Propagation of a Short Pulse in a Semiconductor Bounded by Two Hole-Electron Transistors

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Herein is presented a calculation of the propagation of a short pulse in a semiconductor bounded by two hole-electron transistors, one of which ($x = 0$) is positive and the other ($x = w$) has a negative bias such that a semiconducting triode system exists. The form of the impulse is approximated by a δ-function. With the aid of Laplace transforms we obtained the current in p-representation at any semiconductor section. Transforming to the original representation at $x = w$ permits one to obtain the expression for the collector current $i(w, t)$ and clarifies the influence of diffusion and recombination of the nonequilibrium current carriers in a semiconductor on the form of the impulse at the collector.

1. GENERAL SOLUTION OF THE PROBLEM

LET us consider a semiconductor doped with a conducting impurity, bounded at $x = 0$ (emitter) and $x = w$ (collector) by hole-electron transistors. No electric field exists within a semiconductor and the changes in the concentration of the current carriers (deficit type of conduction) can be specified only through two mechanisms: 1. diffusion and 2. disturbance of the thermodynamic equilibrium between thermal creation and electron-hole pair recombinations. The conservation equation and the expression for the current have the following form:

$$\frac{\partial n_h}{\partial t} + \frac{1}{q}\operatorname{div}\mathbf{j} = g - \frac{n_h}{\tau}, \quad \frac{1}{q}j = -D\frac{\partial n_h}{\partial x}, \quad (1)$$

from which it follows that the concentration of deficit carriers (holes) is specified by the equation

$$\frac{\partial n_h}{\partial t} + \frac{n_h - n_0}{\tau} - D\frac{\partial^2 n_h}{\partial x^2} = 0. \quad (2)$$

Here n_0 is the equilibrium concentration of holes; for equilibrium the source function g, representing thermal creation of electron-hole pairs in a unit volume, equals the numbers of recombinations, i.e., n_0/τ.

Separating the stationary solution $n_{st}(x)$, which satisfies the equation

$$\frac{n_{st} - n_0}{\tau} - D\frac{d^2 n_{st}}{dx^2} = 0 \quad (3)$$

with given stationary boundary conditions (constant potentials for the hole-electron transistors), one obtains for

$$n = n_h - n_{st} \quad (4)$$

the equation

$$\frac{\partial n}{\partial t} + \frac{n}{\tau} - D\frac{\partial^2 n}{\partial x^2} = 0, \quad (5)$$

with the boundary conditions*

$$n(w, t) \equiv 0, \qquad \frac{\partial n}{\partial x}(0, t) = -\frac{N}{D}\delta(t) \quad (6)$$

and the initial conditions

$$n(x, 0) \equiv 0 \text{ with } 0 < x \leqslant w. \quad (7)$$

With the help of the assumption[1]

$$n(x, t) = y(x, t)\, e^{-t/\tau} \quad (8)$$

Eq. (5) is transformed to a pure diffusion equation

$$\frac{\partial y}{\partial t} - D\frac{\partial^2 y}{\partial x^2} = 0 \quad (9)$$

with the previous boundary and initial conditions.

Performing the Laplace transformation[2] on the function $y(x, t)$

* N equals the concentration of current carriers in the base produced by the introduced delta impulse

$$-qD\int_0^\infty \frac{\partial n}{\partial x}(0, t)\, dt = qN\int_0^\infty \delta(t)\, dt = qN.$$

[1] E. I. Andirovich, Dokl. Akad. Nauk SSSR **86**, 1085 (1952).

[2] M. A. Lavrent'ev and B. V. Shabat, *Methods of Function Theory of Complex Variable*, GITTL, 1951.

$$y(x, t) \doteq F(x, p) = \int_0^\infty y(x, t)\, e^{-pt}\, dt, \qquad (10)$$

we find that $F(x, p)$ satisfies the equation

$$\frac{d^2F}{dx^2} - \frac{p}{D} F = 0 \qquad (11)$$

with the conditions

$$F(w, p) \equiv 0, \quad \frac{dF}{dx}(0, p) = -\frac{N}{D}. \qquad (12)$$

The solution to Eqs. (11) and (12) is

$$F(x, p) = \frac{N \operatorname{sh} k(w-x)}{Dk \operatorname{ch} kw}, \qquad (13)$$

where

$$k = \sqrt{p/D}. \qquad (14)$$

The dispersion in the concentration of non-equilibrium carriers, specified by a δ-current pulse on the emitter, can be represented by the general formula

$$n_h(x, t) \qquad (15)$$

$$= n_{st}(x) + \frac{e^{-t/\tau}}{2\pi i} \frac{N}{D} \int_{a-j\infty}^{a+j\infty} \frac{\operatorname{sh} k(w-x)}{k \operatorname{ch} kw} e^{pt}\, dp.$$

Here

$$n_{st}(x) \qquad (16)$$

$$= n_0 \left(1 + \frac{(i_1/n_0) \operatorname{sh} \varkappa(w-x) - qD\varkappa \operatorname{ch} \varkappa x}{qD\varkappa \operatorname{ch} \varkappa w}\right)$$

is the stationary distribution satisfying the given bias on the hole-electron transistors.

The corresponding stationary component of the hole current is

$$i_{st}(x) = \frac{i_1 \operatorname{ch} \varkappa(w-x) + qDn_0\varkappa \operatorname{sh} \varkappa x}{\operatorname{ch} \varkappa w}. \qquad (17)$$

At the emitter, $i_{st}(0) = i_1$, and at the collector,

$$i_{st}(w) = \frac{i_1 + qDn_0\varkappa \operatorname{sh} \varkappa w}{\operatorname{ch} \varkappa w}, \qquad (18)$$

where $\kappa = (D\tau)^{-1/2}$.

2. COLLECTOR CURRENT

The variable component of the hole current (deficit type of conduction) specified by a δ-impulse on the emitter equals

$$i(x, t) = -qD \frac{\partial n(x, t)}{\partial x} = -qDe^{-t/\tau} \frac{\partial y(x, t)}{\partial x}. \qquad (19)$$

Transforming to the p-representation we obtain from Eqs. (13) and (19) that

$$-qD \frac{\partial y(x, t)}{\partial x} \doteq \Phi(x, p) \qquad (20)$$

$$\equiv -qD \frac{dF}{dx} = \frac{qN \operatorname{ch} k(w-x)}{\operatorname{ch} kw}.$$

Returning to the original representation for Eq. (20) and inserting it into (19), one obtains the expression for the current* at any section x. At the collector, i.e., at $x = w$,

$$e^{t/\tau} i(w, t) \doteq \Phi(w, p) = \frac{qN}{\operatorname{ch} kw}. \qquad (21)$$

Expressing $\Phi(w, p)$ by an exponential series[3], we have

$$\Phi(w, p) = 2 \sum_{n=1}^{\infty} (-1)^{n+1} e^{-(2n-1)kw} \qquad (22)$$

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* $i(x, t)$ is the excess over the stationary current $i_{st}(x, t)$ and corresponds to the given boundary conditions.

[3] I. M. Ryzhik and T. C. Gradshtein, *Tables of Integrals, Sums, Series and Products*, GITTL, 1951.

and transforming each term of (22) to the original representation[4] we obtain the series

$$I(\xi)=\frac{i(w,t)}{i_0} \tag{23}$$

$$=\frac{e^{-\lambda^2\xi}}{\xi^{3/2}}\sum_{n=1}^{\infty}(-1)^{n+1}(2n-1)e^{-(2n-1)^2/\xi},$$

which represents the collector current in general form. Here

$$\xi=t/\theta,\quad \lambda^2=\theta/\tau, \tag{24}$$

$$i_0=2qN/\sqrt{\pi}\theta,\quad \theta=w^2/4D.$$

We approximate the collector current, in series (23), by the S-sum,

$$I(\xi)\cong I_s(\xi) \tag{25}$$

$$=\xi^{-3/2}e^{-\lambda^2\xi}\sum_{n=1}^{s}(-1)^{n+1}(2n-1)e^{-(2n-1)^2/\xi},$$

and evaluate the region of validity and degree of accuracy of this approximation. It is evident that

$$I(\xi)-I_s(\xi)=P_s(\xi) \tag{26}$$

$$=\xi^{-3/2}e^{-\lambda^2\xi}\sum_{n=s+1}^{\infty}(-1)^{n+1}(2n-1)e^{-(2n-1)^2/\xi}.$$

With

$$\xi<8(s+1)/\ln\frac{2s+3}{2s+1} \tag{27}$$

the remaining term, $P_s(\xi)$, is an alternating series whose terms decrease monotonically with increasing n. Consequently,

$$|P_s|(\xi)|<\xi^{-3/2}e^{-\lambda^2\xi}(2s+1)e^{-(2s+1)^2/\xi} \tag{28}$$

and, for $I(\xi)$ one can write upper and lower limits,

$$I_s(\xi)<I(\xi)<I_{s+1}(\xi),\quad \text{if } s \text{ is even,} \tag{29}$$

$$I_s(\xi)>I(\xi)>I_{s+1}(\xi),\quad \text{if } s \text{ is odd.}$$

To determine the accuracy of the approximation we consider

$$\frac{|I_{s+1}(\xi)-I_s(\xi)|}{I_s(\xi)} \tag{30}$$

$$=\frac{(2s+1)e^{-(2s+1)^2/\xi}}{\sum_{n=1}^{s}(-1)^{n+1}(2n-1)e^{-(2n-1)^2/\xi}}\leqslant\varepsilon,$$

from which we can determine the time interval ξ' during which the collector current can be represented by $I_s(\xi)$ with an accuracy ϵ. Here we must require that

$$\xi'<8(s+1)/\ln\frac{2s+3}{2s+1}; \tag{31}$$

otherwise the duration of the determined interval is not ξ' but the right-hand side of the previous inequality.

We shall show that specification of the collector current for any value of the lifetime τ by a three term formula, $I_3(\xi)$, is sufficient; in the given cases it was possible to restrict onself to approximations with two and, in some cases, with one term of the series (23).

3. CASE OF LARGE LIFETIMES FOR THE NON-EQUILIBRIUM CURRENT CARRIERS

Let us consider the case $\lambda^2\to 0$, or correspondingly, $4D\tau/w^2\to\infty$. In Fig. 1 we have shown the curves I_1^∞, I_2^∞ and I_3^∞ which are one, two, and three term approximations [see Eq. (23)] to the collector current I^∞.

The superscript ∞ indicates that these solutions represent currents obtained in the absence of recombination ($\tau=\infty$). The maxima of all three

[4] V. A. Ditkin and P. I. Kuznetsov, *Handbook of Operational Calculus*, GITTL, 1951.

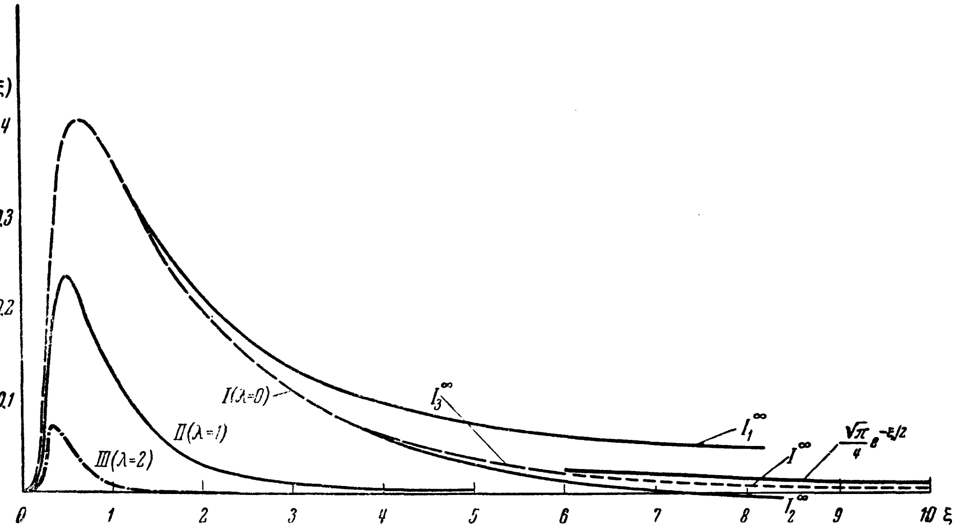

FIG. 1

approximations are identical and occur at the value $\xi = 0.67$. The value of I_1^∞ falls to half maximum at $\xi_1 = 2.1$; I_2^∞ and I_3^∞ at the value $\xi_{2,3} = 2$. From Fig. 1 and Eq. (30) it is seen that the first term of the series (23) approximates the collector current with an accuracy of $\epsilon = 0.01$ down to $\xi = 1.4$; the first two terms down to $\xi = 3.7$; the first three terms down to $\xi = 6.1$. For I_1^∞, condition (27) is satisfied up to $\xi = 3.1$, i.e., no additional restriction is imposed. At $\xi = 6$ the value of the current is 26 times smaller than the maximum. Consequently, the three term formula $I_3^\infty(\xi)$ gives a sufficiently accurate expression for the collector current over the total range. The two term formula $I_2^\infty(\xi)$ is a satisfactory representation of $i(w, t)$ for values of the time less than the time of maximum. In those cases, when the problem is confined to the half-maximum of the curve $i(w, t)$, i.e., to times for which the current, after passing through its maximum value, has decreased by no more than a factor of two, then the value of the collector current agrees with $I_1^\infty(\xi)$ to within 5%.

For greater times, the actual curve should be approximated by a larger number of terms of the series (23) since, with $\xi \gg 1$, i.e., for $t \gg w^2/4D$, the series (23) converges slowly. According to Tauber's theorem[5] the form of the asymptotic solution as $\xi \to \infty$ can be obtained from the series expansion of $\Phi(w, p)$ in powers of kw,

[5] I. M. Ryzhik and I. C. Gradshtein, loc. cit.

$$\Phi(w, p)|_{p\to 0} = qN/\left(1 + \frac{k^2w^2}{2}\right). \qquad (32)$$

Transforming this equation to the original representation, we find

$$I^\infty(\xi)|_{\xi\to\infty} = \frac{\sqrt{\pi}}{4} e^{-\xi/2}. \qquad (33)$$

Equation (33) is shown in Fig. 1 beginning at $\xi = 6$. For all larger ξ, the asymptotic expression (33) approximates $I^\infty(\xi)$ from above, since in the transition to the asymptotic representation (32), positive terms were truncated in the denominator. Consequently, for later times, when $I_3^\infty(\xi)$ does not give a good approximation to the actual current, $I^\infty(\xi)$ is bounded between the axis and the exponential (33). In Fig. 1, $I^\infty(\xi)$ is shown in this region as a dashed curve.

We shall write the final formula in different variables. With relative accuracy to $\epsilon = 0.01$, we have

$$i(w, t) = \frac{qNw}{\sqrt{\pi D t^3}}\left(e^{-w^2/4Dt} - 3e^{-9w^2/4Dt} + 5e^{-25w^2/4Dt}\right) \qquad (34)$$

with $t \leq 1.52\, w^2/D$. Up to $t = 0.93\, w^2/D$ one can, with the same degree of accuracy, use the first two terms, and, to $t = 0.35\,w^2/D$, the first term. Indeed, this last gross approximation permits one to obtain the major part of the current impulse on the collector, up to the point where the curve has fallen to approximately half its maximum value.

4. CASE OF MODERATE AND SMALL LIFETIMES

In those cases in which it is not permissible to disregard the recombination of carriers, one obtains for the collector current,

$$I(\xi) = e^{-\lambda^2\xi} I^{\infty}(\xi), \tag{35}$$

where $I^{\infty}(\xi)$ is the value of the collector current with $\tau = \infty$ and is given by the series expression on the right in (23). Inasmuch as the evaluation of the values of ξ' is independent of λ, the results are correct for any value of τ. But, because of the exponential factor in (35), i.e., $e^{-\lambda^2\xi}$, $I(\xi)$ falls faster for the same value of ξ. The three term approximation $I_3^{\infty}(\xi)$, which was accurate to $\epsilon = 0.01$, was good for ξ's up to the point where the current is 1/26 its peak value; here, however, for $\lambda = 1$ ($\tau = w^2/4D$), the current in the same interval, $\xi \leq 6.1$, falls to 1/1000 its peak value. The same situation holds for the other approximations; the smaller the recombination time, the more complete the expression for the collector current. Thus it follows that the approximate expressions for the collector current derived above, i.e., $I_3(\xi)$, $I_2(\xi)$, $I_1(\xi)$, hold to the same accuracy for any value of τ.

To find the magnitude and location of the peak current for the general case $\lambda \neq 0$ one can use the condition resulting from (35) that

$$\left.\frac{d\ln I^{\infty}(\xi)}{d\xi}\right|_{\xi=\xi_{max}} = \lambda^2. \tag{36}$$

From (36) it is seen that the recombination process hastens the occurrence of the maximum current at the collector, since we have, with $\lambda \neq 0$

$$(dI^{\infty}/d\xi)|_{\xi_{max}} > 0.$$

In the vicinity of the maximum

$$I^{\infty}(\xi) \cong I_1^{\infty}(\xi) = \xi^{-3/2} e^{-1/\xi}. \tag{37}$$

Consequently,

$$\xi_{max} = \frac{3}{4\lambda^2}\left[\sqrt{1 + \frac{16}{9}\lambda^2} - 1\right]. \tag{38}$$

Likewise,

$$I_{max} = \xi_{max}^{-3/2} \exp\{-\lambda^2\xi_{max}\} \exp\{-1/\xi_{max}\}. \tag{39}$$

The asymptotic expression for $I(\xi)$ for large times is, for $\tau \neq 0$,

$$I(\xi)|_{\xi\to\infty} \cong \frac{\sqrt{\pi}}{4} e^{-(1/4+\lambda^2)\xi}. \tag{40}$$

Curve *II* in Fig. 1 represents the time history of the collector current for $\lambda = 1$, i.e., for $\tau = w^2/4D$. As was mentioned above, in this case the current does not fall from its peak value in the interval $t \leq \frac{1.52\,w^2}{D}$ by a factor of 26 as for $\lambda = 0$ (curve *I*), but by a factor of 1000. At $t = 0.93\,w^2/D$ the current has decreased to 1/14 its peak value. Consequently, for moderate values of the lifetimes ($\lambda \simeq 1$), the two term formula I_2 fully serves as a sufficient approximation to the collector current.

For small τ ($\lambda > 1$), the current, over its entire range, for all practical purposes is given by

$$I(\xi) \approx I_1(\xi) = \xi^{-3/2} e^{-\lambda^2\xi} e^{-1/\xi}. \tag{41}$$

Thus, for example, with $\lambda = 2$ the collector current at $\xi = 1.4$, where the approximation given by (41) is accurate to 1%, is 1/25 its peak value (curve *III* in Fig. 1).

Curves *I*, *II* and *III* in Fig. 1 demonstrate the dependence of the form of the impulse at the collector on the lifetime of the nonequilibrium current carriers in a semiconductor.

5. THE INFLUENCE OF BOUNDARY CONDITIONS ON THE ELECTRON PROCESSES IN THE VOLUME OF A SEMICONDUCTOR

To clarify the influence of boundaries we shall compare the results obtained above with the results for the propagation of a δ-impulse in a semi-infinite semiconductor ($0 \leq x \leq w$). The mathematical formulation of the problem differs from the preceding only in the fact that the condition $n = 0$ occurs not at $x = w$ but at $x = \infty$. For this case, the concentration and current of nonequilibrium carriers have the following representation

$$e^{t/\tau} n^*(x, t) \doteqdot F^*(x, p) = \frac{N}{Dk} e^{-kx}, \quad (42)$$

$$e^{t/\tau} i^*(x, t) \doteqdot \Phi^*(x, p) = qNe^{-kx}.$$

Comparing these with the corresponding representations for a semiconducting layer of thickness w [Eqs. (13) and (20)] we see that the two agree when $k \gg (w - x)^{-1}$, i.e., when

$$p \gg D(w - x)^{-2}. \quad (43)$$

Thus as $p \to \infty$, $t \to 0$, which indicates that for all $x < w$ the diffusion process proceeds from the start the same in a semi-infinite semiconductor as in a bounded semiconductor. Departures occur when $t \simeq t_0$, corresponding to $p = D(w-x)^{-2}$, i.e., that much earlier the smaller the separation $(w - x)$. The inequality (43) indicates that at the collector ($x = w$), the process differs from that in a semi-infinte semiconductor right from the beginning (i.e., at $t = 0$, corresponding to the establishment of a δ-impulse on the emitter). In particular, [consider Eq. (21)],

$$\Phi(w, p)|_{p \to \infty} = 2qNe^{-kw} = 2\Phi^*(w, p), \quad (44)$$

i.e., at the beginning of the process, the collector current is twice the current which would be obtained at the distance w from the emitter in a semi-infinite medium. The reason for this is the absence of a partial counter current of deficit carriers which is specified by the assigned potential on the collector. Associated with the maintenance of a small constant concentration of deficit carriers, $n(w, t) \simeq 0$, is a faster diffusion rate which involves a more rapid decrease of current, so that as $t \to \infty$, the collector current becomes much smaller than the current at $x = w$ in a semi-infinite conductor. In Fig. 2, the currents $i(w, t)$ and $i^*(w, t)$ for $\tau = \infty$ are represented, corresponding respectively to the bounded and semi-infinite case. The given potential which specifies $n(w, t) \simeq 0$ not only doubles the maximum and sharpens it but, in addition, introduces a sharper decrease for later times. Instead of the current falling off like $t^{-3/2}$ (for $t \gg w^2/4D$)

$$i^*(w, t) \equiv \frac{wN}{2\sqrt{\pi D t^3}} e^{-w^2/4Dt} \approx \frac{\text{const}}{t^{3/2}}, \quad (45)$$

as in a semi-infinite semiconductor, the current behaves exponentially

$$i = 2qNDw^{-2}e^{-2Dt/w^2}. \quad (46)$$

Figure 2 illustrates these effects.

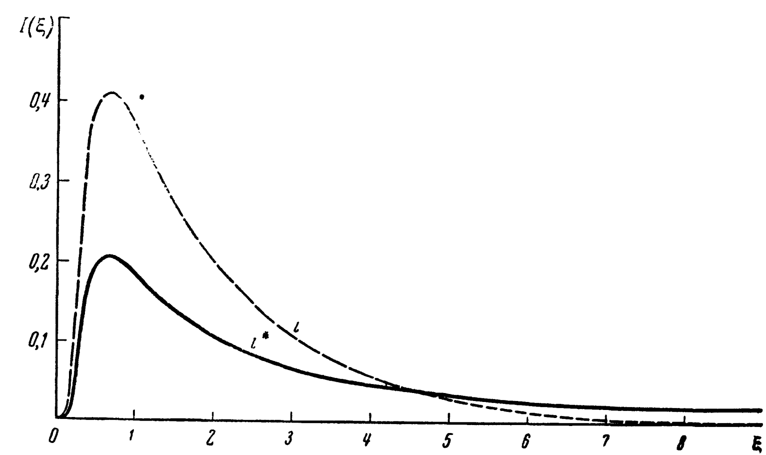

FIG. 2

We note that the recombination of non-equilibrium current carriers also produce a change in the time dependence through the exponential factor $e^{-t/\tau}$ for late times (i. e., $e^{-t/\tau}$ predominates over $t^{-3/2}$). Likewise, the main portion of the impulse is decreased (i.e., the value of i_{max}).

Within the volume of the semiconductor the same effects as at the collector are realized (i.e., greater currents at first and a greater decrease near the tail of the passing impulse) but they are smaller in magnitude the greater the distance from the collector and begin to be noticed only at large intervals of time after the onset of the diffusion process.

Correct to a constant factor, Eq. (23) represents the transient characteristics[6] of a semiconducting triode

$$g(\xi) = \frac{dh(\xi)}{d\xi} = \frac{2}{\sqrt{\pi}} J(\xi) . \quad (47)$$

We note further that the influence of finite lifetime on the electronic processes in a semiconducting layer is represented by a multiplicative factor $e^{-t/\tau}$ only for a δ-impulse. For other kinds of signal shapes, assumption (8) implies that τ appears in the second boundary condition of (6), and consequently the diffusion and recombination process cannot be separated.

[6] V. P. Siforov, *Radio Principles*, p. 58, 1954.

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