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On the Theory of Nucleons

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A study is made of the hypothesis that the pion-nucleon interaction is indirect (transmitted by an intermediary field). This leads to a representation of the nucleon as an inner core surrounded by a pion cloud.

1. INTRODUCTION

THE experiments on pion-nucleon scattering[1-3] show clearly the existence of so-called isobars, i.e., excited states of nucleons. Also the peculiar decay of the Λ^0 – particle compels us to consider it as an excited state of the proton[4].

It is therefore very probable that at the present time we have already observed two levels of the proton, one with a short life-time which is responsible for the meson scattering, and one with a long life-time which is observed as the Λ^0-particle.

In this connection there have recently appeared some papers[5,6] treating isobaric states of nucleons phenomenologically, without penetrating into the internal structure of the nucleon. A theory describing isobaric states as manifestations of the internal dynamics of the nucleon was worked out long ago, namely, the "strong-coupling" theory of Wentzel, Pauli and Dancoff[7]. The chief obstacle to the development of this theory was its lack of relativistic invariance, arising from the "smearing-out" of the nucleon by a hypothetical form-factor. Because of the presence of such a form-factor $K(\mathbf{x}-\mathbf{x}') \neq \delta(\mathbf{x}-\mathbf{x}')$, signals are transmitted within the nucleon with greater than light velocity.

One way and another, all the existing proposals for escaping from infinite particle self-energies run up against the same fundamental difficulty. For example, up to now nobody has found a logically consistent formulation of a non-local theory[8]. In the development of non-linear theories[9,10], there are also fundamental difficulties; the infinite zero-point energy appears non-additively in the energies of excited states of a non-linear field, and the attempt to quantize the field by means of a discrete lattice-space[11] leads again, as it is easy to show, to signals propagating with infinite velocity.

Our purpose in the present work is to explain a point of view which justifies the introduction of a form-factor in strong-coupling theory, while remaining within the framework of contemporary quantum field theories. The questions connected with infinite particle self-energies are not resolved but are circumvented. The essence of our proposal is that the nucleon interacts with the pi-meson not directly but only through a hypothetical K-meson with a large mass M.

There are two ways of looking at this model. One may say that a short-range force with a range (h/Mc) is acting between nucleon and pion. Or one may say that the nucleon has a composite structure, consisting of a small core of size (h/Mc) and a pion cloud, just as an atom consists of a nucleus and an electron cloud. Furthermore, just as a relativistic theory of the electronic shell of an atom can be constructed without solving the problems of nuclear structure, a relativistic theory of the pion shell around a nucleon ought to be possible without a complete theory of the core. In what follows we present an outline of such a theory.

2. CALCULATION OF THE INTERACTION BETWEEN NUCLEON AND PION FIELD

We postulate the existence of K-mesons with mass M and charge $\pm e$ and 0. Their wave-function is denoted by θ_s(s = 1, 2, 3 corresponding to the

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charges $\pm e$, 0). For definiteness we suppose θ_s to be a pseudoscalar.

The pion wave-function we represent in the usual way by ϕ_s ($s = 1, 2, 3$), the nucleon wave-function by ψ. The interactions between K- and π-meson, and between K-meson and nucleon, cannot at this stage be uniquely determined. Since we are only explaining the general structure of the theory, it is reasonable to start with the simplest assumptions about the form of these interactions.

We suppose the interactions to have the form

$$W = \int dx \left[g \sum_{s=1}^{3} \theta_s \varphi_s + \frac{\sqrt{4\pi} f}{\varkappa} \sum_{s=1}^{3} \psi^+ \gamma_\mu \gamma_5 \tau_s \psi \frac{\partial \theta_s}{\partial x_\mu} \right]. \tag{1}$$

Here g and f are two coupling constants, $\kappa = Mc/h$, τ_s are the isotopic spin matrices and γ_μ, γ_5 are the Dirac matrices. Thus we assume a charge-symmetric scalar interaction between K- and π-meson, and a charge-symmetric pseudovector interaction between K-meson and nucleon. The complete Hamiltonian of the system is

$$H = H_1^0 + H_2^0 + H_3^0 + W = H^0 + W, \tag{2}$$

where H_1^0, H_2^0 and H_3^0 are the Hamiltonians for free nucleons, pions and K-mesons, respectively. The theory with the Hamiltonian (2) is obviously relativistically invariant, but it is not free from self-energy divergences produced by the (θ, ψ) and (θ, ϕ) interactions. However, these divergences lie at a deeper level, since they are related to the nucleon core and not to the pion cloud.

To find the pion-nucleon interaction, we try to eliminate the K-meson field from (2). We do this by means of a chain of Tamm equations. Let Ω be the wave-functional, depending on the fields θ, ϕ and ψ. For the variables describing the field θ, we take the number N_q^s of K-mesons in the charge-state s with momentum q. We expand the field θ in a Fourier series

$$\theta_s = \sum_q \frac{1}{\sqrt{2\varepsilon_q}} \left[\theta_s(q) e^{i\mathbf{qx}} + \theta_s^+(q) e^{-i\mathbf{qx}} \right], \tag{3}$$

where $\theta_s(q)$ is an annihilation operator and $\theta_s^+(q)$ is a creation operator for a K-meson of energy ϵ_q. Further, we write $\Omega(0)$ for a state without K-mesons, $\Omega(1_q^s)$ for a state with one K-meson (s, q), etc. The equation

$$(H_0 - E)\Omega = -W\Omega \tag{4}$$

then expands into the following chain of equations:

$$\begin{aligned}(H_0 - E)\Omega(0) = &-\left\{ \int d\mathbf{x} \left[g \sum_{s=1}^{3} \varphi_s(x) \sum_q \frac{1}{\sqrt{2\varepsilon_q}} e^{-i\mathbf{qx}} \right.\right. \\ &\left.\left. - i \frac{\sqrt{4\pi} f}{\varkappa} \sum_{s=1}^{3} D_\mu^s(x) \sum_q \frac{q_\mu}{\sqrt{2\varepsilon_q}} e^{-i\mathbf{qx}} \right] \right\} \Omega(1_q^s), \\ (H_0 + \varepsilon_q - E)\Omega(1_q^s) = &-\left\{ \int d\mathbf{x}' \left[g\varphi_s(x') \frac{2}{\sqrt{2\varepsilon_q}} e^{i\mathbf{qx}'} \right.\right. \\ &\left.\left. + i \frac{\sqrt{4\pi} f}{\varkappa} D_\mu^s(x') \frac{q_\mu}{\sqrt{2\varepsilon_q}} e^{i\mathbf{qx}'} \right] \right\} \Omega(0) + \cdots,\end{aligned} \tag{5}$$

where $D_\mu^s(x) = \psi^+ \gamma_\mu \gamma_5 \tau_s \psi$. Denoting the operator $(H_0 + \epsilon_q - E)^{-1}$ by Δ^{-1} (it is approximately equal to ϵ_q^{-1}) we obtain

$$\begin{aligned}(H_0 - E)\Omega(0) \\ = -\left\{ \iint d\mathbf{x}\, d\mathbf{x}' \left[g^2 \sum_{s=1}^{3} \varphi_s(x) \varphi_s(x') K(\mathbf{x} - \mathbf{x}') \right.\right. \\ + \frac{4\pi f^2}{\varkappa^2} \sum_{s=1}^{3} D_\mu^s(x) D_\nu^s(x') \frac{\partial^2 K(\mathbf{x} - \mathbf{x}')}{\partial x_\mu \partial x'_\nu} \\ \left.\left. + \frac{2\sqrt{4\pi} fg}{\varkappa} \sum_{s=1}^{3} D_\mu^s(x) \varphi_s(x') \frac{\partial K(\mathbf{x} - \mathbf{x}')}{\partial x'_\mu} \right] \right\} \Omega(0),\end{aligned} \tag{6}$$

with

$$K(\mathbf{x}-\mathbf{x}') = \int (e^{i\mathbf{q}(\mathbf{x}-\mathbf{x}')} / 2\varepsilon_q^2)\, d\mathbf{q} \tag{7}$$

where we use the approximation $\Delta^{-1} \approx \epsilon_q^{-1}$. The differentiation of $K(\mathbf{x}-\mathbf{x}')$ with respect to x_4' is to be first performed with $x_4' \neq x_4$, and then x_4' is to be set equal to x_4. When $x_4' = x_4$ the function $K(\mathbf{x}-\mathbf{x}')$ becomes

$$K(\mathbf{x}-\mathbf{x}') = e^{-\varkappa r} / r, \qquad r = |\mathbf{x}-\mathbf{x}'|. \tag{7'}$$

Equation (6) is an approximate equation for $\Omega(0)$. The first term in the square bracket, proportional to g^2, gives an interaction of pions among themselves. The second term gives a short-range interaction between nucleon cores. The third terms may be written in the form

$$V = \frac{\sqrt{4\pi}\, F}{\varkappa} \iint d\mathbf{x}\, d\mathbf{x}' \tag{8}$$

$$\times \sum_{s=1}^{3} \psi^+(x)\gamma_\mu\gamma_5\tau_s\psi(x) \frac{\partial K(\mathbf{x}-\mathbf{x}')}{\partial x_\mu'} \varphi_s(x')$$

where $F = 2fg$, and represents a pseudovector interaction of pions with extended nucleons, exactly as in the strong coupling theory. The function $K(\mathbf{x}-\mathbf{x}')$ now plays the role of form-factor. This factor has the effect that pions and nucleons do not interact at a point but over a region of size (h/Mc). For the theory to be consistent, it is necessary that $(h/Mc) \ll (h/\mu c)$, where μ is the pion mass.

3. CONCLUSIONS

Our proposal of an indirect pion-nucleon interaction thus actually implies the existence of a core of size (h/Mc) within the nucleon. The core is surrounded by a pion shell with size of order $(h/\mu c)$. In addition there appears automatically a very short-range interaction between nucleon cores, acting over a region of size (h/Mc), and a similar short-range interaction between pions. The core itself is described non-relativistically, just like the nucleus in a relativistic theory of the atom.

We intend later to develop the theory in a more consistent way, starting from the Lagrangian method, which will allow a more symmetrical relativistic treatment of the interaction. It would also be interesting to introduce an indirect interaction through K-mesons, renormalized so as to remove completely all the divergences of meson theory. However, it is doubtful whether any indirect interaction exists which would lead to an unrenormalized pseudovector renormalized interaction between pion and nucleon.

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