross sections of excitation of a level with momentum l and projection m in the direction of the vector k, are calculated with the help of the addition theorem for spherical functions. There also follow from Eq. (8) certain conclusions on the angular correlation between the scattered neutron and the photon produced in the transition $l \to 0$. Thus, for example, for $l=2$, the photon has an angular momentum equal to two and a projection of this momentum in the direction of the vector k′ − k equal to zero; therefore, we get for the angular distribution of photons[7]:

$$I(\alpha) = (15/8\pi)(\cos^2\alpha - \cos^4\alpha), \tag{10}$$

where α is the angle between the direction of the photon and the vector k′ − k.

Starting out with Eq. (8) and the expression for the total cross section of elastic scattering and of the excitation of all rotational levels $\sigma_s(\Omega) = (4\pi)^{-1}\int d\omega\, |f(\omega,\Omega)|^2$, one can show that the angular distributions just obtained are the mean of the angular distributions, taken with a weighting factor for elastic scattering on spherical nuclei having radii from $R_1 = a$ to $R_2 = b$.

For small eccentricity ($|\epsilon| \ll 1$) we get from Eqs. (8) and (9):

$$\sigma_l(\Omega) = \varepsilon^l A_l \left|\left(2kb\sin\frac{\theta}{2}\right)^{(l-3)/2} J_{(l+3)/2}\left(2kb\sin\frac{\theta}{2}\right)\right|^2, \tag{11}$$

$$A_l = 2^{l+1}(2l+1)\pi\left|\frac{\mu ab^2 U\,(l!)^2}{\hbar^2 (l/2)!\,(2l+1)!}\right|^2.$$

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Possibility of Using Artificial Earth Satellites for the Experimental Verification of the Theory of General Relativity

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AT the present time it is possible to assert that experimental tests have brought about a convincing verification of the general theory of relativity.[1] However, an even further verification of the theory does not appear superfluous. Therefore it is appropriate at this time to point out the possibility of an experimental verification of the general theory of relativity by utilizing artificial earth satellites.

The reception of the radio signals on earth from the satellite can be used to determine the gravitational shift of frequency in the earth's field[2]. The relativistic gravitational change of frequency ν is