of increase of magnetic field in interstellar medium that the interstellar magnetic field will be sufficiently homogeneous in regions of dimensions of the pulsation L_k, which have a kinetic energy density equal to the magnetic energy density of the medium[2-4]. Charged particles moving in an interstellar magnetic field will go from one homogeneous region to another. Because of the turbulent character of the magnetic field, it can be assumed that the directions of the homogeneous regions of the magnetic field are distributed randomly.

We want to examine the dependence of the diffusion coefficient on the particle energy. If the energy is such that the mean radius of curvature of the particle trajectory is much smaller than L_k, then, due to the chaotic structure of the magnetic field, the particle will be moving randomly. For the mean free path we can take the mean dimension of the homogeneous regions of the magnetic field. In this energy interval, we can therefore consider the diffusion coefficient as being constant and equal to

$$D \approx cL_k. \tag{1}$$

Consider now the case when the particle energy is such that the mean radius of curvature of the trajectory of the particle in the magnetic field is much larger than the dimension of the homogeneous regions of the magnetic field. The scattering will be mostly in the forward direction, and we have first to evaluate the transport mean free path of the particle in the interstellar magnetized medium. It is known[5,6] that the transport mean free path is the mean distance travelled by the particle after it has passed through an infinite number of randomly distributed homogeneous regions of the magnetic field:

$$l = L_k(1 + \overline{\cos\theta_k} + \overline{\cos\theta_k^2} + \ldots) = L_k/(1 - \overline{\cos\theta_k}), \tag{2}$$

where θ_k is the scattering angle due to one homogeneous region of the magnetic field.

The radius of curvature of the trajectory of a particle with momentum P in a magnetic field H is equal to

$$R_h = cp/eH\sin\theta, \tag{3}$$

where θ is the angle between the momentum and the magnetic field. The mean dimension of a homogeneous region of the magnetic field is L_k; hence,

$$\overline{\theta_k^2} \simeq \frac{L_k^2 e^2 H^2 \overline{\sin^2\theta}}{c^2 p^2}, \tag{4}$$

and

$$l = \frac{L_k}{1 - \overline{\cos\theta_k}} \simeq \frac{2L_k}{\overline{\theta_k^2}} = \frac{2c^2p^2}{L_k e^2 H^2 \overline{\sin^2\theta}}; \tag{5}$$

but for extreme relativistic energies $p \sim E/c$, and

$$l \simeq E^2/L_k e^2 H^2, \tag{6}$$

therefore, for high energies the diffusion coefficient will be:

$$D \approx cE^2/L_k e^2 H^2. \tag{7}$$

If one assumes that the dependence of the diffusion coefficient on the energy is monotonic, it is easy to determine the dependence for intermediate energies. It is clear that for some energy interval in the intermediate region, this dependence may be considered as linear. This is to some extent a complementary argument for our assumption of linear dependence for the diffusion coefficient.

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Determination of the Dielectric Constant of Superconductors

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ABRIKOSOV[1] has obtained formulas for the determination of the dielectric constant of superconductors, taking into account its abnormally large value (see, for example, Ginzburg[2]). However, the question of the existence of an anomalously large polarizability in superconductors has not yet received final settlement, since the corresponding calculations, carried out on the results of the measurements by Galkin[3] at a frequency of $\omega = 2.8 \times 10^{11}$ sec^{-1}, have not indicated any change in the sign of the dielectric constant $\epsilon = \epsilon_0 - (c^2/\omega^2\delta_0^2)$ (δ_0 is the penetration depth of the static magnetic field in the superconductor). Galkin and Kaganov[4] have attributed this to the weak dependence of δ_0 on the frequency.

It is shown in the present work that the results of Galkin's experiments[3] can be understood if we take into account the anisotropy of superconductors. As is seen below, consideration of the anisotropy makes an essential change in the expression for the surface impedance of superconductors.

In this investigation we consider the surface impedance of the superconducting metal for an arbitrary dispersion law for the normal electrons $\epsilon = \epsilon(\mathbf{p})$ (ϵ and $\mathbf{p}$ are their energy and quasimomentum). The complete set of equations in this case has the form

$$E''_\alpha(z) + \frac{\omega^2\varepsilon_{\alpha k}}{c^2}E_k = \frac{4\pi i\omega}{c^2}j_\alpha \quad (\alpha = x, y); \tag{1}$$

$$(\varepsilon_{\alpha k}E_k = \varepsilon_{\alpha x}E_x + \varepsilon_{\alpha y}E_y + \varepsilon_{\alpha z}E_z); \tag{2}$$

$$j_z = \frac{\omega}{4\pi i}D_z;$$

$$\mathbf{j}e^{i\omega t} = \frac{2e}{(2\pi\hbar)^3}\int \mathbf{v}f\,dp_x dp_y dp_z; \tag{3}$$

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial z}v_z + \frac{\partial f}{\partial \mathbf{p}}e\mathbf{E}e^{i\omega t} + \frac{f-\bar f}{\tau} = 0; \tag{4}$$

$$\varepsilon_{ik} = \varepsilon^0_{ik} - \frac{c^2}{\omega^2}(\delta^{-2})_{ik} = \varepsilon^0_{ik} - \frac{(\omega_0^2)_{ik}}{\omega^2}; \tag{5}$$

$$\bar f = \int_{\varepsilon(\mathbf{p})=\varepsilon} f\frac{dS}{v} \Big/ \int_{\varepsilon(\mathbf{p})=\varepsilon}\frac{dS}{v}; \tag{6}$$

$$f\Big|_{\substack{z=0\\ v_z>0}} = (1-q)f_0 + qf\Big|_{\substack{z=0\\ -v_z}} \tag{7}$$

Here ϵ_{ik} is the dielectric constant tensor, δ_{ik} the tensor of the penetration depth of a static magnetic field in a superconductor, τ the relaxation time of the normal electrons at a given temperature, $\mathbf{v}$ the velocity of the electron, ω the frequency of the electromagnetic field, ϵ_0 is the limiting energy, f the distribution function of the electrons, $\mathbf{E}$ the variable electric field intensity, $\mathbf{j}$ is the current density, q the reflection coefficient of the electrons from the surface; the axis Oz coincides with the inwardly drawn normal to the surface of the metal.

We linearize the kinetic equation (4) by setting

$$f = f_0 - e\tau^*\frac{\partial f_0}{\partial \varepsilon}e^{i\omega t}\psi; \quad \tau^* = \frac{\tau}{1+i\omega t};$$

[$f_0(\epsilon)$ is the equilibrium Fermi distribution function]. Taking into account that $D_i = \epsilon^0_{ik}E_k$ and

$$\int \psi dS = \frac{(2\pi\hbar)^3}{2e^2\tau^*}\rho' = \pi^2\hbar^3\varepsilon^0_{zk}E'_k(z) \Big/ e^2\tau \int_{\varepsilon(\mathbf{p})=\varepsilon_0}\frac{dS}{v},$$

we get from Eqs. (1)-(7):

$$(1-\delta_{i3})E''_i(z) + \frac{\omega^2\varepsilon_{ik}}{c^2}E_k(z) \tag{8}$$

$$= \frac{3i}{l\delta^2}\Big\{\int_{-\infty}^{\infty}K_{ik}(z-\mu)E_k(\mu)\,d\mu + \int_{-\infty}^{\infty}Q_{ik}(z-\mu)E'_k(\mu)\,d\mu$$

$$-[1-(-1)^{\delta_{k3}}q]\int_0^\infty P_{ik}(|z|+\mu)E_k(\mu)\,d\mu$$

$$+(1+q)\int_0^\infty R_{ik}(|z|+\mu)E'_k(\mu)\,d\mu\Big\};$$

$$\delta_{ik} = \begin{cases}1 & (i=k)\\ 0 & (i\neq k),\end{cases}$$

where

$$K_{ik}(\omega) = \frac{4}{S}\int_{n_z\geqslant 0}\frac{n_i n_k}{n_z}\exp\left\{-\frac{|\omega|}{e^* n_z}\right\}dS;$$

$$Q_{ik}(\omega) = \operatorname{sign}\omega\,\frac{4}{S}\int_{n_z\geqslant 0}\frac{n_i a_k}{n_z}\exp\left\{-\frac{|\omega|}{e^* n_z}\right\}dS;$$

$$P_{ik}(\omega) = \frac{4}{S}\int_{n_z\geqslant 0}\frac{n_i n_k}{n_z}\exp\left\{-\frac{\omega}{e^* n_z}\right\}dS;$$

$$R_{ik}(\omega) = \frac{4}{S}\int_{n_z\geq 0}\frac{n_i a_k}{n_z}\exp\left\{-\frac{\omega}{e^* n_z}\right\}dS;$$

$$l\delta^2 = 3(2\pi\hbar)^3c^2/4\pi\omega e^2 S;$$

$$a_k = \pi^2\hbar^3\epsilon^0_{zk} \mid e^2 l\int\frac{dS}{v}; \; l = v\tau.$$

In Eq. (8) the field function remains even in the region outside the metal: $\mathbf{E}(-z) = \mathbf{E}(z)$.

Transferring from the equations for the function $\mathbf{E}_i(z)$ to the equations for its Fourier transform, we find that in the zeroth approximation, for $\delta/|l^*| \ll 1$, Eq. (8) can be written in the form:

$$E''_\alpha(z) + (\omega^2\epsilon^{\text{eff}}_{\alpha\beta}/c^2)E_\beta(z) = (3i/l\delta^2) \tag{9}$$

$$\times\int_{-\infty}^{\infty}K_{\alpha\beta}(z-\mu)E_\beta(\mu)\,d\mu \quad (\alpha, \beta = x, y),$$

where

$$\epsilon^{eff}_{\alpha\beta} = \epsilon_{\alpha\beta} + \frac{1}{2} i S_\alpha \frac{\omega^{0\,2}_{z\beta}}{\omega^2} \frac{1 - (\omega^{0\,2}_{zz}/\omega^{0\,2}_{z\beta})(\epsilon^0_{z\beta}/\epsilon^0_{zz})}{\omega\tau(\epsilon_{zz}/\epsilon^0_{zz}) + 4i}; \quad (10)$$

$$S_\alpha = \frac{8}{\overline{v}} \frac{1}{\int \frac{dS}{v}} \int_0^{2\pi} N_\alpha \, d\varphi \int_0^{\pi/2} tg\,\theta \left\{ \frac{\overline{v}}{v(\varphi, \theta)} - \frac{\overline{v}}{v\left(\varphi, \frac{\pi}{2}\right)} \right\} d\theta; \quad \overline{v} = \frac{1}{S} \int v\, dS;$$

$$\mathbf{v} = (v \sin\theta \cos\varphi,\ v \sin\theta \sin\varphi,\ v\cos\theta);$$

$$N_\alpha = \begin{cases} \cos\varphi\ (\alpha = x) \\ \sin\varphi\ (\alpha = y). \end{cases} \quad (11)$$

The integration in Eq. (11) is carried out over the Fermi surface $\epsilon(\mathbf{p}) = \epsilon_0$.

In the simplest case of perfect reflection ($q=1$) of the electrons from the metal surface the tensor of surface impedance is

$$Z_{\alpha\beta} = -\frac{4\pi i\omega}{c^2} \frac{\partial E_\alpha(0)}{\partial E'_\beta(0)} \frac{8 i\omega}{c^2 (l\delta^2)^{4/3}} \quad (12)$$

$$\times \int_0^\infty \frac{t\,dt}{t^3 \delta_{\alpha\beta} + 3ik_{\alpha\beta} - (\omega^2 \epsilon^{eff}_{\beta\alpha}/c^2)(l\delta^2)^{2/3}};$$

$$k_{\alpha\beta} = \frac{8\pi}{S} \int_0^\pi \frac{M_\alpha N_\beta \, d\varphi}{K\left(\varphi, \frac{\pi}{2}\right)}$$

[$K(\phi, \theta)$ is the Gaussian curve of the Fermi surface].

It is seen from Eq. (10) that the effective dielectric constant is a complex quantity. Hence, it is not sufficient to know X and R for the measurement of ϵ^0 and the determination of the sign of ϵ. We note that the direction of the principal axes of the surface impedance tensor $z_{\alpha\beta}$ depends on the frequency.

These results [Eqs. (10)-(12)] apply not only to single crystals but also to polycrystals with sufficiently large crystal dimensions.

In conclusion, I consider it my pleasant duty to express my thanks to I. M. Lifshitz for his discussions of the results of the research.

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The Role of Spin in the Study of the Radiating Electron

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IN his recently published paper, Nelipa[1] claims that the ratio of the magnitude of the integrated radiation of the electron to the magnitude of the integrated radiation of a spinless particle is equal to $1 + (mc^2/E)^2$. Our calculations[2], which take into account quantum corrections of all orders, show that this is not so.

We obtained the following formulas, which may be applied to the radiation of all the spectrum, for arbitrary energies of the electron or of a spinless particle.

$$dW^{(1/2)} = \frac{ce^2}{\pi\sqrt{3}} \left(\frac{mc}{h}\right)^2 \quad (1a)$$

$$\times\ \xi d\xi \left[\int_{\frac{\xi}{1-\xi}\frac{1}{\zeta}}^{\infty} K_{5/3}(x)\,dx + \frac{\xi^2}{1-\xi} K_{2/3}\left(\frac{\xi}{1-\xi}\frac{1}{\zeta}\right)\right];$$

$$dW^{(0)} = \frac{ce^2}{\pi\sqrt{3}} \left(\frac{mc}{h}\right)^2 \xi d\xi \int_{\frac{\xi}{1-\xi}\frac{1}{\zeta}}^{\infty} K_{5/3}(x)\,dx;$$

$$\xi = \frac{h\omega}{E}, \quad \zeta = \frac{3}{2}\frac{h}{Rmc}\left(\frac{E}{mc^2}\right)^2. \quad (1b)$$

For $h = 0$ these formulas give the classical formula for the differential spectrum:

$$dW = \frac{3\sqrt{3}}{4\pi} \frac{ce^2}{R^2} \left(\frac{E}{mc^2}\right)^4 y\,dy \int_y^\infty K_{5/3}(x)\,dx, \quad (2)$$

$$y = \frac{\omega}{\omega_c}, \quad \omega_c = \frac{3}{2}\frac{c}{R}\left(\frac{E}{mc^2}\right)^3,$$

obtained by Ivanenko and Sokolov[3], and later by Schwinger[4]. If one considers only quantities of first order in h (first quantum correction), one obtains the quantum-theoretical formulas for the differential spectrum obtained by Sokolov and Ternov[5] and Schwinger[6] which are exact to the first order in h.

Formulas for total radiation energy, which are exact for arbitrary energies of the radiating particles, have the following form (see also reference 7):