renormalized charge is equal to zero. In the case, however, where the "bare" charge is large (which apparently happens in quantum electrodynamics [2]), all the diagrams of higher order than e^2, not considered in section I, become extremely important and may radically alter the asymptote found. Using Eq. (1) and property I, and neglecting terms of the type $(\gamma_\nu\gamma_\mu - \gamma_\mu\gamma_\nu) f(p^2)$ in the expression for $\left.\frac{\delta\Gamma_\mu}{\delta x_\nu}\right|_{k=0}$, one can show that $\Pi(0)$ remains infinite in the general case‡. Therefore, it seems likely that in quantum electrodynamics the renormalized charge is equal to zero. Since the experimental charge in quantum electrodynamics is small, one could eliminate the difficulty of the zero charge by a correction to the interaction at very large momenta, and a calculation of gravitational effects could make it, generally speaking, not inconsistent with the mathematical theory[5,1]

* The proof of this relation will be given separately.
** $\Pi(0)$ is always ≥ 0.
‡ A discussion of the use of the fact that $\Pi(0)$ is infinite is also given in reference 4

[1] L. D. Landau, A. A. Abrikosov and I. M. Khalatnikov, Dokl. Akad. Nauk SSSR **95**, 773 (1954); **96**, 261 (1954)

[2] M. Gell-Mann and F. E. Low, Phys. Rev **95**, 1300 (1954)

[3] E. S. Fradkin, J. Exper. Theoret. Phys. USSR **26**, 753 (1954)

[4] G. Kallen, Dansk. Mat. Phys. Med. **27**, 12 (1953)

[5] M. A. Markov, J. Exper. Theoret. Phys. USSR **17**, 9 (1947)

Translated by M. Rosen
140

The Optimum Length of an Undulator

A. N. MATVEEV
Moscow State University

(Submitted to JETP editor January 24, 1955)
J. Exper. Theoret. Phys. USSR **28**, 760 (June, 1955)

As has been shown by Ginsburg [1], only the utilization of coherent radiation in an undulator [2,3], makes it possible to obtain considerable radiated power. At the same time, the power radiated from an undulator is directly proportional to its length (see, for example, reference 2); therefore it is desirable to increase the length of the undulator. However, these two contributions to the power radiated from an undulator are in contradiction with one another. As the electrons pass through the undulator, the conditions for coherent radiation become worse, in view of the spreading of the bunch. The resolution of this contradiction appears to be the imposition of a restriction on the length of the undulator.

Consider the definiteness, a spherical bunch of N electrons, in a coordinate system moving with the center of the bunch. Making use of the integral of motion $\frac{mv_r^2}{2} + \frac{e^2N}{r} = \frac{e^2N}{r_0}$, we find that the increase of the radius of the bunch by a factor p takes place during proper time $\Delta\tau$, according to the relation

$$\Delta\tau = \frac{1}{e}\sqrt{\frac{m}{2N}}\left[\sqrt{p(p-1)} + \ln\left(\sqrt{p} + \sqrt{p-1}\right)\right] r_0^{3/2}, \tag{1}$$

where r_0 is the initial radius of the bunch, with the initial rate of expansion assumed to be zero, and m and e are the rest mass and charge of the electron, respectively.

The condition for coherent radiation is that the dimensions of the electron bunch be small in comparison with the wavelength of the radiated waves. In order to obtain the wavelength λ in the laboratory system of coordinates, it is necessary to generate the wavelength $\lambda/\sqrt{1-\beta^2}$ in the coordinate system (with velocity $v = c\beta$) moving with the center of the oscillating bunch. In view of this, the condition for coherence is well fulfilled when $r_0 \sim \lambda$, and is completely violated when $p \sim E/mc^2$. When, as was assumed in references 1-3, the fields in the undulator are not too large, the velocities of the electrons are small with respect to a coordinate system moving with the center of the oscillating bunch, and therefore, an interval of proper time of the bunch is approximately equal to an interval of time in the coordinate system moving with the center of the oscillating bunch. Therefore, on the basis of Eq. (1), the time for complete loss of coherence in the laboratory system of coordinates is

$$\Delta t \approx \frac{1}{e}\sqrt{\frac{m}{2N}}\left(\frac{E}{mc^2}\right)^2 \lambda^{3/2}. \tag{2}$$

Denoting the length of the spatial period of the field in the undulator by l_0, and taking into account the equality $\lambda \sim l_0\left(\frac{mc^2}{E}\right)^2$, we find that the number q of spatial periods of the field in the undulator must in any case be no greater than

$$q_{\max} = \frac{c}{e}\sqrt{\frac{m}{2N}}\sqrt{\lambda} \approx \frac{10^6}{\sqrt{N}}\sqrt{\lambda} \quad (\lambda \text{ cm}). \tag{3}$$

For example, with $\lambda \sim 1$mm and $N \sim 10^8$, we get $q_{max} \sim 30$. The existence of an initial velocity spread of the bunch when entering the undulator, and the impossibility of providing equally good injection conditions for each of the electrons of the bunch, shortens considerably the maximum possible length of the undulator. Therefore, an undulator with too great a number of spatial periods of the field is undesirable.

[1] V. L. Ginsburg, Izv. Akad. Nauk SSSR, Ser. Fiz. **11**, 165 (1947)

[2] H. Motz, J. Appl. Phys. **22**, 527 (1951)

[3] H. Motz, W. Thon and R. N. Whitehurst, J. Appl. Phys. **24**, 826 (1953)

Translated by D. Lieberman
145

On the Paper "The Excitation Spectrum of a System of Many Particles"

V. P. SILIN
P. N. Lebedev Institute of Physics, Academy of Sciences, USSR

(Submitted to JETP editor February 7, 1955)
J. Exper. Theoret. Phys. USSR **28**, 749-750 (June, 1955)

In considering the correlation of identical particles, one should distinguish between the correlation of particles which are in the same spin state and the correlation of particles in different spin states. In reference 1 an approximation to the binary distribution was used appropriate for the calculation of the correlation of identical particles in the same spin state. Strictly speaking, this is realized only in the case of particles having no spin. Hence the results of reference 1 are completely valid in the case of spinless Bose particles. However, in the case of electrons, for example, it is necessary, generally speaking, to make several further considerations.

Let $\rho^{(+)}(q_1', q_1)$ and $\rho^{(-)}(q_1', q_1)$ be the density matrices for electrons with spin projections $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively. For particles in the same spin state, the coordinate part of the wave function is anti-symmetric. Hence the binary density matrix can be approximated as follows:

$$\rho_2^{(+,+)}(q_1', q_2'; q_1, q_2) = \rho^{(+)}(q_2', q_1)\,\rho^{(+)}(q_2', q_2) - \rho^{(+)}(q_1', q_2)\,\rho^{(+)}(q_2', q_1). \quad (1)$$

In the case of particles in different spin states, the coordinate part of the wave function is symmetric; therefore*

$$\rho_2^{(+,-)}(q_1', q_2'; (q_1, q_2) = \rho^{(+)}(q_1', q_1)\,\rho^{(-)}(q_2', q_2) + \rho^{(+)}(q_1', q_2)\,\rho^{(-)}(q_2', q_1). \quad (2)$$

Relations (1) and (2) allow one to obtain the following equation for the quantum distribution function $f^{(+)}(\mathbf{q},\mathbf{p})$ of an electron with a positive spin projection. (The notation used is that adopted in reference 1):

$$\frac{\partial f^{(+)}}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f^{(+)}}{\partial \mathbf{q}} + \frac{i}{\hbar}\frac{1}{(2\pi)^3} \times \int d\mathbf{q}' d\vec{\tau} d\mathbf{p}' d\mathbf{p}'' \left[U\left(\left|\mathbf{q}-\mathbf{q}'+\frac{\hbar\vec{\tau}}{2}\right|\right) - U\left(\left|\mathbf{q}-\mathbf{q}'-\frac{\hbar\vec{\tau}}{2}\right|\right)\right] \times \Big\{ e^{i\vec{\tau}(\mathbf{p}''-\mathbf{p})} f^{(+)}(\mathbf{q},\mathbf{p}'')\,[f^{(+)}(\mathbf{q}',\mathbf{p}') + f^{(-)}(\mathbf{q}';\mathbf{p}')] + \exp\left[i\vec{\tau}\left(\frac{\mathbf{p}'+\mathbf{p}''}{2} - \mathbf{p}\right) + \frac{i(\mathbf{q}'-\mathbf{q})(\mathbf{p}'-\mathbf{p}'')}{\hbar}\right] f^{(+)}\left(\frac{\mathbf{q}+\mathbf{q}'}{2} - \frac{\hbar\vec{\tau}}{4}, \mathbf{p}'\right) \times \left[f^{(-)}\left(\frac{\mathbf{q}+\mathbf{q}'}{2} + \frac{\hbar\vec{\tau}}{4}, \mathbf{p}''\right) - f^{(-)}\left(\frac{\mathbf{q}+\mathbf{q}'}{2} + \frac{\hbar\vec{\tau}}{4}, \mathbf{p}''\right)\right]\Big\} = 0. \quad (3)$$

The equation for $f^{(-)}$ is obtained from Eq. (3) by making the substitutions $(+) \to (-)$, and $(-) \to (+)$. Being interested in the fluctuations of the density, let us look at the equation for the function $f = f^{(+)} + f^{(-)}$. This equation is:

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f}{\partial \mathbf{q}} + \frac{i}{\hbar}\frac{1}{(2\pi)^3}\int d\mathbf{q}' d\vec{\tau} d\mathbf{p}' d\mathbf{p}'' \left[U\left(\left|\mathbf{q}-\mathbf{q}'+\frac{\hbar\vec{\tau}}{2}\right|\right) - U\left(\left|\mathbf{q}-\mathbf{q}'-\frac{\hbar\vec{\tau}}{2}\right|\right)\right] \times \Big\{ e^{i\vec{\tau}(\mathbf{p}''-\mathbf{p})} f(\mathbf{q},\mathbf{p}'') f(\mathbf{q}',\mathbf{p}') - \Phi\left(\frac{\mathbf{q}+\mathbf{q}'}{2} - \frac{\hbar\vec{\tau}}{4}, \mathbf{p}'\right) \times \Phi\left(\frac{\mathbf{q}+\mathbf{q}'}{2} + \frac{\hbar\vec{\tau}}{4}, \mathbf{p}''\right) \times \exp\left[i\vec{\tau}\left(\frac{\mathbf{p}'+\mathbf{p}''}{2} - \mathbf{p}\right) + i(\mathbf{q}'-\mathbf{p})(\mathbf{p}'-\mathbf{p}'')/\hbar\right]\Big\} = 0, \quad (4)$$

Where $\Phi = f^{(+)} = f^{(-)}$. Equation (4) differs from