The Theory of Λ^0 Particles

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A class of equations possessing internal degrees of freedom is investigated from the formal point of view for its possibilities in describing the Λ^0 particle as an excited state of the nucleon.

In his works, I. E. Tamm repeatedly returned to the idea of excited nucleon states[1].

Discoveries of new particles in recent years (hyperons, heavy mesons) make natural further investigations of different possibilities for introducing some kind of internal degrees of freedom for the elementary particles. The discovery of a hyperon heavier than the Λ^0 particle --- the X hyperon --- and the latter's disintegration cascade $X \to \pi^- + \Lambda^0 \to \pi^- + p + \pi^-$ makes plausible, if not conclusive, the hypothesis that the hyperons can be represented as excited states of nucleons.

We shall investigate the possibility of one such hyperon model from this point of view.

One of us has made an attempt to write the equations of an elementary particle with an increasing mass spectrum[2]. One of these equations has the form

$$\left\{ \gamma_\mu k_\mu + m_0 + a\left[-\frac{\partial^2}{\partial\xi_\mu\partial\xi_\mu} + \xi_\mu\xi_\mu + 2\,\frac{\left(k_\mu \frac{\partial}{\partial\xi_\mu}\right)^2 - (k_\mu\xi_\mu)^2}{k_\mu k_\mu}\right]^n + g\gamma_5\tau_i\varphi_i(x+r)\right\}\psi = 0, \quad (1)$$

where $\xi_\mu = r_\mu/r_0$ is the dimensionless internal degree of freedom, r_0 is the length constant, x is the "center of mass" coordinate of the particle, k_μ is its momentum, φ is the pseudoscalar meson field, and n is an integer.

Equation (1) is outside the framework of the class of equations investigated in reference 3.

[1] V. L. Ginzburg and I. E. Tamm, J. Exper. Theoret. Phys. USSR **17**, 227 (1947)

[2] M. A. Markov, Dokl. Akad. Nauk SSSR **101**, 51 (1955)

[3] I. M. Gel'fand and A. Iaglom, J. Exper. Theoret. Phys. USSR **18**, 703 (1948)

We shall regard the Λ^0 particle then as an excited state of the nucleon. We should like to state clearly that our purpose is merely to discuss the possibilities of a class of similar equations, and that this investigation at the present time might have only a formal significance. The selection of a definite form of Eq. (1) itself is associated with a wide freedom of choice.

For simplicity we shall let $n = 1$ in Eq. (1).

Now the following properties are characteristic of the Λ^0 particle:

a) The disintegration of the Λ^0 particle into a proton and a π^- meson:

$$\Lambda^0 \to p + \pi^- + (Q = 35 \text{ mev}). \quad (2)$$

b) A long lifetime: $\tau \sim 3 \times 10^{-10}$ sec.

c) A Λ^0 particle, apparently, may be found in a complex nucleus in a bound state.

d) The creation of Λ^0 particles in a gas from Θ^0 particles.

e) A relatively high probability for the creation of Λ^0 particles in the reaction $\pi^- + p \to \Lambda^0 + \Theta^0$.

We shall investigate the disintegration of a Λ^0 particle at rest. The constant a in Eq. (1) is determined from the observation a) for the disintegration of Λ^0: $a = \mu_\pi + Q \sim 180$ mev. It characterizes the energy interval between two levels of the excited nucleon.

The unperturbed wave function of Eq. (1) has the form

$$\Psi = U_m(x)\,\chi_{n_1 n_2 n_3 n_0}(k_1\xi), \quad (3)$$

where $U_m(x)$ is the solution of the usual Dirac equation in the form of a plane wave corresponding to a mass m. The function χ for the unexcited state of the nucleon, for example, has the form

$$\chi_{0000} = \text{const}\exp\left\{-\left(\frac{\xi_\mu\xi_\mu}{2} + \frac{(k_\mu\xi_\mu)^2}{m^2}\right)\right\}, \quad (4)$$

where $\xi_\mu\xi_\mu = \xi_1^2 + \xi_2^2 + \xi_3^2 - \xi_0^2$; $n_1 = n_2 = n_3 = n_0 = 0$.

In the non-relativistic approximation the function (4) becomes

$$\chi_{0000} = \text{const} \exp\{-(\xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_0^2)\}. \quad (5)$$

The mass operator in Eq. (1) in this case admits the simple expression

$$\left\{\ldots m_0 + a\left\{\xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_0^2 - \frac{\partial^2}{\partial\xi_1^2} - \frac{\partial^2}{\partial\xi_2^2} - \frac{\partial^2}{\partial\xi_3^2} - \frac{\partial^2}{\partial\xi_0^2}\right\}\Psi = 0, \right. \quad (6)$$

That is, in the rest system we have a four-dimensional oscillator in the literal sense of the word, inasmuch as the real time component ξ_0 enters in Eqs. (5) and (6) with the same sign as the other three components.

Let us consider initially the Λ^0 particle as the first excited state of the nucleon. The internal wave function χ_{1000} of the Λ^0 particle at rest, for example, assumes the form

$$\chi_{1000} = \frac{1}{\pi r_0^2}\sqrt{2}\,\xi_1 \times \exp\left\{-\frac{1}{2}(\xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_0^2)\right\}. \quad (7)$$

In the present variation of the theory, four different states of the Λ^0 particle are possible: χ_{1000}, χ_{0100}, χ_{0010}, χ_{0001}.

In the variation described by Eq. (15)* the state χ_{0001} is excluded.

Neglecting the recoil of the proton upon disintegration of the particle at rest, we obtain for the final state of the proton the internal function

$$\chi_{0000} = \frac{1}{\pi r_0^2}\exp\left\{-\frac{1}{2}(\xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_0^2)\right\}. \quad (8)$$

We shall write the interaction, under whose influence the transition from state (7) to state (8) takes place, in the form

$$H' = g\gamma_5\frac{\hbar c}{\sqrt{2E_\pi V}}\exp\{-i(k_\mu^\pi\xi_\mu r_0 + k_\mu^\pi x_\mu\}. \quad (9)$$

The matrix element corresponding to the reaction $\Lambda^0 \to p + \pi^-$ will be written in the following manner:

$$M_{01} = \sim g\frac{(U_p^* \beta\gamma_5 U_{\lambda^0}(k_1 r_0)}{\sqrt{E_\pi}} \times \exp\left\{-\frac{r_0^2}{4}(k_1^2 + k_2^2 + k_3^2 + k_0^2)\right\}, \quad (10)$$

where $\hbar k_i$ is the component of the momentum of the π^- meson (or proton). The form-factor in Eq. (10) is retained by us in the non-relativistic approximation. This factor would have the relativistic form

$$\exp\left\{-\frac{r_0^2}{4}\left(k_\mu^\pi k_\mu^\pi + 2\frac{(k_\nu^p k_\nu^\pi)^2}{k_\mu^p k_\mu^p}\right)\right\}; \quad (10')$$

If the momentum of the proton $\hbar k^p = 0$, and $\hbar k_0^p = m_p$, we obtain the form-factor (10). For the probability of the process discussed we obtain the expression

$$\omega = \frac{1}{\tau} \sim \frac{g^2}{\hbar c}\frac{P^2}{m_p^2 c^2}\left(\frac{Pr_0}{\hbar}\right)^2\frac{Pc}{\hbar} \times \exp\left\{-\frac{r_0^2}{2\hbar^2}(P_1^2 + P_2^2 + P_3^2 + P_0^2)\right\}, \quad (11)$$

where P is the momentum of the decay proton (or meson). The multiplier $P^2/m_p^2c^2$ in Eq. (11) appears as a consequence of the assumed pseudo-scalar interaction; for scalar mesons it is replaced by unity.

From the given lifetime of the Λ^0 particle it is possible to determine the second constant in Eq. (1), the "sizes" r_0 of the oscillator, assuming that the relative smallness of ω in Eq. (11) is completely a consequence of the smallness of r_0. Putting $k \sim \mu_\pi c$ ($k_i \sim \mu_\pi c/\sqrt{3}$), $m_p \sim 6\mu_\pi$, we obtain for $r_0^{(1)}$ the estimate

$$r_0^{(1)} \sim \frac{\hbar}{\mu_\pi c}\left(\frac{\hbar}{\mu_\pi c c\tau}\right)^{1/2}\sqrt{\frac{\hbar c}{g^2}}; \quad (12)$$

Assuming that $(\hbar c/g^2)^{1/4} \sim 1$, we obtain

$$r_0 \sim 10^{-19} \text{ cm}. \quad (13)$$

Let us now suppose that the Λ^0 particle is associated with the nth excited state of a nucleon. Then, using similar crude estimates, we obtain

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* Taking into account the recoil gives certain additional possibilities for the formation of short-lived excited states with small spin and long-lived ones with large spin. The point is that if the recoil is taken into account, it turns out that transitions from the excited to the ground state are allowed if $n_0 = n_1 + n_2 + n_3$. We are indebted for this observation to L. G. Zastavenko.

$$r_0^{(n)} \sim \frac{\hbar}{p}\left(\frac{\hbar}{pc\tau}\,\frac{\hbar c}{g^2}\,\frac{m_p^2 c^2}{p^2}\,2^n n!\right)^{1/2n} \quad (14)$$

or

$$r_0^{(n)} \sim \frac{\hbar}{\mu_\pi c}\left(\frac{\hbar\, 100}{\mu_\pi c c\tau}\right)^{1/2n}\left(2^n n!\,\frac{\hbar c}{g^2}\right)^{1/2n}. \quad (14')$$

For $n = 6$ we obtain for $r_0^{(6)}$ the estimate

$$r_0^{(6)} \sim 10^{-13}\ \text{cm}.$$

In other words, the quantities $r_0^{(n)}$ for the nucleon rapidly increase as the excitation number assumed to be characteristic of the nucleon increases, until for $n = 6$ the critical size, $r \sim 10^{-13}$ cm, is reached. These magnitudes depend only slightly on the size of the interaction constant $(g^2/\hbar c)^{1/2n}$.

The smallness of $(k_1, r_0^{(1)})$ in Eq. (10) also determines the smallness of the probability for the inverse process of the creation of a single particle for the interaction of a Λ^0 meson with a proton. It can be noted here that if the Λ^0 particle in the present model is associated with the first excited state of the nucleon, then single* creation of the Λ^0 particle is forbidden up to very high energies of the incident meson $(k^\pi \sim 1/r_0^{(1)})$. If we assume that the Λ^0 particle is represented by a nucleon of higher excitation, then $r_0^{(n)}$ becomes larger, and the energy region for the single creation of a Λ^0 particle moves toward smaller energies $(k^\pi \sim 1/r_0^{(n)})$. For $n = 5, 6$, $r_0^{(n)} \sim 10^{-13}$cm.

In other words, the creation of single Λ^0 particles from π^- mesons obtained from the cosmotron is not forbidden for energies of the π^- meson ~ 1.5 bev, $\lambda\!\!\!^{-} \sim 2 \times 10^{-14}$cm for the case of a large n factor $(k^\pi r_0^{(n)})^{2n}$. If n were sufficiently large it would be possible to produce single Λ^0 particles by the scattering of such mesons on nucleons. We are concerned here with cross sections not much smaller than the cross sections for the scattering of mesons on nucleons.

The energy threshold for the creation of a single Λ^0 particle might be indicated within the framework of the present theory for the magnitude of r_0.

All these remarks apply also to those formulations which, while not giving a detailed mathematical description of the particle, explain its long lifetime by a large self-momentum . Unfortunately, correlation experiments, which in principle might give an answer to the question of the magnitude of the self-momentum of the Λ^0 particle are, as yet, given with poor statistics.

The indicated considerations are capable of explaining only some of the listed properties of the Λ^0 particles, that is, a) and b). Inasmuch as the Λ^0 particle is considered as an excited nucleon with all the resultant consequences from this for the interaction of the Λ^0 particle with the π^- meson field (one and the same $g^2/\hbar c$), then it is evident that the Λ^0 particle might be found in complex nuclei in a bound state (property c). Quantitative data concerning the energy binding of the Λ^0 particle in nuclei might here prove to be essential.

It is evident that properties d) and e) demand a new hypothesis, since here we encounter a new field (the Θ^0 field). In reference 4 an attempt is made to interpret these properties of the Λ^0 particles also, proceeding from the idea of excited nucleon states.

* That is, without the accompaniment of Θ^0 particles: for example, $\pi^- + p \rightarrow \Lambda^0 + \pi^0$.

[4] M. A. Markov, Dokl. Akad. Nauk SSSR 101, 449 (1955)

Translated by E. V. Ivash
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