Letters to the Editor

The Theory of Multiple Production of Particles at High Energy

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EXPERIMENTAL data obtained to date allows us to assert that the "nuclear charge" is conserved in all nuclear phenomena. In the investigation of phenomena of multiple production of nucleons at high energies[1] the conservation of the "nuclear charge" has been taken into account only for the assumption that particles and antiparticles are produced in equal numbers. Actually, in the collisions of nucleons with nuclei there are several initial nucleons (not less than two). We consider here in more detail the influence of the "nuclear charge" conservation on the production of heavy particles at high energies*.

Recent theoretical research has been carried out on the multiple production of particles, based on the methods of thermodynamics and hydrodynamics [3-5]. In nucleon-nucleon or nucleon-nucleus collisions, a system is formed in which a high energy is concentrated in a very small region. Then this system expands very rapidly and when its size has become sufficiently large, it decays into separate particles. The stage of decomposition depends on the temperature kT of the system with $kT \approx m_\pi c^2$, where m_π is the π-meson mass. The density of particles of different kinds is given by the equations

$$n_{NN} = \frac{g_N}{2\pi^2}\left(\frac{kT}{\hbar c}\right)^3 F^+(z, y_{NN}), \quad (1)$$

$$n_{AN} = \frac{g_N}{2\pi^2}\left(\frac{kT}{\hbar c}\right)^3 F^+(z, y_{AN}), \quad (2)$$

$$n_\pi = \frac{g_\pi}{2\pi^2}\left(\frac{kT}{\hbar c}\right)^3 F^-(z, 0). \quad (3)$$

Here n_{NN} and n_{AN} are the densities of nucleons and antinucleons, n_π is the density of π-mesons, $g_N = 4$, $g_\pi = 3$,

$$F^\pm(z, y) = z^3 \int_0^\infty \frac{x^2\, dx}{\exp\{-y + z\sqrt{1+x^2}\} \pm 1}; \quad (4)$$

$z = Mc^2/kT$ for nucleons and $z = m_\pi c^2/kT$ for π-mesons, $y = \mu/kT$ where μ is the chemical potential.

The equilibrium condition with respect to pair production and pair annihilation will be $y_{NN} + y_{AN} = 0$. Thus, if we denote y_{NN} by y, then y_{AN} will be equal to $-y$.

We shall consider the case where $z > 1$ and $y < z$. In Eq. (4) we expand the denominator in a power series of $\exp\{y - z\sqrt{1+x^2}\}$, perform the integration and limit ourselves to the first term of the expansion, We obtain

$$F^+(z, y) = F^+(z, 0)\, e^y, \quad (5)$$

$$F^+(z, -y) = F^+(z, 0)\, e^{-y}.$$

Now it is not difficult to establish the following relations

$$\sinh y = \frac{N_{\pi 0}}{N_{N_0}} \frac{N_0}{N_\pi}; \qquad \cosh y = \frac{N_{\pi 0}}{N_{N_0}} \frac{N_N}{N_\pi}.$$

Here $N_{\pi 0}$ and N_{N0} are the total number of π-mesons and of nucleons and antinucleons produced in the system with the condition that the initial nucleons are not included, N_0 is the number of initial nucleons, N_π and N_N are the total numbers of π-mesons and of nucleons and antinucleons, where the existence of the initial nucleons is taken into account. Hence we obtain

$$N_N / N_\pi = \sqrt{(N_{N_0} / N_{\pi 0})^2 + (N_0 / N_\pi)^2}. \quad (6)$$

The number of nucleons N_{NN} and the number N_{AN} of antinucleons are

$$N_{NN} / N_\pi = {}^1\!/_2 \left[\sqrt{(N_{N_0} / N_{\pi 0})^2 + (N_0 / N_\pi)^2} + N_0 / N_{\pi 0}\right], \quad (7)$$

$$N_{AN} / N_\pi = {}^1\!/_2 \left[\sqrt{(N_{N_0} / N_{\pi 0})^2 + (N_0 / N_\pi)^2} - N_0 / N_{\pi 0}\right].$$

*Schiff[2] has indicated that it is necessary to take into account the existence of the initial nucleons in the Fermi theory of multiple production; however he himself has not done so. It will be remembered that, according to Fermi, the decay of the system into separate particles takes place at the temperature $kT > Mc^2$ where M is the nucleon mass. Therefore in the frame of the Fermi theory the influence of the initial nucleons is not essential.

[1] E. Fermi, Prog. Theor. Phys. **5**, 570 (1950)
[2] L. Schiff, Phys. Rev. **85**, 374 (1952)
[3] L. D. Landau, Izv. Akad. Nauk. SSSR Ser. Fiz. **17**, 51 (1953)
[4] I.L. Rozental and D.S. Chernavskii, Usp. Fiz. Nauk **52**, 185 (1954)
[5] I. Ia. Pomeranchuk, Doklady Akad. Nauk SSSR **78**, 889 (1951)

We turn now to the energy of the nucleons and π-mesons. It is easy to see that the ratio of the total energy density E_N of the nucleons and the antinucleons to E_π, the energy density of the π-mesons is equal to:

$$E_N \; E_\pi = \sqrt{1 + (N_{\pi 0} / N_{N_0})^2 (N_0 / N_\pi)^2} E_{N_0} / E_{\pi 0}. \quad (8)$$

where E_{N0} and $E_{\pi 0}$ are the energy density of the nucleons and π-mesons for $N_0 = 0$. Equation (8) gives the ratio of the energy taken away by the nucleons and the π-mesons.

Let the critical temperature T_k at which the decay of the system into separate particles takes place, be equal to $1.2_\pi mc^2$. Then, according to a previous paper [6], $E_{N0} / E_{\pi 0} = 0.3$, $N_{N0} / N_{\pi 0} = 0.13$. If we take $N_0 / N_\pi = 0.15$, then $E_N / E_\pi = 0.42$. If we suppose $N_0/N_\pi = 1$, then $E_N/E_\pi = 2.3$. This means that, for $N_\pi = N_0$, the nucleons carry away about 70% of the total energy. Thus the consideration of the initial nucleons gives a larger share of energy for the nucleons. This effect is particularly large for not too high energy values, i.e. when the numbe fof π-mesons produced is small. Qualitatively, the results obtained are in agreement with the experimental data obtained by Grigorov et al.[7] for energies of the order 10^{10} to 10^{11} eV. It is necessary however to emphasize that this theory represents only a rough approximation for such energies.

Equation (8) for the energy ratio contains two parameters: the temperature T_k of decay of the system into separate particles, and the ratio N_0/N_π. Both parameters are unknown. It is possible, however, to form a quantity which does not depend on N_0 / N_π and the measurement of which would permit a direct determination of the decay temperature T_k of the system. It is not difficult to see that the energy attributed to each nucleon does not depend on the chemical potential, i.e., on the number of initial nucleons. (This is correct if we use a relativistic Maxwell-Boltzmann distribution instead of a Fermi distribution.) Let us consider now the following ratio of energies: the mean energy of the nucleons divided by the mean energy of the π-mesons. We call it $\propto$ with $\propto = (E_N/E_\pi)(n_\pi/n_N)$. This quantity depends only on the decay temperature T_k. Table 1 gives the value of $\propto$ computed from our previous paper [6] for different temperatures T_k.

Collisions with nuclei can also produce heavier particles (the Λ-particles), possessing nucleon charges. These particles can be included in our consideration. If we suppose that these particles, like the nucleons, have spin ½, we obtain in particular that Eqs. (6) to (8) are still valid in the presence of Λ-particles, if we understand by N_N the sum of the numbers of nucleons, antinucleons, Λ-particles and anti-Λ-particles produced for $N_0 = 0$, and so on....In Table 2 we give the ratios N_{NN}/N_π, $N_{\Lambda N}/N_\pi$, N_{AN}/N_π, $N_{A\Lambda}/N_\pi$ (where $N_{\Lambda N}$ is the number of Λ-particles and $N_{A\Lambda}$ the number of anti-Λ-particles) for different values of N_0/N_π. For this computation, g_Λ has been taken equal to 4 and M_Λ (the Λ-particle mass) equal to 2200 m_e.

Table 1

kT in units of $m_\pi c^2$	$\propto$	kT in units of $m_\pi c^2$	$\propto$
$kT \ll m_\pi c^2$	6.8	1.5	2.16
0.5	3.76	2	1.83
0.7	3.28	3	1.6
0.8	2.92	4	1.4
1	2.65	$kT \gg m_\pi c^2$	1.17*

*This last value has been computed using a Fermi distribution for the nucleons and a Bose distribution for the π-mesons.

Table 2

kT in units of $m_\pi c^2$	N_0/N_π	N_{NN}/N_π	$N_{\Lambda N}/N_\pi$	N_{AN}/N_π	$N_{A\Lambda}/N_\pi$
	0	0.063	0.027	0.063	0.027
1.2	0.15	0.135	0.057	0.029	0.013
	0.30	0.23	0.096	0.018	0.008

In Table 2 we see that the existence of initial nucleons modifies essentially the ratio between the numbers of Λ-particles and antinucleons produced. For instance, for $kT_k = 1.2 m_\pi c^2$ and $N_0 = 0$, the number of antinucleons produced is 2.3 times larger than the number of Λ-particles produced. But for $N_0 / N = 0.15$ (which corresponds to 3 initial nucleons when 20 π-mesons are produced) the number of Λ-particles is already twice the number of antinucleons. If the spin of the Λ-particles is larger than 1/2 and particles are formed at $T > T_k$, our quantitative relations are changed, but our qualitative deductions are still valid.

[6] S. Z. Belenkii, Doklady Akad. Nauk SSSR 99, 523 (1954)

[7] N. L. Grigorov and V. S. Murzin, Izv. Akad. Nauk SSSR, Ser. Fiz. 17, 21 (1953)

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