Nucleon-Nucleon Scattering According to the Theory of Damping

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Nucleon-nucleon scattering cross sections have been calculated on the basis of the theory of damping. It is shown that the calculated dependence of the total cross sections on the energy cannot be made to agree with experiment.

IT is well known that theoretical attempts to explain the behavior of nucleon-nucleon scattering cross sections at high energies have not been successful to date. In particular, the examination of this problem within the framework of perturbation theory[1,2] also has not led to satisfactory results insofar as the angular dependence of the proton-proton and proton-neutron scattering cross sections turned out to be in sharp disagreement with experiment. At the same time it was shown that[1] the total scattering cross sections found on the basis of perturbation theory depend only weakly on the energy over a wide energy range and remain approximately constant, in agreement with experimental data. However, inasmuch as the perturbation theory results cannot be trusted, in view of the large value of the nuclear interaction constant, it seeemed to be of methodological interest to investigate the problem of nucleon-nucleon scattering within the framework of the theory of damping, which would enable one formally to deal with large interaction constants and to compare results obtained by different methods. The results of such a calculation are presented below.

2. We write the basic integral equation of the theory of damping in the form[3,4]:

$$R(\mathbf{k}'s_1's_2'\tau_1'\tau_2' \mid \mathbf{k}s_1s_2\tau_1\tau_2) \quad (1)$$
$$= K(\mathbf{k}'s_1's_2'\tau_1'\tau_2' \mid \mathbf{k}s_1s_2\tau_1\tau_2)$$
$$-\frac{ikE_k}{(4\pi)^2}\int d\Omega_q K(\mathbf{k}'s_1's_2'\tau_1'\tau_2' \mid \mathbf{q}s_1''s_2''\tau_1''\tau_2'')$$
$$\times R(\mathbf{q}s_1''s_2''\tau_1''\tau_2'' \mid \mathbf{k}s_1s_2\tau_1\tau_2),$$

where R is the scattering amplitude, $K = iS_2$. Here S_2 is the matrix element of the S-matrix[5] quadratic in the interaction constant. We shall consider the case of nucleons interacting through a pseudoscalar meson field subject to the sum of two couplings: a pseudoscalar coupling characterized by the constant g, and a pseudovector coupling characterized by the constant g'. In this case the quantity K occurring in Eq. (1) may be represented in the form:

$$K(\mathbf{k}'s_1's_2'\tau_1'\tau_2' \mid \mathbf{k}s_1s_2\tau_1\tau_2) \quad (2)$$
$$= \frac{G^2}{(2E_k)^2}\left\{\frac{(\vec{\sigma}\cdot\mathbf{k}-\mathbf{k}')_{s_2's_2}(\vec{\sigma}\cdot\mathbf{k}-\mathbf{k}')_{s_1's_1}\vec{\tau}_{\tau_2'\tau_2}\vec{\tau}_{\tau_1'\tau_1}}{2k^2(1-\cos\vartheta_{kk'})+\mu^2}\right.$$
$$\left.-\frac{(\vec{\sigma}\cdot\mathbf{k}+\mathbf{k}')_{s_1's_2}(\vec{\sigma}\cdot\mathbf{k}+\mathbf{k}')_{s_2's_1}\vec{\tau}_{\tau_1'\tau_2}\vec{\tau}_{\tau_2'\tau_1}}{2k^2(1+\cos\vartheta_{kk'})+\mu^2}\right\}.$$

Here $G = g + (2M/\mu)g'$, σ, τ are Pauli matrices, $\mathbf{k}$ is the momentum of the colliding nucleons in the center of-mass system s_1, s_2, τ_1, τ_2 are the spins and isotopic spins of the two colliding nucleons; $\mathbf{k}'$, s_1', s_2', τ_1', τ_2' are the corresponding quantities after scattering, μ is the meson mass, M is the nucleon mass and $E_k = (k^2 + M^2)^{1/2}$.

3. In order to solve the integral equation (1) we first of all separate out the isotopic spin variables by setting

$$R = \Psi^{*T=0}R^{T=0}\Psi^{T=0} + \Psi^{*T=1}R^{T=1}\Psi^{T=1}, \quad (2a)$$

where $\Psi^{T=0}$ and $\Psi^{T=1}$ are the isotopic spin functions of the system of two nucleons which correspond to the total isotopic spin of zero or one,

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respectively. Following this, Eq. (1) separates into two independent equations which determine the functions $R^{T=0}$ and $R^{T=1}$. We solve the equation for $R^{T=0}$ by expanding the unknown function into a series with respect to the total angular momentum eigenfunctions. Here q denotes the totality of all the angle and spin coordinates $q = \{\Omega, s_1, s_2\}$.

$$R^{T=0}(\mathbf{k}'s_1's_2' \mid \mathbf{k}s_1s_2) = \sum \{W_M^{J*}(q) A_1 W_M^J(q') + W_M^{J,J*}(q) A_2 W_M^{J,J}(q') + W_M^{J,J-1*}(q) B_1 W_M^{J,J-1}(q') + W_M^{J,J+1*}(q) B_2 W_M^{J,J-1}(q') + W_M^{J,J-1*}(q) B_3 W_M^{J,J+1}(q') + W_M^{J,J+1*} B_4 W_M^{J,J+1}(q')\}. \tag{3}$$

The normalized eigenfunctions of the total angular momentum in our case have the form:

$$W_M^J(q) = Y_M^J(\Omega) S_0(s_1 s_2); \tag{4}$$

$$W_M^{J,J} = \sqrt{\frac{(J+M)(J-M+1)}{2J(J+1)}}\, Y_{M-1}^J\, {}^3S_1 + \frac{M}{\sqrt{J(J+1)}}\, Y_M^J\, {}^3S_0 - \sqrt{\frac{(J+M+1)(J-M)}{2J(J+1)}}\, Y_{M+1}^J\, {}^3S_{-1};$$

$$W_M^{J,J-1} = \sqrt{\frac{(J+M)(J+M-1)}{2J(2J-1)}}\, Y_{M-1}^{J-1}\, {}^3S_1 - \sqrt{\frac{(J+M)(J-M)}{J(2J-1)}}\, Y_M^{J-1}\, {}^3S_0 + \sqrt{\frac{(J-M)(J-M-1)}{2J(2J-1)}}\, Y_{M+1}^{J-1}\, {}^3S_{-1};$$

$$W_M^{J,J+1} = \sqrt{\frac{(J-M+2)(J-M+1)}{(2J+2)(2J+3)}}\, Y_{M-1}^{J+1}\, {}^3S_1 + \sqrt{\frac{(J+M+1)(J-M+1)}{(J+1)(2J+3)}}\, Y_M^{J+1}\, {}^3S_0 + \sqrt{\frac{(J+M+1)(J+M+2)}{(2J+2)(2J+3)}}\, Y_{M+1}^{J+1}\, {}^3S_{-1};$$

where $S_0(s_1, s_2)$ and ${}^3S_m(s_1, s_2)$ are the singlet and triplet spin functions for a system of two nucleons. The spherical harmonics $Y_m^l(\Omega)$ are defined in accordance with reference 6.

[6] H. Bethe, Quantum Mechanics of the Simplest Systems, Handbuch der Physik, vol. XXIV-1

On substituting the expansion (3) into the equation determining $R^{T=0}$, one finds the expansion coefficients A_i and B_i. We shall not give here the explicit form of these coefficients insofar as we shall be interested only in the total nucleon scattering cross section, into which enter only the squares of the absolute values of these coefficients given below.

The total nucleon-nucleon scattering cross section is determined by the formula

$$\sigma = \frac{1}{4} \sum_{\text{spin}} \frac{E_k^2}{(4\pi)^2} \int d\Omega_k \, |R|^2. \tag{5}$$

Substituting the expansion (3) into Eq. (5) we obtain the total nucleon scattering cross section in the state $T = 0$

$$\sigma^{T=0} = \frac{4\pi}{k^2} \sum_{J\geqslant 1} (2J+1) \left\{ \frac{a_1^2 s_0}{1+4a_1^2} + \frac{a_2^2 s_1}{1+4a_2^2} + \frac{b_1^2 + b_3^2 + 2b_2^2 + 8(b_1 b_3 - b_2^2)^2}{1 + 4(b_1^2 + b_3^2 + 2b_2^2) + 16(b_1 b_3 - b_2^2)^2} s \right\}. \tag{6}$$

Here the following notation has been used:

$$a_1 = U\left(\frac{J+1}{2J+1} K_{J+1} + \frac{J}{2J+1} K_{J-1} - K_J\right); \tag{7}$$

$$a_2 = U\left(K_J - \frac{J+1}{2J+1} K_{J-1} - \frac{J}{2J+1} K_{J+1}\right);$$

$$b_1 = U \frac{1}{2J+1} (K_{J-1} - K_J);$$

$$b_2 = U \frac{\sqrt{J(J+1)}}{2J+1} (K_{J-1} + K_{J+1} - 2K_J);$$

$$b_3 = U \frac{1}{2J+1} (K_J - K_{J+1});$$

$$U = \frac{k}{E_k} \frac{G^2 t_0}{2^5 \pi}; \quad K_J = \int_{-1}^{+1} \frac{P_J(x)\, dx}{z - x}; \quad z = 1 + \frac{\mu^2}{2k^2};$$

$$s_0 = \frac{1}{2}[1 - (-1)^J]; \quad s_1 = \frac{1}{2}[1 + (-1)^J];$$

$$s = \frac{1}{2}[1 + (-1)^{J+1}]; \quad t_0 = -3;$$

$P_j(x)$ is the Legendre polynomial normalized by the condition

$$\int_{-1}^{+1} P_J^2(x)\, dx = \frac{2}{2J+1}. \tag{7a}$$

4. The equation for $R^{T=1}$ is solved by a quite analogous method and the following expression is obtained for the total nucleon scattering cross section in the state $T = 1$

$$\sigma^{T=1} = \frac{4\pi}{k^2}\sum_{J\geqslant 1}(2J+1)\left\{\frac{a_1^2 s_0}{1+4a_1^2} + \frac{a_2^2 s_1}{1+4a_2^2} + \frac{b_1^2+b_3^2+2b_2^2+8(b_1b_3-b_2^2)^2}{1+4(b_1^2+b_3^2+2b_2^2)+16(b_1b_3-b_2^2)^2}\,s\right\} + \frac{4\pi}{k^2}\left\{\frac{a_1^2(J=0)}{1+4a_1^2(J=0)}\right.$$

$$\left. + \frac{b_3^2(J=0)+4[b_1(J=0)\,b_3(J=0)-b_2^2(J=0)]^2}{1+4[b_1^2(J=0)+b_3^2(J=0)+2b_2^2(J=0)]+16[b_1(J=0)\,b_3(J=0)-b_2^2(J=0)]^2}\right\}. \qquad (8)$$

The quantities a_i, b_i and s_i occurring in formula (8) differ from the corresponding quantities given in Eq. (7) by the fact that in (7) the following substitutions should be made:

$$t_0 \to t_1 = 1;\ s_0 \to {}^1/_2[1+(-1)^J]; \qquad (8a)$$

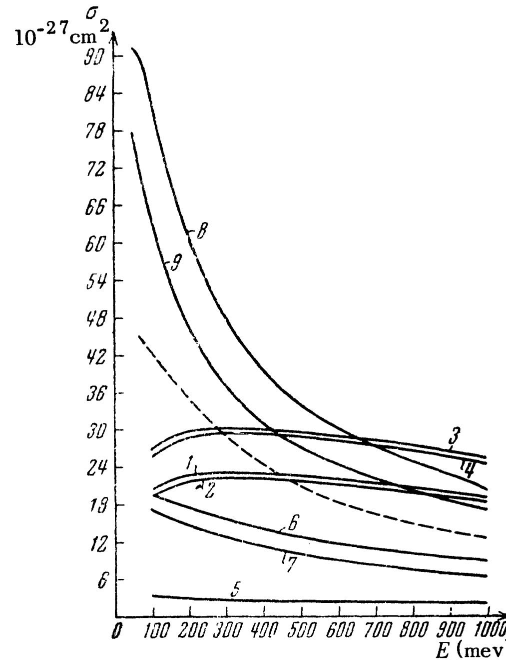
$$s_1 \to {}^1/_2[1-(-1)^J];$$

$$s \to {}^1/_2[1-(-1)^{J+1}].$$

5. Knowing the cross sections $\sigma^{T=0}$ and $\sigma^{T=1}$, it is easy to obtain the total proton-proton and proton-neutron scattering cross sections with the aid of formulas

$$\sigma_{pp} = \sigma^{T=1};\ \sigma_{pn} = {}^1/_4(\sigma^{T=0} + \sigma^{T=1}). \qquad (9)$$

The Figure shows graphs of the dependence of σ_{pp} and σ_{pn} on the energy for various values of the interaction constant $G = g + (2M/\mu)g'$. For comparison, the same Figure also gives curves obtained by neglecting damping, i.e., corresponding to the perturbation theory results*[1]. From the Figure it may be seen that while the total cross sections calculated by perturbation theory for $G^2 \sim 70$ represents quite well the experimentally observed independence of the total cross sections on the energy, the curves which correspond to the more consistent method of calculation on the basis of the theory of damping disagree sharply with the experimental data**. As may be seen from the Figure, for large values of the interaction constant the theoretical curves depend strongly on the energy, while for smaller values of the interaction constant, although a weaker dependence of the cross sections on the energy is obtained, nevertheless, the absolute values of the cross sections turn out to be too small. Therefore, in the case



The dependence on the nucleon energy (in the laboratory system) of the total cross sections σ_{pp} and σ_{pn} for various values of the constsnt G^2. Curves 1-4 correspond to perturbation theory, 5-9 to damping theory. *1.* σ_{pn}, $G^2 = 70$; *2.* σ_{pp}, $G^2 = 70$; *3.* σ_{pn}, $G^2 = 80$; *4.* σ_{pp}, $G^2 = 80$; *5.* $\sigma_{pp} = \sigma_{pn}$, $G^2 = 80/3$; *6.* σ_{pp}, $G^2 = 80$; *7.* σ_{pn}, $G^2 = 80$; *8.* σ_{pp}, $G^2 = 240$; *9.* σ_{pn}, $G^2 = 240$; the dotted curve shows the approximate energy dependence of the cross sections σ_{pp} and σ_{pn} for a value of the constant G^2 intermediate between 80 and 240. The experimental data for σ_{pp} correspond to a cross section which is independent of the energy, and is approximately equal to 24×10^{-27} cm^2.

* Our Eqs. (9) agree with the corresponding formulas of reference 1 in the case that damping is neglected. However, it should be noted that in reference 1 a factor equal to 1/4 has been left out which corresponds to averaging over the spins of the initial nucleons, in view of which the formulas of reference 1 must be multiplied by 1/4 before the cross sections are compared.

** We also note that at sufficiently high energies, processes of inelastic scattering of nucleons accompanied by production of mesons become possible. We have not taken such processes into account.

of nucleon-nucleon scattering, the theory of damping in the approximation considered cannot be made to agree with experiment, as was also the situation in the case of the scattering of mesons by nucleons[7].

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The Interaction of Fast Deuterons with Nuclei

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In the interaction of fast deuterons with nuclei, the processes usually considered are stripping, deuteron capture, electric dissociation and diffraction scattering of the deuteron as a whole. It is shown that, in addition, as a result of the diffraction of deuterons, dissociation of the deuteron takes place outside the nucleus, the nucleus recoiling only slightly in the process. A general formula is derived which can serve as basis for calculations on diffraction dissociation induced by either purely nuclear forces or by nuclear and electric forces.

1. IN the analysis of the interaction between nuclei and fast, nonrelativistic deuterons (energy $E_0 \sim 30 - 300$ mev), it is usual to consider only three processes: a) capture of the deuteron by the nucleus (followed by a nuclear reaction), b) capture by the nucleus of the neutron (or proton) in the deuteron, with an accompanying nuclear reaction and emission of a fast proton (or neutron) --- the so-called stripping process and c) dissociation of the deuteron into a free proton and neutron under the influence of the electric field of the nucleus.

The first of these processes, deuteron capture, has the largest cross section. For a black nucleus, $\sigma_{\text{capt}} = \pi(R + R_d)^2$, where R and R_d are some effective radii for the nucleus and deuteron. For medium and heavy nuclei, the stripping cross section is less by a factor of order $R_d/(R + R_d)$ (for a black nucleus, $\sigma_{\text{st}} \approx \pi/2RR_d$)[1]. The cross section σ_{el} for dissociation in an electric field is still smaller, except for the heaviest nuclei (this cross section cannot be expressed by such a simple formula).

To this list we should add d) diffraction scattering of the deuteron as a whole --- a process which at the present time can be considered trivial. It is a necessary consequence of the existence of all the other processes depleting the initial deuteron beam, and has a cross section of order of magnitude the same as the largest indicated above, i.e., $\sigma^{\text{diff}}_{\text{scatt}} \sim \pi(R + R_d)^2$ (for a black nucleus).

However, there is yet another process, of very unusual character, and with a cross section, as we shall show, comparable to the cross sections of processes b) and c). This process is associated with the fact that in the diffraction scattering of deuterons, a certain momentum is given to the nucleus as a whole. As a consequence, not only can the deuteron be scattered, but it may also dissociate and give rise to a free proton and neutron. The characteristic feature of this diffraction dissociation is that the nucleus absorbs the momentum as a whole, and undergoes no nuclear reaction. Similar physical phenomena have been investigated in several recent papers on the theory of diffraction processes[2,3]. These phenomena are characterized by the fact that when the colliding particles have large energies, the momentum

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