The de Haas-van Alphen Effect in Thin Metal Layers

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The magnetic properties of electrons in thin metal layers are discussed for an arbitrary law of dispersion. The energy levels of quasi-particles with an arbitrary law of dispersion in a magnetic field in the presence of a transverse potential field are determined. The oscillating part of the magnetic moment of a gas of such quasi-particles is calculated and general formulas are used for investigating the de Haas-van Alphen effect in thin metal layers. It is shown that the periods and amplitudes of the oscillations are determined by the form of the limiting Fermi surface and depend appreciably on the ratio of the thickness of the layer to the "radius of the classical orbits" of the quasi-particles.

1. IN previous papers by the authors[1] a method of quasi-classical quantization of the motion of charged particles with an arbitrary law of dispersion $\mathcal{E}(p_x, p_y, p_z)$ in a uniform magnetic field was proposed. This method can be extended to the quantization in a magnetic field in the presence of an additional field $U(y)$.

For a magnetic field **H** along the z axis, the components of the kinetic momentum operator **P** may be conveniently chosen as

$$\check{P} = \check{p}_x + \frac{e}{c} Hy = -i\hbar \frac{\partial}{\partial x} + \frac{e}{c} Hy,$$

$$\check{P}_y = -i\hbar \frac{\partial}{\partial y}, \quad \check{P}_z = \check{p}_z = -i\hbar \frac{\partial}{\partial z}$$

i.e., taking $y = (\check{P}_x - \check{p}_x)c/eH$.

Thus, the Hamiltonian of the particles has the form

$$\mathcal{E}^*(P_x, P_y, P_z) = \mathcal{E}(P_x, P_y, P_z) + U[(P_x - p_x)c/eH]$$

Classical motion of the particles is described by the equations

$$\mathcal{E}^*(P_x, P_y, P_z) = \text{const}, \quad P_z = p_z = \text{const}, \quad p_x = \text{const},$$

and quasi-classical quantization is given by[1]

$$\iint dP_x dP_y \equiv S(E, p_z, p_x; H) = (n+\gamma)\frac{2\pi e\hbar H}{c}, \quad 0 < \gamma < 1 \tag{1}$$

where the integral is to be taken over the region bounded by the plane curve

$$\mathcal{E}(P_x, P_y, P_z) + U[(P_x - p_x)c/eH] = E = \text{const} \tag{2}$$

$$P_z = p_z = \text{const}, \quad p_x = \text{const}.$$

Equation (1) implicitly determines the dependence of the energy level E on the quantum number n and the components p_z and p_x.

$$E = E_n(p_z, p_x; H). \tag{3}$$

Having in mind the application of the formulas obtained to the study of the electron gas in a metal layer of finite thickness, we take $U(y)$ as having the form of infinitely high potential walls at the metal surface:

$$U(y) = 0, \quad |y| < a; \quad U(y) = \infty, \quad |y| \geqslant a \tag{4}$$

where a is the half-thickness of the layer. For this case the region (2) is given by the conditions

$$\mathcal{E}(P_x, P_y, P_z) = E, \quad |P_x - p_x| < \frac{|e|H}{c} a, \quad P_z = p_z = \text{const}, \quad p_x = \text{const}. \tag{5}$$

[1] I. M. Lifshitz and A. M. Kosevich, Dokl. Akad. Nauk SSSR **96**, 963 (1954); A. M. Kosevich and I. M. Lifshitz, J. Exper. Theoret. Phys. USSR **29**, 730 (1955).

Let $R_2(p_z, E)$ be the right-hand extreme value of the P_x coordinate of the plane closed curve

$$\mathscr{E}(P_x, P_y, P_z) = E, \quad P_z = p_z = \text{const}, \tag{6}$$

and $R_1(p_z, E)$ the left-hand extreme value of the P_x coordinate of the curve (Fig. 1). If

$$R_2(p_z, E) < p_x + \frac{|e|H}{c}a, \tag{7}$$

$$R_1(p_z, E) > p_x - \frac{|e|H}{c}a,$$

then the closed curve (6) lies completely between the straight lines $P_x = p_x \pm eHa/c$ (Fig. 1), and the quasi-particle can be considered as "free" and its energy levels are determined by the formulas given earlier[1].

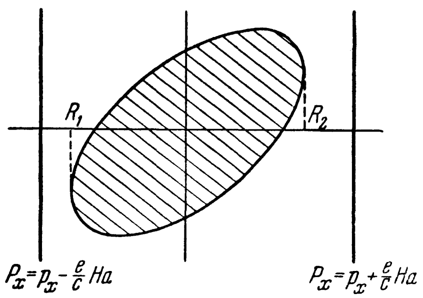

FIG. 1.

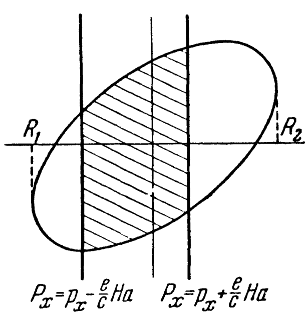

FIG. 2

If the condition (7) is not satisfied, then the area of the region (5) entering into (1) is the area bounded by the curve (6) and the straight lines (or only one of them) $P_x = p_x \pm eHa/c$ (Fig. 2). Since this area depends on a, the energy levels in this case will also depend on a:

$$E = E_n(p_z, p_x; H, a). \tag{8}$$

2. Knowing the values of the energy levels (3) or (8) of a single particle, we can determine the oscillating part of the magnetic moment of a Fermi gas consisting of such particles. The magnetic moment of the gas contained in a volume having linear dimensions L in the x and z directions is given by

$$M = -2\frac{L^2}{(2\pi\hbar)^2}\sum_n \int dp_x \times \int dp_z \frac{\partial E_n(p_z, p_x; H)}{\partial H} f\left(\frac{E-\zeta}{\Theta}\right) \tag{9}$$

where $f(\epsilon)$ is the Fermi distribution function $f(\epsilon) = (e^\epsilon + 1)^{-1}$, ζ is the chemical potential and $\Theta = kT$.

To separate out the oscillating part of the magnetic moment, we use the Poisson formula, replacing the summation with respect to n by an integration. The relevant part of the formula has the form

$$M_{osc} = -\frac{L^2}{\pi^2\hbar^2}\sum_{k=1}^{\infty} \operatorname{Re}\left\{\int_0^\infty \varphi(E_n)\exp(2\pi kin)\,dn\right\}; \tag{10}$$

where $\varphi(E_n)$ denotes the expression under the summation sign in (9).

Taking account of (1) and the equation $\partial E/\partial H = (S - H\frac{\partial S}{\partial H})/H\frac{\partial S}{\partial E}$ which follows from it, we can go over in the integrals of (10) to an integration with respect to E

$$I_k = \int_0^\infty \varphi(E_n)e^{2\pi kin}\,dn \tag{11}$$

$$= \frac{c}{2\pi e\hbar H^2}e^{-2\pi ki\gamma}\int_0^\infty dE f\left(\frac{E-\zeta}{\Theta}\right) \times \left\{\iint_{S>0} dp_x dp_z\left(S - H\frac{\partial S}{\partial H}\right)\exp[ikcS/e\hbar H]\right\},$$

where $S = S(E, P_z, P_x; H)$. Since the argument of the exponential in (11) is, according to our assumptions, a very large number, we can apply the method of steepest descent to calculate the integral in curly brackets. If we assume the stationary point ($\partial S/\partial p_x = \partial S/\partial p_z = 0$) to be isolated, we obtain

$$I_k \approx \frac{1}{kH} \exp\left\{-2\pi k i\gamma \pm i\frac{\pi}{4} \pm i\frac{\pi}{4}\right\} \times \int_0^\infty dE \frac{S_m - H(\partial S/\partial H)_m}{\left|\left(\frac{\partial^2 S}{\partial p_z^2}\right)\left(\frac{\partial^2 S}{\partial p_x^2}\right) - \left(\frac{\partial^2 S}{\partial p_x \partial p_z}\right)^2\right|_m^{1/2}} \times f\left(\frac{E-\zeta}{\Theta}\right) \exp\left[ikcS_m(E,H)/e\hbar H\right].$$

where $S_m(E, H)$ is the extreme value of $S(E, p_z, p_x; H)$ for constant E. The sign of the phase $i\pi/4$ is the same as the sign of the derivative $(\partial^2 S/\partial p_z^2)_m$ or $(\partial^2 S/\partial p_x^2)_m$ taken at the extreme point.

Because of the extreme steepness of the function $f[(E-\zeta)/\Theta]$ in the neighborhood of $E \approx \zeta$, the main contribution to I_k for $\Theta \ll \zeta$ comes from the integration close to the limiting energy. This integration finally gives

$$I_k \approx -\frac{1}{k^2}\frac{e\hbar}{c} \frac{S_m(\zeta, H) - H(\partial S/\partial H)_{m,\zeta}}{\left|\left(\frac{\partial^2 S}{\partial p_x^2}\right)\left(\frac{\partial^2 S}{\partial p_z^2}\right) - \left(\frac{\partial^2 S}{\partial p_x \partial p_z}\right)^2\right|_{m,\zeta}^{1/2} \frac{\partial S_m(\zeta, H)}{\partial \zeta}} \Psi(k\lambda) \times \exp\left[i\frac{kcS_m(\zeta,H)}{e\hbar H} + i\frac{\pi}{2} \pm i\frac{\pi}{4} \pm i\frac{\pi}{4} - 2\pi k i\gamma\right], \tag{12}$$

where

$$\Psi(z) = \frac{z}{\sinh z}, \quad \lambda = \frac{\pi c\Theta}{e\hbar H}\frac{\partial S_m(\zeta, H)}{\partial \zeta}.$$

As can be seen from (12) the period of oscillation is given by

$$\Delta\left(\frac{1}{H}\right) = \frac{2\pi e\hbar}{c} / \left|S_m(\zeta, H) - H(\partial S_m(\zeta, H)/\partial H)\right|, \tag{13}$$

which shows that the period $\Delta(1/H)$ of oscillation depends on the value of the magnetic field H.

3. We shall now apply the general formulas obtained for a gas of quasi-particles in an arbitrary potential field to the case of an electron gas in a metal layer of finite thickness whose walls we shall replace by the infinitely high potential walls (4). The calculation of the integral I_k in this case is somewhat changed. This is connected with the fact that for

$$R_2(p_z^0, \zeta) < p_x + \frac{|e|H}{c}a, \qquad R_1(p_z^0, \zeta) > p_x - \frac{|e|H}{c}a,$$

the energy levels are independent of p_x, where p_z^0 corresponds to the maximum value $S_m^0(\zeta)$ of the area of cross section $S^0(\zeta, p_z)$ of the surface $\mathcal{E}(P_x, P_y, P_z) = \zeta$ by planes **PH** = const. Thus the expression for I_k assumes different forms for different ratios of the thickness $2a$ of the layer, to the "diameter" cD/eH of the orbit of the quasi-particle:

1) If $$D(p_z^0, \zeta) \equiv R_2(p_z^0, \zeta) - R_1(p_z^0, \zeta) < 2\frac{eH}{c}a, \tag{14}$$

$$I_k \approx -\left[\frac{eH}{c}2a - D(p_z^0, \zeta)\right] \times \frac{(e\hbar/c)^{1/2} S_m^0(\zeta)\Psi(k\lambda^0)}{k^{3/2}\sqrt{2\pi H}\left|\partial^2 S^0(\zeta, p_z)/\partial p_z^2\right|_m^{1/2}} \times \frac{\exp\left\{ikcS_m^0(\zeta)/e\hbar H - 2\pi k i\gamma + i\left(\frac{\pi}{2} \pm \frac{\pi}{4}\right)\right\}}{\partial S_m^0(\zeta)/\partial\zeta},$$

where

$$\lambda^0 = \frac{\pi c\Theta}{e\hbar H}\frac{dS_m^0(\zeta)}{d\zeta}.$$

2) If $D(p_z^0, \zeta) > 2(eH/c)a$ the expression for I_k is the same as (12), where $S(E, p_z, p_x; H)$ is to be understood as the area bounded by the curve (5) and the straight lines $P_x = p_x \pm eHa/c$.

The corresponding periods of oscillation of magnetic moment in these two cases are given by different formulas, namely,

a) for $D < 2eHa/c$, the period of oscillation is given by

$$\Delta\left(\frac{1}{H}\right) = \frac{2\pi e\hbar}{c} \Big/ S_m^0(\zeta),$$

where $S_m^0(\zeta)$ is the extreme area bounded by the curve (6) for $E = \zeta$;

b) for $D > 2eHa/c$ the period is given by (13), where $S_m(\zeta, H)$ is to be understood as the extreme value of the "cut-off" area (Fig. 2). Since $S_m(\zeta, H)$ depends on a, the period $\Delta(1/H)$ in this case depends both on the magnetic field and the thickness of the layer.

It should be noted that for the electron gas in a metal layer, the dependence of the period $\Delta(1/H)$ on H occurs only for thicknesses smaller than the mean "radius" of the electron orbit. However, the dependence of the amplitude of the oscillations on the layer thickness begins to appear at larger thicknesses [see Eq. (14)].

For an electron gas with a quadratic law of dispersion $\mathcal{E} = p^2/2m$, the curve (6) is a circle and $D(p_z^0, \zeta) = 2\sqrt{2m\zeta}\ (p_z^0 = 0)$. As has been pointed out, the formula for the magnetic moment of such a gas in a metal layer depends on the size of the ratio

$$\sqrt{\delta} = 2a \Big/ \frac{c}{eH} D = eHa / c\sqrt{2m\zeta}.$$

For $\delta > 1$ the oscillating part of the magnetic moment is given by (10) and (14), where

$$S_m^0(E) = 2\pi mE,$$

$$dS_m^0/dE = 2\pi m, \quad |\partial^2 S/\partial p_z^2| = 2\pi,$$

and for $\delta < 1$, by (10) and (12) in which we must use the relations

$$S_m(\zeta, H) = 4m\zeta\left(\sqrt{\delta(1-\delta)} + \sin^{-1}\sqrt{\delta}\right);$$

$$\partial S_m(\zeta, H)/\partial\zeta = 4m\sin^{-1}\sqrt{\delta},$$

$$H(\partial S/\partial H)_{m,\zeta} = 8m\zeta\sqrt{\delta(1-\delta)},$$

$$|\partial^2 S/\partial p_z^2|_{m,\zeta} = 4\sin^{-1}\sqrt{\delta},$$

$$|\partial^2 S/\partial p_x^2|_{m,\zeta} = 4\sqrt{\delta/(1-\delta)}, \quad \partial^2 S/\partial p_x \partial p_z = 0.$$

The expressions obtained for the oscillating part of the magnetic moment of a free electron gas from the general formulas (10), (12) and (14) go over into the expressions obtained earlier by the authors from direct calculations[2]. We give here only the formulas for the periods of oscillation

$$\begin{aligned} \Delta(1/H) &= e\hbar/mc\zeta, & \delta > 1; \\ \Delta(1/H) &= \pi e\hbar/2mc\zeta\left|\sqrt{\delta(1-\delta)} - \sin^{-1}\sqrt{\delta}\right|, & \delta < 1. \end{aligned} \tag{15}$$

Equation (15) gives the explicit dependence of the period of oscillation on H and the layer thickness a for $\delta < 1$.

It should be noted that, in determining the magnitude of the extreme area of cross section of the Fermi surface from experimental data, it should be possible to establish that the dependence of the period $\Delta(1/H)$ on H appears at thicknesses $a \sim 10^{-4}$ to 10^{-5} cm (in fields $H \sim 10^3$ to 10^4 oe).

In order to apply the above results to the investigation of the magnetic properties of metallic films, it is essential that the metallic film, or the packet of metallic films, should be nearly a single crystal or else the scatter of the orientations of the single crystals will lead to smearing out or complete extinction of the oscillations of the magnetic moment.

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[2] I. M. Lifshitz and A. M. Kosevich, Dokl. Akad. Nauk SSSR 91, 795 (1953).

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