Letters to the Editor

Calculation of the Energy Distribution Function of Neutrons by Markov's Method

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AN attempt is made to investigate by Markov's method some questions in the theory of slowing down of neutrons due to elastic collisions with the nuclei of the scatterer[1-6]. The results obtained are also applicable to slowing down by thin foils when the number of elastic collisions is not too large (between 25 and 30).

According to references 1 and 2, the normalized probability $f_0(u_j)\,du_j$, that the quantity $u_j = \ln(E_{j-1}/E_j)$ will lie in the interval $(u_j, u_j + du_j)$ can be written as:

$$f_0(u_j)\,du_j = \frac{(A+1)^2}{4A} e^{-u_j} du_j \text{ if } u_j \leqslant \ln\left(\frac{A+1}{A-1}\right)^2, \qquad f_0(u_j)\,du_j = 0 \text{ if } u_j > \ln\left(\frac{A+1}{A-1}\right)^2, \tag{1}$$

where E_{j-1} and E_j are, respectively, the neutron energy before and after the jth collision, and A is the atomic mass of the scattering nuclei (assuming that $A \neq 1$). We have the identity

$$\frac{E_0}{E_n} = \frac{E_0}{E_1}\frac{E_1}{E_2} \cdots \frac{E_{j-1}}{E_j} \cdots \frac{E_{n-1}}{E_n}, \tag{2}$$

where E_0 is the initial energy of the neutron and E_n the energy after n collisions. From Eq. (2):

$$\ln\frac{E_0}{E_n} = \ln\frac{E_0}{E_1} + \ln\frac{E_1}{E_2} + \cdots + \ln\frac{E_{j-1}}{E_j} + \cdots + \ln\frac{E_{n-1}}{E_n} \tag{3}$$

or

$$U = u_1 + u_2 + \cdots + u_n, \tag{3'}$$

where

$$U = \ln(E_0/E_n).$$

We can consider (3′) as a stochastic equation. All the u_j's are stochastic variables, and we are confronted with the typical stochastic problem of summing n random variables. With the help of references 6 and 7, it is easy to write the function $A_n(\rho)$.

$$A_n(\rho) = \prod_{j=1}^{n} \int_0^{q_A} e^{i\rho\varphi_j} f_0(u_j)\,du_j, \tag{4}$$

where the function[7,8] ϕ_j can simply be defined as $\phi_j = u_j$; we denote $\ln[(A+1)/(A-1)]^2$ by q_A. Taking into account the fact that by definition $f_0(u_j)\,du_j$ does not depend on j, it is easily shown that

$$A_n(\rho) = \left[\frac{(A+1)^2}{4A}\right]^n \frac{(\exp\{(i\rho-1)\,q_A\}-1)^n}{(i\rho-1)^n}. \tag{4'}$$

Then the probability that, after n collisions, the quantity U lies in the previous given interval $(U, U+dU)$ is exactly

$$W_n(U)\,dU = \frac{dU}{2\pi}\int_{-\infty}^{+\infty} e^{-i\rho U} A_n(\rho)\,d\rho,$$

$$W_n(U) = \left[\frac{(A+1)^2}{4A}\right]^n \frac{e^{-U}}{(n-1)!} \times \sum_{m=0}^{n} (-1)^m C_n^m \left[(n-m)\,q_A - U\right]^{n-1}, \tag{5}$$

where the terms in the summation are different from zero when $(n-m)\,q_A - U > 0$.

In the case of scattering by hydrogen, we obtain for $A_n(\rho)$

$$A_n(\rho) = (-1)^n/(i\rho-1)^n \tag{6}$$

and the distribution function becomes

$$W_n(U) = U^{n-1}e^{-U}/(n-1)! \tag{7}$$

The distribution thus found is Poisson's distribution. In the case $n = 1$, we obtain Eq. (1). The results obtained here show that the method of Markov[4-7] permits a rather simple solution of some problems arising in the theory of the slowing down of neutrons.

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On the Angular Distribution of β-Radiation. II

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THE analysis of β-radiation from oriented nuclei is of considerable interest, as relative measurements of angular distribution of β-radiation, in conjunction with theory, can give valuable information about the spins, parities and magnetic moments of β-radioactive nuclei.

A theoretical analysis of angular distribution of β-radiation has been carried out in references 1 and 2. Reference 2 in particular gives the expression $W(E, \theta)$ for the distribution (as a function of energy and of angle) of β-particles emitted by oriented nuclei for the transition $\Delta I = \pm 2$, "yes" (I is the nuclear spin, "yes" denotes that the parity of the nucleus changes upon emission of a β-ray). However the introduction of this expression in reference 2 applies only in the Born approximation.

By comparing the expression for $W(E, \theta)$ obtained in the Born approximation with the expression for the β-spectrum of the corresponding transition (including Coulomb effects[3]), it becomes quite simple to include Coulomb effects in the formula for angular distribution: in order to carry this out it is sufficient to multiply by ϕ_1 the term proportional to p^2 in the square brackets of Eq. (2), reference 3, and to multiply by ϕ_0 the term proportional to q^2; according to reference 3 (using the notation and units of reference 2):

$$\varphi_0 = \frac{1 + s_0}{2} F_0(Z, E), \tag{1}$$

$$\varphi_1 = \frac{2 + s_1}{4} F_1(Z, E). \tag{2}$$

where

$$s_n = \sqrt{(n+1)^2 - (\alpha Z)^2}, \tag{3}$$

$$\xi = Z\alpha / \beta, \tag{4}$$

and the Coulomb factors F_0 and F_1 are given by the formula

$$F_n(Z, E) = \frac{[(2n+2)!]^2}{(n!)^2 [\Gamma(1 + 2s_n)]^2} \times (2pR)^{2(s_n - n - 1)} \exp(\pi\xi) \, |\Gamma(s_n + i\xi)|^2. \tag{5}$$

In Eqs. (1) - (5), α denotes the fine structure constant, R the nuclear radius, β the electron velocity, Z the nuclear charge (in the case of positron emission, ξ must be replaced by $-\xi$).

Finally we obtain for the distribution of β-radiation as a function of energy and angle, for the transition $\Delta I = \pm 2$, "yes":

$$W(E, \vartheta) = {}^1/_3 (2\pi)^{-5} \pi G^2 |B_{ik}|^2 pEq^2 \times \{q^2\varphi_0 + p^2\varphi_1 [1 - a(I) f_2 P_2(\cos\vartheta)]\}. \tag{6}$$

Note that Eq. (6) can be transformed into Eq. (2) of reference 2 by neglecting terms of the order of $(aZ)^2$.

Translated by M. A. Melkanoff
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The Absorption of Ultrasonic Waves in Armco Iron and Plexiglass

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THERE are a number of works devoted to the measurement of the ultrasonic absorption coefficient in solids, although the number is still comparatively small. In many metals and dielectrics, no measurements have as yet been made, and the existing theory of the mechanism of this