Theory of Strong Coupling for Meson Fields. III

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Examination is made of the pseudoscalar meson field which interacts with moving nucleons in the strong coupling approximation. A theory is developed which takes into account the polarization of the nucleonic vacuum.

1. IN earlier works[2,3] we investigated the meson field which interacts with infinitely massive nucleons at rest. We now consider this problem for moving nucleons with finite mass.

The Hamiltonian (1) and (29) in reference 1, for nucleons at rest,

$$H=\frac{1}{2}\int\sum_{\alpha=1}^{3}\Big[\Pi_\alpha^2+c^2(\nabla\Phi_\alpha)^2+c^2\varkappa^2\Phi_\alpha^2 -2\sqrt{4\pi}\,gc\sum_i^n O_\alpha(\mathbf{r}-\mathbf{r}_i)\,\Phi_\alpha\Big]d\mathbf{r} \quad (1)$$

$[O_{\alpha i}=\tau_\alpha\vec{\sigma}_i\ \nabla U(\mathbf{r}-\mathbf{r}_i)$ in the case of a pseudoscalar field $]$ is obtained from the exact Hamiltonian, which has the following form for a pseudoscalar charged field:

$$H=\sum_i^n[c(\vec{\alpha}_i,\mathbf{p}_i)+\beta_i mc^2] +\frac{1}{2}\int\sum_\alpha^3\Big[\Pi_\alpha^2+c^2(\nabla\Phi_\alpha)^2+c^2\varkappa^2\Phi_\alpha^2 -2c\sqrt{4\pi}\sum_i^n(G_{\alpha i}\Phi_\alpha + F_{\alpha i}\Pi_\alpha)+4\pi c^2\sum_{ij}^{n,n}F_{\alpha i}F_{\alpha j}\Big]d\mathbf{r}; \quad (2)$$

$$G_{\alpha j}=-i\beta_j\gamma_{5j}\tau_{\alpha j}g_{ps}U(\mathbf{r}-\mathbf{r}_j)+\frac{g_{pv}}{\varkappa}\tau_{\alpha j}(\vec{\sigma}_j,\nabla U(\mathbf{r}-\mathbf{r}_j));$$

$$F_{\alpha j}=\frac{g_{pv}}{\varkappa c}\gamma_{5j}\tau_{\alpha j}U(\mathbf{r}-\mathbf{r}_j);$$

$2\pi c^2\int\sum_{\alpha,i,j}F_{\alpha i}F_{\alpha j}\,d\mathbf{r}$ are the so-called contact terms[3]. The Hamiltonian (1) is obtained from (2) with the help of a transition to the nonrelativistic approximation(i.e., a neglect of terms of order E/nmc^2) in comparison with those entering into (1), and then with neglect of the kinetic energy of the nucleons. Neglect of the kinetic energy of the nucleons in the zeroth approximation corresponds to expansion in powers of μ/m, as it does in molecular theory. The motion of the nucleons can easily be taken into account by means of adiabatic perturbation theory, analogous to the theory which applies to molecules[4].

However, if we take into consideration in the Hamiltonian (1) the subsequent terms in the expansion of (2) in powers of (E/nmc^2), then, as was shown in reference 4, these terms give the following correction for φ_α^0 in the case of a single nucleon:

$$\delta\varphi_\alpha^0=h\left(1+\frac{g_{ps}^2}{mc^2}\int hU\,d\mathbf{r}\right. - B\langle\vec{\tau}\rangle^2\left.\int hU\,d\mathbf{r}\right)^{-1}\left\{\left[B\langle\vec{\tau}\rangle^2-\frac{g_{ps}^2}{mc^2}\right]\times\int f_\alpha U\,d\mathbf{r}-\langle\tau_\alpha\rangle B\sum_\beta\langle\tau_\beta\rangle\int f_\beta U\,d\mathbf{r}\times\left(1+\frac{g_{ps}^2}{mc^2}\int hU\,d\mathbf{r}\right)^{-1}\right\}; \quad (3)$$

$$f_\alpha=(g/c\sqrt{4\pi})\int\langle O_\alpha(\mathbf{r}')\rangle G(\mathbf{r},\mathbf{r}')\,d\mathbf{r}';$$

$$h=\int U(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\,d\mathbf{r}';\qquad \langle\tau_\alpha\rangle=\frac{(\psi_{\lambda\lambda}^{0*},\tau_\alpha\psi_{\lambda\lambda}^0)}{(\psi_{\lambda\lambda}^{0*},\psi_{\lambda\lambda}^0)};$$

$$B=4\pi\left(\frac{g_{ps}g_{pv}}{m\varkappa c^3}\right)^2\int U^2\,d\mathbf{r}\left(1+\frac{4\pi g_{pv}^2}{m\varkappa^2c^2}\int U^2\,d\mathbf{r}\right)^{1};$$

[1] T. Geilikman, J. Exper. Theoret. Phys. USSR 29, 417 (1955); Soviet Phys. 2, 509 (1956).

[2] B. T. Geilikman, J. Exper. Theoret. Phys. USSR 29, 430 (1955).

[3] N. Kemmer, Proc. Roy. Soc. (London) 166A, 127 (1938).

[4] B. T. Geilikman, Dokl. Akad. Nauk SSSR 91, 39 (1953).

Equation (3) keeps terms of order $g^2(amc^2)^{-1}$ in comparison with φ_α^0 for $m = \infty$ (a = nuclear "radius"). The correction for the energy E_λ^0 is of the same order of magnitude (i.e., it is of order E/mc^2). If we assume that the eigenvalue of the energy of the nucleon in the meson field amounts to only a small part of its rest energy, then $g^2/a \sim mc^2$ and $g^2/\kappa^2 a^3 \sim mc^2$ (for gradient coupling); if $a \to 0$, then the passage of m to infinity must be removed by renormalization. In this case the usual transition from the exact Hamiltonian to the nonrelativistic approximation and then to nucleons at rest (1) is invalid. For two or more nucleons, the condition for the applicability of the non-relativistic approximation $(E_\lambda^0 - E_{\lambda'}^0)/mc^2 \ll 1$ is not satisfied in the case of small separation of the nucleons, even for $\delta\varphi_\alpha^0 = 0$, i.e., without taking into account the subsequent terms in (E/nmc^2). Indeed, it was shown in reference 2 that the distance between two levels $E_{33}^0 - E_{11}^0$ for a system of two nucleons is equal to $2K$, i.e., $\sim g^2/a$ for $r \sim a$ (nongradient coupling) and $(g^2/\kappa^2 a^3)$ (gradient coupling). Thus, $E_{33}^0 - E_{11}^0$ is of order mc^2 if $g^2/a \sim mc^2 (g^2/\kappa^2 a^3 \sim mc^2)$. Therefore, if $mc^2 \sim g^2/a (mc^2 \sim g^2/\kappa^2 a^3)$ we must consider the Hamiltonian (2) directly. In this case, only the solution of (2) for the nonrelativistic case $v \ll c$ makes sense, so long as the nucleons are assumed to be extended.

For pseudoscalar coupling the energy of interaction of the nucleons with the field depends not only on Φ_α but also on the momenta Π_α. Therefore, $\dot{\Phi}_\alpha = \frac{i}{\hbar}[H, \Phi_\alpha] \neq \Pi_\alpha$. As was shown in reference 1, the zeroth approximation corresponds to a neglect of the kinetic energy of the field, but since $\Pi_\alpha \neq 0$ for $\dot{\Phi}_\alpha = 0$, then $\Pi_\alpha \neq$ for the zero field also (potential momentum). We assume that $\Phi_\alpha = \varphi_\alpha^0 + \varphi_\alpha$; $\Pi_\alpha = \pi_\alpha^0 + \pi_\alpha$. The Hamiltonian of zeroth approximation is then

$$H^0 = \sum_i^n [c(\vec{\alpha}_i, \mathbf{p}_i) + \beta_i mc^2] + \frac{1}{2}\int \sum_\alpha \Big[(\pi_\alpha^0)^2 + c^2\varphi_\alpha^0(\varkappa^2 - \Delta)\varphi_\alpha^0 - 4c\sqrt{\pi}\sum_i^n (G_{\alpha i}\varphi_\alpha^0 + F_{\alpha i}\pi_\alpha^0) + 4\pi c^2 \sum_{i,j}^{n,n} F_{\alpha i}F_{\alpha j}\Big] d\mathbf{r}$$

The eigenfunctions of H^0 ψ_n^0 depend on the coordinates $\mathbf{r}_1, \ldots, \mathbf{r}_n$, and on the spin variables $s_1, \ldots, s_n$. The perturbation is

$$H' = H - H^0 = H^{(1)} + H^{(2)};$$

$$H^{(1)} = \int \sum_\alpha \Big\{ c^2 \Big[(\varkappa^2 - \Delta)\varphi_\alpha^0 - \sqrt{4\pi}/c \sum_i^n G_{\alpha i}\Big]\varphi_\alpha + \Big[\pi_\alpha^0 - c\sqrt{4\pi}\sum_i^n F_{\alpha i}\Big]\pi_\alpha\Big\} d\mathbf{r};$$

$$H^{(2)} = \frac{1}{2}\int \sum_\alpha [\pi_\alpha^2 + c^2\varphi_\alpha(\varkappa^2 - \Delta)\varphi_\alpha] d\mathbf{r}.$$

The equation for φ_α^0 and π_α^0 must be found from the condition

$$H_{nn}^{(1)} = \int (\psi_n^{0*}, H^{(1)}\psi_n^0) \prod_i^n d\mathbf{r}_i = 0.$$

as was shown previously in reference 1 [see Eq. (12)]. This condition will be satisfied if φ_α^0 and π_α^0 obey the equations[4,5]:

$$(\varkappa^2 - \Delta)\varphi_\alpha^0 = \frac{\sqrt{4\pi}}{c}\sum_i^n \langle G_{\alpha i}\rangle_{nn}^s; \qquad (4)$$

$$\pi_\alpha^0 = c\sqrt{4\pi}\sum_i^n \langle F_{\alpha i}\rangle_{nn}^s.$$

Here

$$\langle G_{\alpha i}\rangle_{nn}^s = \frac{(\psi_n^{0*}(\mathbf{r}_1, \ldots, \mathbf{r}_n, s_1, \ldots, s_n), G_{\alpha i}\psi_n^0(\mathbf{r}_1, \ldots, s_n))}{(\psi_n^{0*}, \psi_n^0)}$$

is the mean value of $G_{\alpha i}$ over the variables of ordinary and isotopic spin of the nucleons. Equations (4) are the natural generalizations of the equations for the zero field in the case of nucleons at rest (reference 1). We substitute the solution of Eq. (4) in the expressions for H^0 and $H^{(1)}$:

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[5] B. T. Geilikman, Dokl. Akad. Nauk SSSR 91, 225 (1953).

$$H^0 = \sum_i^n [c(\vec{\alpha}_i, \mathbf{p}_i) + \beta_i mc^2] \tag{5}$$

$$+ \frac{1}{2} \sum_{\alpha, i, j}^{3, n, n} \int [\langle G_{\alpha i}(\mathbf{r}')\rangle_{nn}^s$$

$$-2G_{\alpha i}(\mathbf{r}')] \langle G_{\alpha i}(\mathbf{r})\rangle_{nn}^s G(\mathbf{r}, \mathbf{r}')\, d\mathbf{r}\, d\mathbf{r}'$$

$$+ 2\pi c^2 \sum_{\alpha, i, j}^{3, n, n} \int [(\langle F_{\alpha j}\rangle_{nn}^s$$

$$- 2F_{\alpha i}) \langle F_{\alpha j}\rangle_{nn}^s + F_{\alpha i}F_{\alpha j}]\, d\mathbf{r};$$

$$G(\mathbf{r}, \mathbf{r}') = \frac{e^{-\varkappa|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}; \tag{6}$$

$$H^{(1)} = c\sqrt{4\pi} \sum_{\alpha, i}^{3, n} \int [(\langle G_{\alpha i}\rangle_{nn}^s - G_{\alpha i})\varphi_\alpha$$

$$+ (\langle F_{\alpha i}\rangle_{nn}^s - F_{\alpha i})\pi_\alpha]\, d\mathbf{r}.$$

If the eigenfunctions ψ_n^0 and eigenvalues E_n^0 of the nonlinear equation $H^0\psi_n^0 = E_n^0\psi_n^0$ are found, it is possible to form the Hamiltonians H_n^0 from H^0 by substituting in $\langle F_{\alpha i}\rangle_{nn}^S$ and $\langle G_{\alpha i}\rangle_{nn}^S$ the function $\psi_n^0(\mathbf{r}_1, \ldots, s_n)$ in explicit form. The eigenfunctions ψ_{nl}^0 of the equation $H_n^0\psi_{nl}^0 = E_{nl}^0\psi_{nl}^0$ form a complete orthogonal set. We now seek the total function for the Hamiltonian (2) in the form

$$\Psi_n = \sum_l [\chi_n^0(\varphi_\alpha)\delta_{nl} + \chi_{nl}'(\varphi_\alpha)]\,\psi_{nl}^0(\mathbf{r}_1, \ldots s_n).$$

Assuming that $\varphi \ll \varphi^0$, $\chi_{nl}' \ll \chi_n^0$, we can develop the theory of perturbations as in reference 1, Sec. 2 (only now the spectrum of the unperturbed Hamiltonian is continuous or quasi-discrete if a finite volume V is considered). In this case we obtain an equation for χ_n^0 (in the absence of degeneracy for E_{nn}^0: $E_{nl}^0 \neq E_{nn}^0$ of $l \neq n$; the degenerate case will be considered below):

$$(H_{n2} + H_{n3}' + H_{n3}'')\chi_n^0 = E_n'\chi_n^0; \tag{7}$$

$$E_n' = E_n - E_{nn}^0;$$

$$H_{n2} = H_{nn}' + \sum_{l \neq n} H_{nl}^{(1)} H_{ln}^{(1)} (E_{nn}^0 - E_{nl}^0)^{-1};$$

$$H_{n3}' = \sum_l H_{nl}^{(1)} H_{ln}^{(1)} [(E_{nn}^0 - E_{nl}^0 + E_n')^{-1}$$

$$- (E_{nn}^0 - E_{nl}^0)^{-1}];$$

$$H_{n3}'' = \sum_l \sum_m H_{nl}^{(1)} H_{lm}^{(1)} H_{mn}^{(1)} (E_{nn}^0 - E_{nl}^0)^{-1}$$

$$\times (E_{nn}^0 - E_{nm}^0)^{-1};$$

$$H_{lm}' = \int (\psi_{nl}^{0*}, H'\psi_{nm}^0) \prod_i^n d\mathbf{r}_i;$$

$$H_{nn}' = H_{nn}^{(2)}, \text{ так как } H_{nn}^{(1)} = 0.$$

If we generalize the usual method of reducing to the sum of squares a Hamiltonian which is quadratic in momenta p_S and coordinates q_S (see reference 6) to the case of π_α and φ_α which depend on the continuous index $\mathbf{r}$, we can show that H_{n2} takes the form:

$$H_{n2} = \frac{1}{2} \sum_k (p_k^2 + \omega_k^2 q_k^2)$$

after the canonical transformation:

$$\pi_\alpha = \sum_k \left[\pi_{k\alpha}^+(\mathbf{r})(q_k - ip_k/\omega_k) + \frac{1}{2}\pi_{k\alpha}^-(\mathbf{r})(p_k - i\omega_k q_k)\right];$$

$$\varphi_\alpha = \sum_k \left[\varphi_{k\alpha}^+(\mathbf{r})(q_k - ip_k/\omega_k) + \frac{1}{2}\varphi_{k\alpha}^-(\mathbf{r})(p_k - i\omega_k q_k)\right];$$

Here $\pi_{k\alpha}^+$, $\varphi_{k\alpha}^+$ are the solutions of the field equations

$$\dot{\varphi}_\alpha \doteq -i\omega\varphi_\alpha$$

$$= \frac{i}{\hbar}[H_{n2}, \varphi_\alpha] \text{ and } -i\omega\pi_\alpha = \frac{i}{\hbar}[H_{n2}, \pi_\alpha],$$

[6] E. T. Whittaker, *Analytical Dynamics.*

which correspond to the frequency $+\omega_k$, and $\pi^{-}_{k\alpha}$, $\varphi^{-}_{k\alpha}$ to the frequency $-\omega_k$.

Inasmuch as $H^{(2)}$ for $m \neq \infty$ coincides with $H^{(2)}$ for $m=\infty$, the frequencies $\omega_k = c\sqrt{\kappa^2+k^2}$ as before, and do not depend on φ^0_α. Since the field $\varphi^0_\alpha \sim g^1$ and $\varphi_\alpha \sim g^0$, as was the case for $m=\infty$, the nonlinearity of the field of real mesons φ_α remains of the order g^{-2}.

The conditions of applicability of the perturbation theory $\varphi \ll \varphi^0$, $\chi'_{nl} \ll \chi^0_n$ have the same form as in the case of nucleons at rest, since the estimates of φ and $\varphi^0 (\pi^0 \approx \kappa c\, \varphi^0)$ remain unchanged: $g^2/\hbar c \gg 1$ and $g^2/\hbar c \gg \kappa^2 a^2$ (in the absence of mesons).

2. The solution of the zeroth order approximation of the equation with the Hamiltonian (5) presents the greatest difficulty in the case of moving nucleons. Equation (5) is then a nonlinear differential equation, and not an algebraic equation as was the case for nucleons at rest. The problem is simplified in the case of a single nucleon. There the solution has the form: $\psi^0(\mathbf{r}, s) = [u^0 \times (s) e^{i\,\mathbf{k}\cdot\mathbf{r}}$ [$u^0(s)$ is a factor which depends on the spin variable s]. Substituting $u^0(s)e^{i\,\mathbf{k}\cdot\mathbf{r}}$ in Eq. (5), we get an algebraic equation for the eight component function $u^0(s)$:

$$\Big\{\hbar c(\vec{\alpha}, \mathbf{k}) + \beta mc^2 + \frac{1}{2}\sum_\alpha \int [|G_\alpha(\mathbf{r}')|_{\lambda\lambda} - 2G_\alpha(\mathbf{r}')]\,|G_\alpha(\mathbf{r})|_{\lambda\lambda} G(\mathbf{r}, \mathbf{r}')\,d\mathbf{r}\,d\mathbf{r}' + 2\pi c^2 \sum_\alpha \int [(|F_\alpha|_{\lambda\lambda} - 2F_\alpha)\,|F_\alpha|_{\lambda\lambda} + F_\alpha^2]\,d\mathbf{r}\Big\}\, u^0_\mu = E^0_\mu u^0_\mu. \tag{8}$$

An analogous equation is obtained in the case of a scalar field with scalar coupling:

$$\Big[\hbar c(\vec{\alpha}, \mathbf{k}) + \beta mc^2 + \frac{g^2}{2}\sum_\alpha \int [|\tau_\alpha\beta|_{\lambda\lambda} - 2\tau_\alpha\beta]\,|\tau_\alpha\beta|_{\lambda\lambda} U(\mathbf{r})\,U(\mathbf{r}')\,G(\mathbf{r}, \mathbf{r}')\,d\mathbf{r}\,d\mathbf{r}'\Big] \times u^0_\mu = E^0_\mu u^0_\mu.$$

As an example we take the solution of the equation for u^0_μ with a neutral field:

$$[c(\vec{\alpha}, \mathbf{p}) + \beta mc^2 + (J_s\xi^2/2) - \beta J_s \xi]\, u^0_\mu = E^0_\mu u^0_\mu;$$
$$J_s = g^2 \int U(\mathbf{r})\,U(\mathbf{r}')\,G(\mathbf{r}, \mathbf{r}')\,d\mathbf{r}\,d\mathbf{r}';$$
$$\xi = |\beta|_{\lambda\lambda};\quad \mathbf{p} = \hbar\mathbf{k}.$$

If we equate the determinant of this set of four equations to zero, we get:

$$\Big[\Big(mc - \frac{J_s\xi}{c}\Big)^2 + p^2 - \Big(\frac{E^0}{c} - \frac{J_s\xi^2}{2c}\Big)^2\Big]^2 = 0;$$
$$E^0 = \frac{J_s\xi^2}{2} \mp c\sqrt{\Big(mc - \frac{J_s\xi}{c}\Big)^2 + p^2}.$$

We see that both the level with $E^0 > 0$ and the level with $E^0 < 0$ are doubly degenerate. The same degeneracy is obtained for a pseudoscalar field with pseudoscalar coupling. Because of the degeneracy, we cannot use Eq. (4), since the expression $\langle G_\alpha\rangle^S_{nn}$ remains indeterminate in the case of the two functions ψ^0_{n1} and ψ^0_{n2}. With degeneracy, therefore, it is more appropriate to use the first, and not the second, variant of the adiabatic perturbation theory (see reference 1, Sec. 2). In such a case

$$H^0 = c(\vec{\alpha}, \mathbf{p}) + \beta mc^2 + \frac{1}{2}\sum_\alpha^3 \int [c^2\Phi_\alpha(\varkappa^2 - \Delta)\,\Phi_\alpha - 4c\sqrt{\pi}G_\alpha\Phi_\alpha]\,d\mathbf{r};$$
$$H' = \frac{1}{2}\sum_\alpha^3 \int \Pi_\alpha^2 d\mathbf{r};$$

for the scalar field $G_\alpha = \beta\tau_\alpha g U(\mathbf{r} - \mathbf{r}_1)$. (For pseudovector and mixed coupling, the degeneracy is absent; see below.) The ψ functions of zeroth approximation depend upon Φ_α:

$$H^0\psi^0_{\mu\mathbf{k}}(\mathbf{r}, s, \Phi_\alpha) = E^0_{\mu\mathbf{k}}(\Phi_\alpha)\,\psi^0_{\mu\mathbf{k}}(\mathbf{r}, s, \Phi_\alpha).$$

We seek the total Ψ function of the system in the form[1]:

$$\Psi_{\lambda\mathbf{k}} = \sum_{\mu, \mathbf{k}'} \chi_{\mu\mathbf{k}'}(\Phi_\alpha)\,u^0_{\mu\mathbf{k}'}(s, \Phi_\alpha)\,e^{i(\mathbf{k}', \mathbf{r})}.$$

If the degeneracy is twofold (as was the case for neutral field with nongradient coupling), then for $\mathbf{k}' = \mathbf{k}$, $E^0_{1\mathbf{k}}(\varphi^0) = E^0_{2\mathbf{k}}(\varphi^0)$. In the first approximation in the sum over $\mathbf{k}'$ and μ, there remain only two components with $\mathbf{k}' = \mathbf{k}$ and $\mu = 1$ and $\mu = 2$. If we substitute in the equation

$$H\Psi = E\Psi \quad \Psi = (\chi^0_{1\mathbf{k}} u^0_{1\mathbf{k}} + \chi^0_{2\mathbf{k}} u^0_{2\mathbf{k}})\,e^{i(\mathbf{k}, \mathbf{r})},$$

we get two equations for $\chi^0_{1\mathbf{k}}$ and $\chi^0_{2\mathbf{k}}$ (below,

$\chi^0_\lambda \equiv \chi^0_{\lambda k'}$ $V^{-1}\int E^0_{1k} d\mathbf{r}_1 = V^{-1}\int E^0_{2k} d\mathbf{r}_1 \equiv E^0(\Phi)$, $u^0_{\lambda k} \equiv u^0_\lambda$):

$$\left[\frac{1}{2}\int \Pi^2 d\mathbf{r} + E^0(\Phi)\right]\chi^0_1 + f_{11}(\Pi, \Phi)\chi^0_1 + f_{12}(\Pi, \Phi)\chi^0_2 = E\chi^0_1; \quad (9)$$

$$\left[\frac{1}{2}\int \Pi^2 d\mathbf{r} + E^0(\Phi)\right]\chi^0_2 + f_{21}(\Pi, \Phi)\chi^0_1 + f_{22}(\Pi, \Phi)\chi^0_2 = E\chi^0_2;$$

Here

$$f_{\mu\lambda}(\Pi, \Phi) = \frac{1}{2}\left(u^{0*}_\mu, \left[\int \Pi^2, d\mathbf{r}, \quad u^0_\lambda\right]\right);$$

$$([A, B] = AB - BA).$$

We can now introduce the 2×2 Pauli matrices σ_x, σ_y, σ_z and the two component function $\chi^0 = \dfrac{\chi^0_1}{\chi^{0*}_2}$. Then the set (9) is written in the form

$$H_1\chi^0 = E\chi^0; \; H_1 = \frac{1}{2}\int \Pi^2 d\mathbf{r} + E^0(\Phi) + (\vec{\sigma}, \mathbf{a}(\Phi, \Pi)) + b(\Phi, \Pi); \quad (10)$$

$$a_x = \frac{1}{2}(f_{12} + f_{21});$$

$$a_y = \frac{i}{2}(f_{12} - f_{21}); \; a_z = \frac{1}{2}(f_{11} - f_{22});$$

$$b = \frac{1}{2}(f_{11} + f_{22}).$$

We calculate

$$\varepsilon = \left[\int \Pi^2 d\mathbf{r}, \; u^0_\lambda(\Phi)\right] = \int (\Pi[\Pi, u^0_\lambda(\Phi)] + [\Pi, u^0_\lambda(\Phi)]\Pi)\, d\mathbf{r};$$

$[\Pi, u^0_\lambda(\Phi)] = g_\lambda(\Phi)$ = functional of Φ. Therefore

$$\varepsilon = \int ([\Pi, g_\lambda] + 2g_\lambda \Pi)\, d\mathbf{r} = \int (h_\lambda(\Phi) + 2g_\lambda(\Phi)\Pi)\, d\mathbf{r};$$

$$f_{\mu\lambda} = \frac{1}{2}\int [(u^{0*}_\mu, h_\lambda) + 2(u^{0*}_\mu, g_\lambda)\Pi]\, d\mathbf{r}.$$

Thus b and a_x, a_y, a_z are linear functionals of Π.

Equation (10) can be solved by the usual method. We set $\Phi = \varphi^0 + \varphi$, $\Pi = \pi^0 + \pi$ and expand H_1 in a power series in φ, π: $H_1 = \sum_{k=0} H_1^{(k)}$, limiting ourselves to the quadratic terms. The equations for φ^0 and π^0 are found from the conditions[1]: $\langle H_1^{(1)}\rangle = 0$; they have the form:

$$\delta E / \delta\varphi^0 = \delta\langle H_1^0\rangle / \delta\varphi^0 = 0; \quad \delta\langle H_1^0\rangle / \delta\pi^0 = 0.$$

In first approximation for Eq. (10) we write $\chi^0 = \psi^0(q, \varphi^0, \pi^0)\chi^0(\varphi)$, we obtain an equation for the two component function $\chi^0(q)$ (q is the spin variable $\vec{\sigma}$; $q = q_1, q_2$):

$$\left[\frac{1}{2}\int (\pi^0)^2 d\mathbf{r} + E^0(\varphi^0) + (\vec{\sigma}, a(\varphi^0, \pi^0)) + b(\varphi^0, \pi^0)\right]\psi^0_\nu(q) = E^0_{0\nu}\psi^0_\nu(q) \quad (11)$$

and the equation for $\chi^0(\varphi)$ is

$$\left[H_1^{(2)} + \sum_\rho^2 |H_1^{(1)}|_{\nu\rho} |H_1^{(1)}|_{\rho\nu} (E^0_{0\nu} - E^0_{0\rho})^{-1}\right]\chi^0(\varphi) = E'_\nu \chi^0(\varphi);$$

$$|H_1^{(1)}|_{\nu\rho} = (\psi^{0*}_\nu(q), \; H_1^{(1)}\psi^0_\rho(q)).$$

It is evident that the degeneracy is removed because of the terms in Eq. (11) which contain $\vec{\sigma}$, and we obtain $E^0_{01} \neq E^0_{02}$ for a given $\mathbf{k}$.

For χ'_{nl} with $l \neq n$, i.e., $\mathbf{k}' \neq \mathbf{k}$ or $\mu = 3, 4$ for $\mathbf{k}' = \mathbf{k}$, we can make use of the usual equation (see reference 1, Sec. 2). The appropriate solution of Eq. (10) will be given in another work.

In the case of fourfold, rather than twofold, degeneracy, a set of four equations for $\chi^0_{1\mathbf{k}}$, $\chi^0_{2\mathbf{k}}$, $\chi^0_{3\mathbf{k}}$, $\chi^0_{4\mathbf{k}}$ can be written with the help of 4×4 Dirac matrices. The method outlined above, with the introduction of the matrices $\vec{\sigma}$ can also be employed in molecular theory in the case of degeneracy of the electronic level for arbitrary values of the nuclear radius vectors R_i.

We now consider the solution of Eq. (8) for the pseudoscalar neutral field with pseudovector coupling (degeneracy is not present in this case):

$$\left[c(\vec{\alpha}, \mathbf{p}) + \beta mc^2 + \frac{J_p(\vec{\eta})^2}{2} + \frac{J(1+\zeta^2)}{2} - J_p(\vec{\eta}, \vec{\sigma}) - J\zeta\gamma_5\right] u^0_\mu = E^0_\mu u^0_\mu; \quad (12)$$

$$\vec{\eta} = |\vec{\sigma}|_{\lambda\lambda}; \; \zeta = |\gamma_5|_{\lambda\lambda};$$

$$J_p = \frac{g^2_{pv}}{\varkappa^2}\int \frac{\partial^2 G(\mathbf{r}, \mathbf{r}')}{\partial x\, \partial x'} U(\mathbf{r})\, U(\mathbf{r}')\, d\mathbf{r}\, d\mathbf{r}';$$

$$J = \frac{4\pi g^2_{pv}}{\varkappa^2}\int U^2(\mathbf{r})\, d\mathbf{r}'.$$

If we set the determinant of the system (12) to zero, and assume that the z axis is directed along the vector $\mathbf{p}$, we obtain

$$[(p_0+b_z)^2-\gamma^2][(p_0-b_z)^2-\gamma^2]$$
$$+2(p^2+\delta^2)(p_0^2-\gamma^2-b_z^2)$$
$$-2(b^2-b_z^2)(p_0^2+\gamma^2-b_z^2+\delta^2-p^2)$$
$$+(b^2-b_z^2)^2+(p^2-\delta^2)^2$$
$$+8pb_z\delta\gamma=0;\quad p_0=mc;$$
$$\mathbf{b}=-\frac{J_p\vec{\eta}}{c};\quad \delta=-\frac{J\zeta}{c};$$
$$\gamma=\left(E-\frac{J}{2}-\frac{J\zeta^2}{2}-\frac{J_p(\vec{\eta})^2}{2}\right)\Big/c.$$

For $\mathbf{p}=0$, the system (12) divides into two systems of two equations, if we assume that the axis $\bar{z}$ is parallel to $\mathbf{b}$ (see reference 2, Sec. 1).

1) For $\lambda=1$ and $\lambda=2$: $u^0_{\mu 2}=u^0_{\mu 4}=0$; $\eta_{\bar{z}}=+1$; $\zeta=0$ for $\lambda=1$

$$E^0_{11}=\frac{J-J_p}{2}+mc^2;$$

$$\text{for}\quad \lambda=2\quad E^0_{22}=\frac{J-J_p}{2}-mc^2.$$

2) For $\lambda=3$ and $\lambda=4$: $u^0_{\mu 1}=u^0_{\mu 3}=0$; $\eta_{\bar{z}}=-1$; $\zeta=0$;

$$\text{for}\quad \lambda=3\quad E^0_{33}=\frac{J-J_p}{2}+mc^2;$$

$$\text{for}\quad \lambda=4\quad E^0_{44}=\frac{J-J_p}{2}-mc^2.$$

The vector $\mathbf{b}$ is oriented in arbitrary fashion for $\mathbf{p}=0$. For $\mathbf{p}\neq 0$, the energy E^0 and u^0_μ can be found by means of a Lorentz transformation. Also, for $\mathbf{p}\neq 0$, $\mathbf{b}\,||\,\mathbf{p}$ and $\zeta\neq 0$.

If the "nonmeson" mass of the nucleon is small in comparison with the "meson" mass: $mc^2\ll J$, we can effect a transition in Eq. (2) for a single nucleon to the nonrelativistic approximation, assuming that $E=\text{const}+J+E'$ and $E'\ll J$ ($cp\ll J$, $mc^2\ll J$). But there is no necessity for this, since, as we have seen, it is possible to solve Eq. (8), which corresponds to the relativistic Hamiltonian (2), and then assume $p\ll J/c$.

For two and more nucleons, the transition to the nonrelativistic approximation ($E'\ll J$) and then to the case of nucleons at rest, is possible for large separations $|\mathbf{r}_i-\mathbf{r}_j|\gg a$; for small separations $|\mathbf{r}_i-\mathbf{r}_j|\sim a$, such a transition is not possible in principle, as was shown above. Thus, in this case the concept of a static potential of interaction loses its meaning. The general form of the energy of zeroth approximation can be found by taking the mean value of H^0 (5) (in terms of the eigenfunctions ψ^0_{nm}):

$$E^0_{nn}=\sum_i^n[c\,|\,(\vec{\alpha}_i,\ \mathbf{p}_i)\,|_{nn}+|\,\beta_i\,|_{nn}mc^2]$$
$$-\sum_{\alpha,i,j}^{3,n,n}\left\{\frac{1}{2}\int|\,G_{\alpha i}(\mathbf{r})\,|_{nn}\,|\,G_{\alpha j}(\mathbf{r}')\,|_{nn}G(\mathbf{r},\mathbf{r}')\,d\mathbf{r}\,d\mathbf{r}'\right.$$
$$\left.+2\pi c^2\int(|\,F_{\alpha i}\,|_{nn}\,|\,F_{\alpha j}\,|_{nn}-|\,F_{\alpha i}F_{\alpha j}\,|_{nn})\,d\mathbf{r}\right\}.$$

Here, in addition to an interaction with effective radius κ^{-1} (due to terms which contain $F_{\alpha_i}F_{\alpha_j}$, we also get forces between nucleons with a radius of action $\sim a$ [this is seen directly from Eq. (4)].

3. Up to the present, we have been assuming the nucleons to be extended, and have not considered the presence of a nucleonic vacuum which, as is known, leads to an additional interaction of nucleons among themselves, of mesons among themselves and of nucleons with mesons.

A systematic consideration of the vacuum is possible only in a relativistically invariant theory, i.e., for the transition from extended nucleons to point nucleons. Therefore, in the following we shall assume that the transition to the limit $a\to 0$ [$U(\mathbf{r}-\mathbf{r}_i)\to\delta(\mathbf{r}-\mathbf{r}_i)$] is carried out. In this case, the eigenvalue of the energy of the nucleon which enters into the energy of zeroth approximation is divergent, and there arises the problem of renormalization, which will be considered elsewhere. It must be observed that a transition to zero nuclear radii, without consideration of the vacuum, gives reasonable results in the case of nongradient coupling. Actually, as was shown in reference 2, Eq. (9), for an infinitely extended nucleon interacting with the scalar field, the scattering cross section of the meson with the nucleon is, at $a=0$, $d\sigma=(\kappa^2+k^2)^{-1}d\Omega$; in the case of gradient coupling, the cross section tends to zero as a approaches zero in the limit. However, it is necessary to take account of the nucleonic vacuum in a systematic relativistic theory, in addition to the transition to point nucleons.

The effect of the vacuum, without application of second quantization to the nucleons was estimated in reference 5. It is more convenient to make use of the method of second quantization.

In this case the Hamiltonian has the form (in charge symmetric form; see reference 7):

$$H=\int\psi^+(c(\vec{\alpha},\ \mathbf{p})+\beta mc^2)\psi\,d\mathbf{r} \tag{13}$$
$$+\frac{1}{2}\int\sum_{\alpha}^{3}\Big\{\Pi_\alpha^2+c^2(\nabla\Phi_\alpha)^2+c^2\varkappa^2\Phi_\alpha^2$$
$$-2c\sqrt{\pi}\int[\bar{\psi}(\mathbf{r}'),$$
$$\beta(G_\alpha\Phi_\alpha(\mathbf{r})+F_\alpha\Pi_\alpha(\mathbf{r}))\psi(\mathbf{r}')]\,d\mathbf{r}',$$
$$+c^2\pi\Big(\int[\bar{\psi}(\mathbf{r}'),\ \beta F_\alpha\psi(\mathbf{r}')]\,d\mathbf{r}'\Big)^2\Big\}d\mathbf{r},$$

$U(\mathbf{r}-\mathbf{r}')$ enters in G_α and F_α [see Eq. (2)]:

$$\bar{\psi}=\psi^+\beta;\quad \psi(\mathbf{r})\psi^+(\mathbf{r}')+\psi^+(\mathbf{r}')\psi(\mathbf{r})=\delta(\mathbf{r}-\mathbf{r}').$$

The Hamiltonian (13) is obtained from the Hamiltonian (2) in the usual way by a transition from the method of configuration space to the method of second quantization. The Ψ function of the state now depends on the occupation number.

We assume, as earlier, that $\Phi_\alpha=\varphi_\alpha^0+\varphi_\alpha$; $\Pi_\alpha=\pi_\alpha^0+\pi_\alpha$. Then $H=H^0+H^{(1)}+H^{(2)}$:

$$H^0=H_f^0+\int\psi^+(c(\vec{\alpha},\ \mathbf{p})+\beta mc^2)\psi\,d\mathbf{r} \tag{14}$$
$$-c\sqrt{\pi}\int\sum_{\alpha}^{3}[\bar{\psi}(\mathbf{r}'),$$
$$\beta(G_\alpha\varphi_\alpha^0(\mathbf{r})+F_\alpha\pi_\alpha^0(\mathbf{r}))\psi(\mathbf{r}')]\,d\mathbf{r}\,d\mathbf{r}';$$
$$H_f^0=\frac{1}{2}\int\sum_{\alpha}\Big\{(\pi_\alpha^0)^2+c^2(\nabla\varphi_\alpha^0)^2+c^2\varkappa^2(\varphi_\alpha^0)^2$$
$$+\pi c^2\Big(\int[\bar{\psi}(\mathbf{r}'),\ \beta F_\alpha\psi(\mathbf{r}')]\,d\mathbf{r}'\Big)^2\Big\}d\mathbf{r};$$
$$H^{(1)}=\int\sum_{\alpha}\Big\{\pi_\alpha^0\pi_\alpha+c^2\varphi_\alpha(\varkappa^2-\Delta)\varphi_\alpha^0$$
$$-c\sqrt{\pi}\int[\bar{\psi}(\mathbf{r}'),\beta(G_\alpha\varphi_\alpha+F_\alpha\pi_\alpha)\psi(\mathbf{r}')]\,d\mathbf{r}'\Big\}d\mathbf{r};$$
$$H^{(2)}={}^1/_2\int\sum_{\alpha}[\pi_\alpha^2+c^2\varphi_\alpha(\varkappa^2-\Delta)\varphi_\alpha]\,d\mathbf{r}.$$

As was shown in Sec. 1, and in reference 1, Eq. (12), the zero field φ_α^0, π_α^0 must be found from the condition $\langle H^{(1)}\rangle=0$ (if degeneracy is absent; see above), i.e., in case of Eq. (13):

$$(\varkappa^2-\Delta)\varphi_\alpha^0=(\sqrt{\pi}/c)\int\langle[\bar{\psi}',\ \beta G_\alpha\psi']\rangle\,d\mathbf{r}'; \tag{15}$$
$$\pi_\alpha^0=c\sqrt{\pi}\int\langle[\bar{\psi}',\ \beta F_\alpha\psi']\rangle\,d\mathbf{r}';$$
$$\psi'=\psi(\mathbf{r}');$$

Here $\langle[\bar{\psi},\ \beta G_\alpha\psi]\rangle$ is the mean value (diagonal matrix element) of $[\bar{\psi},\ \beta G_\alpha\psi]$ as a function of the state, which depends on the occupation number. It is not difficult to show that the condition $\langle H^{(1)}\rangle=0$ will be satisfied in this case also, if the averaging in the equations for φ_α^0, π_α^0 is carried out only over the spin variables:

$$(\varkappa^2-\Delta)\varphi_\alpha^0=(\sqrt{\pi}/c)\int([\bar{\psi}',\ \beta G_\alpha\psi'])^s\,d\mathbf{r}'; \tag{16}$$
$$\pi_\alpha^0=c\sqrt{\pi}\int([\bar{\psi}',\ \beta F_\alpha\psi'])^s\,d\mathbf{r}'.$$

If ψ_m and ϵ_m are eigenfunctions and eigenvalues of the equation

$$\Big[c(\vec{\alpha},\ \mathbf{p})+\beta mc^2-c\sqrt{\pi}\int\langle[\bar{\psi}',\ \beta(G_\alpha\varphi_\alpha^0 \tag{17}$$
$$+F_\alpha\pi_\alpha^0)\psi']\rangle\,d\mathbf{r}'\Big]\psi_m=\varepsilon_m\psi_m$$

with φ_α^0 and π_α^0 from Eq. (15), then, substituting the expansion $\psi=\sum_m(a_m\psi_m^{(+)}+b_m^+\psi_m^{(-)})$ in H^0 (14) (see reference 7), we evidently obtain:

$$H^0=\sum_m(a_m^+a_m\varepsilon_m-b_mb_m^+\varepsilon_m).$$

Here we find from Eq. (15)

$$(\varkappa^2-\Delta)\varphi_\alpha^0=(j_\alpha)_{\text{vac}} \tag{18}$$
$$+\frac{\sqrt{4\pi}}{c}\sum_m\Big\{n_m^+\int(\bar{\psi}_m^{(+)\prime}\beta G_\alpha\psi_m^{+\prime})\,d\mathbf{r}'$$
$$+n_m^-\int(\bar{\psi}_m^{(-)\prime}\beta G_\alpha\psi_m^{(-)\prime})\,d\mathbf{r}'\Big\}$$

(with an analogous equation for π_α^0): $n_m^+=a_m^+a_m$; $n_m^-=b_m^+b_m$. The mean value of the current in the vacuum $(j_\alpha)_{\text{vac}}$, as is known, is given by[8]

$$(j_\alpha)_{\text{vac}}=\frac{\sqrt{\pi}}{c}\,\mathrm{sp}\langle[\bar{\psi},\ \beta\bar{G}_\alpha\psi]\rangle_{\text{vac}};$$

In the equation for $(j_\alpha)_{\text{vac}}$ we have undergone a transition from $U(\mathbf{r}-\mathbf{r}')$ to the δ-function $\delta(\mathbf{r}-\mathbf{r}')$, since such a transition, as is well

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[7] A. I. Akhiezer and V. B. Berestetskii, *Quantum Electrodynamics*, GTTI, 1953, pp. 150, 130, 442, 126.

[8] J. Schwinger, Phys. Rev. 82, 664 (1952).

known, does not lead to additional divergences (sp denotes the diagonal sum over the spin indices $G_\alpha = U(\mathbf{r}-\mathbf{r}')\,\overline{G}_\alpha$).

Since $<[\overline{\psi},\ \beta G_\alpha \psi]>^S$ has a complicated form if the method of second quantization is used, then it is expedient in the case of Eq. (16) to return to the method of configuration space:

$$(\kappa^2 - \Delta)\,\varphi_\alpha^0 = (j_\alpha)_{\text{vac}} + \frac{\sqrt{4\pi}}{c}\sum_i^n <G_{\alpha i}>^S \qquad (19)$$

(and analogous equation for π_α^0):

$(j_\alpha)_{\text{vac}}$ has the same form as in (18), and $<G_{\alpha i}>^S$ as in (4).

It is not difficult to show that the interaction of the nucleons (and anti-nucleons) in Eq. (18) is substituted for the self-consistent field, in contrast to Eqs. (16) and (19). This is explained by the fact that the operators ψ^+, ψ, as well as the spin operators τ_α, α_k, β enter into H^0. Therefore, the averaging in the equations for φ_α^0, π_α^0 in the case of Eq. (15) is carried out not only over the spin variables, but also over the occupation numbers. In Eqs. (16) and (19), the averaging is carried out only over the spin variables; they entirely correspond to Eqs. (4). If the system consists of a large number of nucleons (for example, the nucleons in the nucleus), it is expedient to use Eqs. (15) for φ_α^0, π_α^0. In this case, however, we must add the usual condition for the applicability of the self-consistent field to the condition of applicability of strong coupling, given in Sec. 1 and in reference 1. In our problem we can obtain this condition by comparing the correction for the Ψ-functions with the Ψ-functions of zero approximation.

The perturbation theory for second quantization of the Hamiltonian $H = H^{(0)} + H^{(1)} + H^{(2)}$ is entirely similar to the perturbation theory developed in Sec. 1 and in reference 1. Expressing the complete Ψ-function of the system in terms of the eigenfunctions of the Hamiltonian H^0:

$$\Psi_n = \sum_l [\chi_n^0(\varphi_\alpha)\,\delta_{nl} + \chi'_{nl}(\varphi_\alpha)]\ \Psi_{nl}^0(n_m),$$

we obtain an equation for χ_n^0 which is similar to Eq. (7). It is evident that the anharmonicity of the field of real mesons φ_α is connected with the presence of the vacuum as well as with the absence of the vacuum, and appears only in the second approximation $1/g^2$. Consequently, the anharmonicity of φ_α is small in the case of strong coupling just as in the case of weak coupling[5].

We consider the zeroth approximation of perturbation theory for one nucleon which is located at a positive level. In this case the field φ_α^0, π_α^0 is defined by the Eqs. (19).

The eigenfunctions of the Hamiltonian H^0 can easily be found, since Eq. (17) for the function ψ_m, by which we can expand the second quantized functions ψ and ψ^+, depends on the operators $\mathbf{p}$, $\mathbf{r}$ and the spin operators τ_α, α_k, β just as Eqs. (5) and (8) in the absence of the vacuum. Therefore, diagonalization of (17) and the spin variables is obtained in the same way as the diagonalization of Eq. (8) in Sec. 1, and the solution has the form $u_\mu e^{i\,\mathbf{k}\cdot\mathbf{r}}$. However, inasmuch as the equation for the field φ_α^0, π_α^0, because of the term with $(j_\alpha)_{\text{vac}}$ have some other form than in the absence of the vacuum, then the integrals analogous to J_S and J_p are also proportional to g^2; hence they will diverge for $a \to 0$ differently than was shown in reference 1, where the vacuum was not considered. If we introduce the corresponding integrals in the form of numerical parameters, ignoring the character of their divergence, then it is not difficult to carry out calculations in the first and second approximation of perturbation theory, i.e., to compute the scattering cross section of the mesons with nucleons and the interaction of the mesons among themselves. This problem will be considered in another work. It should be noted that the greatest difficulty is presented by the calculation of the interaction forces between nucleons (in zeroth approximation of perturbation theory). In order to estimate the internuclear forces, we must solve Eq. (19) for $n \neq 1$ and find the form of the function $\varphi_\alpha^0(\mathbf{r})$.

We now return to Eq. (19) for a single nucleon. The average current in the vacuum can be expressed by the Green's function of the Dirac equation[8]

$$G(x, x') = i\,(c\hbar)^{-1}\,\langle T\,(\psi(x)\,\overline{\psi}(x'))\rangle; \qquad x = \{\mathbf{r}, ct\};$$

T is the symbol of the T-product[7]:

$$[c\gamma p + mc^2 - c\sqrt{4\pi}\beta(\bar{G}_\alpha\Phi_\alpha + \bar{F}_\alpha \Pi_\alpha)]\, G(x, x') = \delta(x - x'); \tag{20}$$

$$\mathrm{sp}\, \tfrac{1}{2}\langle[\bar{\psi}, \beta\bar{G}_\alpha \psi]\rangle_{\mathrm{vac}} = ic\hbar\, \mathrm{sp}\, [\beta\bar{G}_\alpha G(x, x')]_{x'\to x};$$

$$\gamma p = \sum^4 \gamma_k p_k;$$

$$\gamma_k = \beta\alpha_k;\ \gamma_4 = i\beta;$$

$$p_4 = -\frac{\hbar}{c}\frac{\partial}{\partial t}.$$

Schwinger[8] has pointed out the method for the computation of the mean current in the vacuum, i.e., the Green's function of the Dirac equation without application of the theory of perturbation for the case of a constant electromagnetic field. This method was used by Malenka[9] in the case of a constant neutral meson field with pseudoscalar coupling in the presence of a given current. We now attempt to take into account the change of the field Φ_α in space. By way of an example, we consider a charged field with pseudoscalar coupling. In this case the equation $\delta H_1/\delta\varphi_\alpha^0 = 0$ (see Sec. 1) for φ_α^0 has the same form as Eq. (19), but the current of the nucleon $j_{\alpha n}$, in view of the presence of degeneracy, will not equal $\frac{2\sqrt{\pi}}{c}\langle G_\alpha\rangle^S$. We first find $G(x, x')$:

$$G(x, x') = c^{-1}(\gamma p + mc + ig'\gamma_5 \sum \tau_\alpha\Phi_\alpha)^{-1}\delta(x - x')$$
$$= c^{-1}(mc - \gamma p - ig'\gamma_5 \sum \tau_\alpha \Phi_\alpha)(m^2c^2 + p^2 + \hbar g'\gamma_5 \sum \gamma_k\tau_\alpha\partial_k\Phi_\alpha + (g')^2 \sum \Phi_\alpha^2)^{-1}\delta(x - x');$$

$$\partial_k\Phi_\alpha \equiv \frac{\partial\Phi_\alpha}{\partial x_k};\qquad g' = g\sqrt{4\pi};\quad \gamma_5^2 = 1.$$

[9] B. Malenka, Phys. Rev. 85, 686 (1952).

Expanding $\hbar\gamma_5 g'\Sigma\gamma_k\tau_\alpha\partial_k\Phi_\alpha(m^2c^2 + p^2 + (g')^2 \times \Sigma\Phi_\alpha^2)^{-1}$ in a power series and noting that $(AB)^{-1} = B^{-1}A^{-1}$, we find

$$G(x, x') = c^{-1}(mc - \gamma p - ig'\gamma_5\Sigma\tau_\alpha\Phi_\alpha)\sum_{n=0}^{\infty}(a^{-1}b)^n a^{-1}\delta(x - x'); \tag{21}$$

$$a = m^2c^2 + p^2 + (g')^2\,\Sigma\Phi_\alpha^2;$$

$$b = -\hbar g'\gamma_5\sum\gamma_k\tau_\alpha\partial_k\Phi_\alpha;$$

$$j_\alpha(x, x') = i\hbar\sqrt{4\pi}\,\mathrm{sp}\,[\bar{\beta}G_\alpha G(x, x')] = \frac{g'\hbar}{c}\mathrm{sp}\left[\gamma_5\tau_\alpha(mc - \gamma p - ig'\gamma_5\,\Sigma\tau_\beta\Phi_\beta)\sum_n(a^{-1}b)^n a^{-1}\delta(x - x')\right]. \tag{22}$$

In the calculation of the spur in Eq. (22), after multiplication by $\gamma_5\tau_\alpha$ all terms of the row are zero, after multiplication by $\tau_\alpha\tau_\beta$, the terms of the row with odd n are zero, and after multiplication by $\tau_\alpha\gamma_5\gamma_k$ the terms with even n are zero. Therefore, we get from Eq. (22):

$$j_\alpha(x, x') = \frac{8}{i}\frac{(g')^2\hbar}{c}\Big[\Phi_\alpha + i\hbar\sum_k p_k(m^2c^2 + p^2 + \sum(g')^2\Phi_\alpha^2)^{-1}\partial_k\Phi_\alpha\Big] \times\Big[p^2 + m^2c^2 + (g')^2\sum\Phi_\beta^2 + \hbar^2(g')^2\sum_{k,\beta}\partial_k\Phi_\beta(p^2 + m^2c^2 + (g')^2\sum\Phi_\beta^2)^{-1}\partial_k\Phi_\beta\Big]^{-1} \times \delta(x - x'). \tag{23}$$

If the field Φ_α is changed slowly, then:

$$j_\alpha(x,x') \simeq f_0 a^{-1} + f_1 a^{-1} + f_0 a^{-1}\delta a^{-1}; \qquad (24)$$

$$f_0 = -i\,8\,(g')^2\hbar c^{-1}\Phi_\alpha;$$

$$f_1 = 8\,(g')^2\hbar^2 c^{-1}\Sigma p_k a^{-1}\partial_k\Phi_\alpha;$$

$$\delta = -\hbar^2 (g')^2 \sum_{k,\beta} \partial_k \Phi_\beta a^{-1}\partial_k\Phi_\beta.$$

Using the identity

$$A^{-1} = \frac{1}{2i}\int_0^\infty (e^{iAs} - e^{-iAs})\,ds$$

(see, for example, reference 7), it is possible to write Eq. (23) in the following form:

$$j_\alpha(x,x') = \frac{8(g')^2\hbar}{ic}\Big\{\Phi_\alpha + i\hbar\,\Sigma p_k \int_0^\infty \sin\,[(p^2 + m^2c^2 + (g')^2\sum_\beta \Phi_\beta^2)\,u]\,du\,\partial_k\Phi_\alpha\Big\}\int_0^\infty \sin\Big\{\Big[p^2 + m^2c^2 + (g')^2\sum_\beta\Phi_\beta^2 + \hbar^2(g')^2\sum_{k,\beta}\partial_k\Phi_\beta\int_0^\infty \sin\Big(\Big(p^2+m^2c^2 + (g')^2\sum_\beta\Phi_\beta^2\Big)v\Big)dv\,\partial_k\Phi_\beta\Big]s\Big\}\,ds\,\delta(x-x').$$

If A and B are noncommutating operators, then we have, with accuracy to terms of first order in $\epsilon = AB - BA$[10]:

$$e^{A+B} \approx e^A e^B + e^A e^B \varepsilon/2. \qquad (25)$$

For a slowly changing field, we can consider $[p^2, \Sigma\Phi_\beta^2]$ to be a small quantity. Then, using Eq. (25), and expressing $\delta(x-x')$ in the form of a Fourier integral, we obtain the zeroth term in Eq. (24), corresponding to a constant field Φ_α:

$$j_\alpha^0(x,x') = -\frac{4(g')^2\hbar\Phi_\alpha}{c(2\pi)^4}\int_0^\infty [\exp\{is(m^2c^2 + (g')^2\Sigma\Phi_\beta^2\}\,e^{isp^2} - \exp\{is(m^2c^2 + (g')^2\Sigma\Phi_\beta^2)\}\,e^{-isp^2}]\,ds\int_{-\infty}^{\infty}\exp \times \Big\{i\sum^4 k_i(x_i - x_i')\Big\}\,d^4k.$$

Evidently,

$$\exp(is\hat{p}^2)\exp\Big(i\sum^4 k_i(x_i-x_i')\Big) = \exp\Big[-i\hbar^2\Big(\sum_i^3 k_i^2 - k_4^2\Big)s + i\sum^4 k_i(x_i - x_i')\Big].$$

Integrating over d^4k, we find:

$$j_\alpha^0(x,x') = \frac{(g')^2\Phi_\alpha}{c\hbar^3\,2\pi^2}\int_0^\infty \sin\Big\{\Big[m^2c^2 + (g')^2\Sigma\Phi_\beta^2 + \frac{i}{4s\hbar^2}\Big((\mathbf{r}-\mathbf{r}')^2 - c^2(t-t')^2\Big)\Big]s\Big\}\frac{ds}{s^2};$$

$$(j_\alpha(x))_{vac} = j_\alpha(x,x')_{x=x'}.$$

For $x = x'$, the integral over s is easily computed with the help of differentiation with respect to the parameter (the small quantity s_0 must be substituted for the lower limit);

$$j_\alpha^0(x) = \frac{(g')^2\Phi_\alpha}{2c\hbar^3\pi^2}\Big(m^2c^2 + (g')^2\sum_\beta\Phi_\beta^2\Big)\Big[1 - \gamma - \ln\Big(s_0\,(m^2c^2 + (g')^2\sum_\beta\Phi_\beta^2)\Big)\Big]; \qquad (26)$$

$$\gamma = 1{,}78.$$

In the case of the field φ_α^0 we must assume $\Phi_\alpha = \varphi_\alpha^0$ in Eq. (26). We see that a logarithmic divergence appears in Eq. (26). The other terms in Eq. (24) can be computed in a similar manner. If we limit ourselves to second derivatives of φ_α^0 (which are necessary for the renormalization of the meson mass), then only in the first member of Eq. (24) -- $f_0 a^{-1}$ -- must one consider the second component

[10] R. Peierls, Z. Physik **80**, 763 (1933).

of Eq. (25) in the expression for a^{-1}; in the remaining members of Eq. (24), one must keep only the first component of Eq. (25). This method of obtaining the mean current in a vacuum can easily be applied to the case of pseudovector coupling, when φ^0_α, π^0_α are defined by Eqs. (19). In the same manner, we find the Green's function $G^0(x, x')$ which corresponds to the constant field Π_α, Φ_α:

$$G^0(x, x') = \frac{1}{(2\pi)^4 c}(mc - \gamma p + f\beta\gamma_5 \Sigma \tau_\beta \Pi_\beta) \int_0^\infty \sin \times \left[\left(m^2c^2 + f^2\Sigma\Pi_\beta^2 + \frac{i}{4s\hbar^2}(x - x')^2\right)s\right]\frac{ds}{s^2};$$

$$f = g\sqrt{4\pi}/(\varkappa c).$$

We can then find the mean current in the vacuum for the momentum $\Pi_\alpha - g_\alpha$:

$$g_\alpha(x) \equiv c\sqrt{\pi}\,\mathrm{sp}\,\langle[\bar\psi, \beta\bar F_\alpha \psi]\rangle_{\mathrm{vac}} = \frac{f^2 c \Pi_\alpha}{2\pi^2\hbar^3}(m^2c^2 + f^2\Sigma\Pi_\beta^2) \times [1 - \gamma - \ln(s_0(m^2c^2 + f^2\Sigma\Pi_\beta^2))]. \quad (27)$$

Close to the nucleon, it is not possible to consider the field φ^0_α, π^0_α to be changing slowly. Therefore, for the investigation of the behavior of φ^0_α, π^0_α in the neighborhood of $\mathbf{r}_1 (\mathbf{r} - \mathbf{r}_1 \to 0)$, it is appropriate in the exact equation for $j_\alpha(x, x')$ (23) and $g_\alpha(x, x')$ or for $G(x, x')$ (21) to substitute the solution of the equation of the field (19) (or $\delta H_1/\delta\varphi^0_\alpha = \delta H_1/\delta\pi^0_\alpha = 0$; see above) in the form of a functional of $j_\alpha(x)$ and $g_\alpha(x)$.

For the computation and later renormalization of the mass of the nucleon, the equations for π^0_α and φ^0_α must be solved. Renormalization of the meson charge of the nucleon can be carried out[8,9] on the basis of the expressions for $j_\alpha(x)$ and $g_\alpha(x)$.

Translated by R. T. Beyer
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