of nucleon-nucleon scattering, the theory of damping in the approximation considered cannot be made to agree with experiment, as was also the situation in the case of the scattering of mesons by nucleons[7].

Translated by G. M. Volkoff
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[7] G. F. Zharkov, J. Exper. Theoret. Phys. USSR **27**, 296 (1954)

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The Interaction of Fast Deuterons with Nuclei

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In the interaction of fast deuterons with nuclei, the processes usually considered are stripping, deuteron capture, electric dissociation and diffraction scattering of the deuteron as a whole. It is shown that, in addition, as a result of the diffraction of deuterons, dissociation of the deuteron takes place outside the nucleus, the nucleus recoiling only slightly in the process. A general formula is derived which can serve as basis for calculations on diffraction dissociation induced by either purely nuclear forces or by nuclear and electric forces.

1. IN the analysis of the interaction between nuclei and fast, nonrelativistic deuterons (energy $E_0 \sim 30 - 300$ mev), it is usual to consider only three processes: a) capture of the deuteron by the nucleus (followed by a nuclear reaction), b) capture by the nucleus of the neutron (or proton) in the deuteron, with an accompanying nuclear reaction and emission of a fast proton (or neutron) --- the so-called stripping process and c) dissociation of the deuteron into a free proton and neutron under the influence of the electric field of the nucleus.

The first of these processes, deuteron capture, has the largest cross section. For a black nucleus, $\sigma_{\text{capt}} = \pi(R + R_d)^2$, where R and R_d are some effective radii for the nucleus and deuteron. For medium and heavy nuclei, the stripping cross section is less by a factor of order $R_d/(R + R_d)$ (for a black nucleus, $\sigma_{\text{st}} \approx \pi/2RR_d)$[1]. The cross section σ_{el} for dissociation in an electric field is still smaller, except for the heaviest nuclei (this cross section cannot be expressed by such a simple formula).

To this list we should add d) diffraction scattering of the deuteron as a whole --- a process which at the present time can be considered trivial. It is a necessary consequence of the existence of all the other processes depleting the initial deuteron beam, and has a cross section of order of magnitude the same as the largest indicated above, i.e., $\sigma^{\text{diff}}_{\text{scatt}} \sim \pi(R + R_d)^2$ (for a black nucleus).

However, there is yet another process, of very unusual character, and with a cross section, as we shall show, comparable to the cross sections of processes b) and c). This process is associated with the fact that in the diffraction scattering of deuterons, a certain momentum is given to the nucleus as a whole. As a consequence, not only can the deuteron be scattered, but it may also dissociate and give rise to a free proton and neutron. The characteristic feature of this diffraction dissociation is that the nucleus absorbs the momentum as a whole, and undergoes no nuclear reaction. Similar physical phenomena have been investigated in several recent papers on the theory of diffraction processes[2,3]. These phenomena are characterized by the fact that when the colliding particles have large energies, the momentum

[1] A. I. Akhiezer and I. Ia. Pomeranchuk, *Some Problems in the Theory of the Nucleus*, Second Edition, Moscow 1952, Secs. 13, 14

[2] L. D. Landau and I. Ia. Pomeranchuk, J. Exper. Theoret. Phys. USSR **24**, 505 (1953); I. Ia. Pomeranchuk, Dokl. Akad. Nauk SSSR **96**, 265 (1954)

[3] I. Ia. Pomeranchuk and E. L. Feinberg, Dokl. Akad. Nauk SSSR **93**, 439 (1953)

transfer in the direction of motion becomes small, and hence, according to the uncertainty principle, the volume in which the process takes place becomes very large (and can be much larger than the nucleus). Thus, diffraction dissociation can be called a process of external dissociation. The probabilities for the process to occur in various nuclei are indicated at the end of reference 3. Its physical basis is the same as that in the emission of gamma rays by fast charged π-mesons[2] and of π-mesons by fast nucleons[3].

2. The following three conditions must be satisfied if diffraction dissociation is to occur: a) the deuteron must receive an impulse of the order of magnitude of the reciprocal of its dimensions, $q \sim 1/R_d \sim 0.8\ \mu$; b) the volume in which the process takes place must be large in at least one of its dimensions (the longitudinal one) compared to the nuclear radius R, i.e., the dissociation is associated with a momentum transfer in the direction of motion which satisfies the relation $q_{\parallel} R \ll 1$; c) the deuteron must receive the necessary impulse in a sufficiently short time, having an order of magnitude of the inverse of the binding in the deuteron (in energy units, the deuteron binding energy).

A large fraction of the diffraction scattering is at an angle of approximately $\lambda\!\!\!^{-}/R$, where $\lambda\!\!\!^{-} = 1/p_d$ and p_d is the momentum of the deuteron as a whole. Hence,

$$q \sim p_d \lambda\!\!\!^{-}/R \sim 1/R. \tag{2.1}$$

In our calculations (see below), we shall consider this to be considerably smaller than $1/R_d$. Hence, only a small fraction of the scattering is effective in the sense of dissociation, and the dissociation cross section is much smaller than the cross section for diffraction scattering. One can easily convince oneself that in this case it is approximately RR_d, although the order of magnitude should be given by this formula even for $R \sim R_d$. Here the momentum transferred is small compared to the momentum of the emitted particles. It follows that the distributions in energy and angle of the emitted particles are given essentially by the momentum distribution inside the deuteron. In particular, the momenta of the proton and neutron upon dissociation are almost equal. In general, the distributions must be similar to those in the stripping reaction.

In order to check whether conditions b) and c) are satisifed, we must calculate $q_{\parallel}$.

Let p_d, p_n and p_p be the momenta of the deuteron before dissociation of the emitted neutron and of the proton respectively, and ϑ_{nd}, ϑ_{pd} be the angles (with respect to the direction of the incident deuteron beam) at which the neutron and proton are emitted; then

$$q_{\parallel} = p_d - p_n \cos \vartheta_{nd} - p_p \cos \vartheta_{pd} = p_d - p_n - p_p + p_n (1 - \cos \vartheta_{nd}) + p_p (1 - \cos \vartheta_{pd}). \tag{2.2}$$

Let M be the mass of a nucleon and ϵ the deuteron binding energy. Then, for sufficiently large deuteron energies, namely,

$$E_d \approx p_d^2/4M \gg \varepsilon$$

conservation of energy,

$$\frac{p_d^2}{2(2M-\varepsilon)} - \varepsilon = \frac{p_n^2}{2M} + \frac{p_p^2}{2M}$$

(where we neglect the recoil energy of the nucleus, see below), leads to the formula

$$p_d \approx \sqrt{2}\sqrt{p_n^2 + p_p^2} + \frac{2M\varepsilon}{p_d} \approx \sqrt{2}\sqrt{p_n^2 + p_p^2}\left(1 + \frac{M\varepsilon}{p_n^2 + p_p^2}\right). \tag{2.3}$$

For small angles ($\vartheta_{nd} \approx \vartheta_{pd} = 0$), which imply a small value for $q_{\parallel} = q_{\parallel \min}$, we obtain the following conditions for the possibility of external dissociation: (R is the nuclear radius, and as always, $h = c = 1$):

$$q_{\parallel \min} R = \left\{\sqrt{2}(p_n^2 + p_p^2)^{1/2} - p_n - p_p + \frac{\sqrt{2}M\varepsilon}{\sqrt{p_n^2 + p_p^2}}\right\} R \ll 1. \tag{2.4}$$

Since, as noted above, $p_n \approx p_p \approx 1/2 p_d$ (more exactly, it is necessary that $|p_n - p_p| \lesssim \sqrt{4M\epsilon}$; subsequent calculations will confirm that this gives the most significant range of p_n, p_p), the condition (2.4) becomes:

$$q_{\parallel} R = (2M\varepsilon/p_d) R \ll 1, \tag{2.4a}$$

or

$$\frac{p_d^2}{4M} \approx E_d \gg R^2 M \varepsilon^2 = A^{2/3} \frac{M}{\mu} \frac{\varepsilon}{\mu c^2} \varepsilon \sim 0.2\, A^{2/3}\,\text{MeV} \tag{2.5}$$

(where for convenience we put $R = A^{1/3} 1/\mu$).

Hence for deuterons with energy in the tens of mev, condition b) is satisfied.

It will be shown later that the effective angles are approximately $\vartheta_{nd}^2 \sim \vartheta_{np}^2 \sim 4M\epsilon/p_d^2 \sim q_{\parallel \min}/p_d$, so that the above holds also for all angles which occur.

Finally, to check whether condition c) holds, we consider the collision time τ. This is evidently the time of flight of a deuteron through a length of order $1/q_{\|}$, i.e., from (2.4a):

$$\tau \sim \frac{1}{q_{\|}}\frac{2M}{p_d} \sim \frac{1}{\varepsilon}. \tag{2.6}$$

Hence, the third condition is also satisfied. We note that at relativistic energies, a calculation of $q_{\|}$ using energy conservation as in (2.1) - (2.4a) leads to a similar result, namely, that in the rest frame of the deuteron, the momentum is transferred in time $1/\epsilon$. This leads to the supposition that the process in question should also go at relativistic energies, and be of similar character.

3. The basic general formula for the cross section of the process being considered can be obtained in several different ways. We shall use the Born approximation in the sense that the interaction U between the emitted proton and neutron and with the electric field W of the nucleus will be considered to be small perturbations. The nucleus will appear as a fixed source of a nuclear potential V acting on the proton and neutron of the deuteron. We neglect exchange effects between these particles and the nuclear protons and neutrons (this is permissable since the energy of the deuteron's nucleons is large compared to the energy of the nuclear nucleons).

Let us, for example, use the formal apparatus and notation of the scattering theory described in reference 4. Let the full energy operator for the system nucleus plus deuteron be written in the form:

$$H = K + U + V + W. \tag{3.1}$$

Here K is the sum of the operators for the deuteron kinetic energy and the internal energy of motion in the nucleus.

We are interested in the following wave functions:

First, the exact wave function for the system, satisfying the initial condition --- a free incident deuteron of energy E_a --- and containing as reaction products only outgoing waves (from the nucleus) $\Psi^{+\epsilon}$. If the state function: incident deuteron plus unperturbed nucleus is denoted by Ψ_{0a}, then evidently $\Psi_a^{(+\varepsilon)} = \Psi_{0a}$

$$+ \frac{1}{E_a - K - U + i\varepsilon}(V + W)\Psi_a^{(+\varepsilon)}, \tag{3.2}$$

$$(E_a - K - U)\Psi_{0a} = 0. \tag{3.2a}$$

In the absence of a nuclear field, $V = W = 0$, and this expression reduces to Ψ_{0a} as it should; for $\epsilon = 0$ it satisfies the equation $H\Psi_a^{(+0)} = E_a\Psi_a^{(+0)}$ while the term $+i\epsilon$, which will go to zero at the end of the calculation. corresponds to outgoing waves. We write $\Psi_a^{(+0)} = e^{-iE_a t}\psi_a^{(+)}$.

Second, the wave function φ_b for the experimentally observed situation, i.e., a free neutron and proton of definite momenta p_n and p_p (product of two plane waves). Evidently

$$(K - E_b)\varphi_b = 0. \tag{3.3}$$

Third, and last, the function $\chi_b^{(-)}$ describing a neutron and proton of momenta p_n and p_p interacting with the nuclear field V, but not with each other, nor with the electric field of the nucleus. We can write

$$\varphi_b = \chi_b^{(-)} - \frac{1}{E_b - K - i\varepsilon}V\chi_b^{(-)}. \tag{3.4}$$

The $\chi_b^{(-)}$ determined in this way reduces to φ_b for $V = 0$, as it should, and for $\epsilon = 0$ it satisfies the equation $(K + V - E_b)\chi_b^{(-)} = 0$. The term $- i\epsilon$ (note the minus sign!) corresponds to an incoming wave in the space co-ordinate, as is necessary to describe the production of particles with a definite momentum.

Given that the full solution is $\Psi^{(+\epsilon)}$, we are interested in the probability of finding the state φ_b, i.e.,

$$w_{ba} = |\langle \varphi_b \exp\{-iE_b t\}| \tag{3.5}$$

$$\times \exp\{-i(K + U + V + W)t\} \,|\, \psi_a^{(+)}\rangle|^2.$$

From this it can be easily calculated[4] that the transition probability per unit time at $t = 0$ is

$$w_{ba}|_{t=0} = 2\pi|\langle\varphi_b| U + V \tag{3.6}$$

$$+ W|\psi_a^{(+)}\rangle|^2\delta(E_a - E_b).$$

Using Eq. (3.4), we have

$$\langle\varphi_b|U + V + W|\psi_a^{(+)}\rangle \tag{3.7}$$

$$= \langle\chi_b^{(-)}|U + V + W|\psi_a^{(+)}\rangle$$

$$- \langle \frac{1}{E_b - K - i\varepsilon}V\chi_b^{(-)}|U + V + W|\psi_a^{(+)}\rangle$$

$$= \langle\chi_b^{(-)}|U + V + W|\psi_a^{(+)}\rangle$$

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[4] M. Gell-man and M. Goldberger, Phys. Rev. **91**, 398 (1953)

$$-\left\langle \chi_b^{(-)} \middle| V \frac{1}{E_b - K + i\varepsilon} (U + V + W) \middle| \psi_a^{(+)} \right\rangle.$$

But according to Eq. (3.2),

$$(E_a + i\varepsilon - K)\,\psi_a^{(+\varepsilon)} = (U + V + W)\,\psi_a^{(+\varepsilon)} + i\varepsilon\psi_{0a},$$

so that for $\epsilon \to 0$ we get

$$\frac{1}{E_a - K + i\varepsilon}(U + V + W)\,\psi_a^{(+\varepsilon)} \to \psi_a^{(+)},$$

and hence,

$$\dot{w}_{ba}|_{t=0} = 2\pi \,|\langle \chi_b^{(-)} | U + W | \psi_a^{(+)} \rangle|^2 \,\delta(E_a - E_b). \qquad (3.8)$$

So far, this formula is exact. Let us consider now the function $\psi_a^{(+1)}$. It is the superposition of the following four states: a) incident plane waves and outgoing diffracted waves for the deuteron as a whole, $\psi_a + \psi_d^{\text{diff}}$; b) emitted proton plus neutron captured by the nucleus, ψ_{prot}; c) emitted neutron and captured proton, ψ_{neut}; d) the state of interest to us, i.e., simultaneously emitted proton and neutron (strictly speaking, interacting with each other, and with the electric and nuclear fields of the nucleus ---ψ_f^{diff}

By Born's approximation we mean the substitution of $\psi_a + \psi_d^{\text{diff}}$ for $\psi_a^{(+)}$ in Eq. (3.8). Our neglect of the other terms is justified as follows: With ψ_d^{diff} we take into account a term which leads to diffraction scattering of the deuteron with cross section $\sim \pi(R + R_d)^2$. The last constituent (d describes diffraction dissociation which is associated with the transfer to the deuteron of a transverse momentum of order $1/R_d$. As in other processes of this kind[2,3], we expect (see above) it to have a cross section $\sim RR_d$ (this will be confirmed by a later calculation). Hence, our approximation is applicable to nuclei sufficiently heavy that

$$R_d/(R + R_d) \ll 1. \qquad (3.9)$$

The stripping process also has a cross section $\sim RR_d$, and on the same grounds we could neglect the functions b) and c). Actually ,the situation here is even better (at least neglecting the effect of the electric field). The point here is that the interaction U is almost a δ-function, so that when one of the particles is captured by the nucleus, and its function is concentrated inside the nucleus, then it cannot interact with particles in the state $\chi^{(-)}$, which penetrate only weakly into the nucleus. For these reasons, we can calculate from the formula

$$w_{ba} = 2\pi \,|\langle \chi_b^{(-)} | U + W | \psi_0 + \psi_d^{\text{diff}} \rangle|^2 \,\delta(E_a - E_b). \qquad (3.10)$$

This formula, with $W = 0$, can be obtained by another method[5] which was used to study the purely electrical dissociation of the deuteron. We need only replace the electric potential of the nucleus by V and write the deuteron function in the form $\psi_{0a} + \psi_d^{\text{diff}}$. However, in this method the nature of the approximations made is not clear.

The function ψ_d^{diff} may be found either for deuteron wavelengths small compared to R, or for a black nucleus, in which latter case the derivation is particularly simple.

We now justify the Born approximation for the proton-neutron interaction in the final state. Since the particles separate with a relative momentum of the same order of magnitude as their relative momentum inside the deuteron, i.e., $p_n - p_p \sim 2\sqrt{M\epsilon}$, the kinetic energy of this motion is of the order $1/M(p_n - p_p)^2 \sim 4\epsilon \sim 8.5$ mev. But it is well-known that in this energy range the proton-neutron scattering can be described in Born's approximation with the help of a certain pseudo-potential, whose absolute value, it is true, depends on the energy. In this way, if U is regarded as a certain effective operator for the interaction (if necessary, an integral operator), then the Born approximation will be applicable. The subsequent calculations will, in some cases, give the explicit form of this potential. Thus we see that even if we neglect the electric field W of the nucleus, the phenomenon of diffraction dissociation must exist; it will depend on diffraction of the deuteron (the term ψ_d^{diff}), and of the reaction products (if $\chi_b^{(-)}$ is replaced by the product of plane waves and ψ_d^{diff} is discarded, then conservation laws will make w_{ba} zero).

The theory of the electric dissociation of fast deuterons[6] would correspond in Eq. (3.10) not only to neglecting U, but also to replacing $\chi_b^{(-)}$ by plane waves φ_b, and discarding ψ_d^{diff}. Thus, we see that the effect of diffraction on purely electrical dissociation can also be taken into account. Hence,

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[5] L. D. Landau and E. M. Lifschitz, J. Exper. Theoret. Phys. USSR 18, 750 (1948) (see reference 1, Sec. 13)

[6] S. M. Dancoff, Phys. Rev. 72, 1017 (1947) (see reference 1, Sec. 13)

working from Eq. (3.10), the theory of electric dissociation can be improved.

Since in order to induce dissociation of the deuteron, it is sufficient to give it an impulse of the order of the reciprocal of its dimensions, the nucleus, as it receives the same impulse, does not absorb an appreciable amount of energy and undergoes no reaction.

The detailed results will be published separately.

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The Connection between the Vibrations of the Surface of a Nucleus and Single Nucleon Excitation

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The conditions of validity of a model of single nucleon excitations in a nucleus are investigated by the method of adiabatic approximation. The effect of the relation between single nucleon excitations and the vibrations of the surface of a nucleus on the excited states of the whole nucleus are established.

1. INTRODUCTION

BECAUSE of strong interaction between the nucleons in a nucleus, one can speak in a strictly definedsense only of a state of a nucleus as a whole and not of the states of an individual nucleon. However, such consideration is still impracticable and one has to apply the approximate methods of study of energy states of nuclei.

In the case of an approximate examination of the lowest energy states, one usually proceeds from the notion that the nucleons in a nucleus are in the form of "shells" which can change their shape and dimensions. Individual nucleons are moving in an average field of the surrounding nucleons. This average field is such that the resultant force acting on one nucleon differs from zero mainly on the surface of a nucleus. Since the average field is caused by many nucleons, its change is connected with the collective movement of the nucleons. Because of the small compressibility of nuclear matter, its density can be considered as being constant. In this approximation, one can picture the "collective" movement only as deformation of nucleus surface without change of the volume.

In a series of cases the frequencies of the collective movements are smaller than the frequencies corresponding to the excitations of individual nucleons in a nucleus. Then, by investigation of the energy states of a nucleus, one can apply the adiabatic approximation.

In this article we will consider the limits of applicability of adiabatic approximation to a nucleus and make clear what effect the connection between the single nucleonic excitations and vibrations of the nuclear surface has on the energy states of a whole nucleus.

Particularly, it will be shown that the probability of a single nucleonic transition under the influence of an external excitation decreases because of a connection between single nuclear states and collective vibrations.

2. THE STRUCTURE OF THE ENERGY SPECTRUM OF A NUCLEUS AT SMALL EXCITATIONS

Let r denote the coordinates and the spins of nucleons in a nucleus and R the configuration of the nuclear surface. Assume that for every value of R the characteristic functions $\varphi_n(r, R)$ and the energy of the nucleons $E_n(R)$ are known. Let the index n denote the set of quantum numbers $\{n, j, m_j\}$, which characterizes the state of all nucleons in a nucleus in the case of a single particle approximation. The functions $\varphi_n(r, R)$ satisfy the equation

$$\{H(r, R) - E_n(R)\}\varphi_n(r, R) = 0. \quad (1)$$

To take into account the change of the shape of the surface of a nucleus, we introduce the kinetic