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# Scattering of Mesons by Nucleons in the Theory of Radiation Damping

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When radiation damping is taken into account, a qualitative improvement is obtained in the picture given by perturbation theory in the lowest nonvanishing order (the appearance of characteristic maxima in the energy dependence of total cross sections, etc.). However quantitative agreement with experiment is still lacking.

EXPERIMENTALLY, scattering of mesons by nucleons has been studied by various methods [1-3] (see also review article of Silin and Fainberg[4]) Mesons of bombarding energies from 35 to 230 MeV have been considered. We shall not enter into the well known results of these investigations, but will only point out two characteristic features, namely, that at low energies ($E_\mu$ = 35 MeV) the following relation is satisfied by the cross sections:

$$\sigma^{(\alpha)} : \sigma^{(\gamma)} : \sigma^{(\beta)} \approx 2:1:1; \;\; \sigma^{(\gamma)} \approx \sigma^{(\beta)},$$

whereas at higher energies ($E_\mu$ = 120 MeV) this relation takes the form:

$$\sigma^{(\alpha)} : \sigma^{(\gamma)} : \sigma^{(\beta)} \approx 9:2:1.$$

Here $\sigma^{(\alpha)}, \sigma^{(\beta)}, \sigma^{(\gamma)}$, represent cross sections for the following processes, respectively,

$$\pi^+ + p \rightarrow \pi^+ + p, \qquad (\alpha)$$

$$\pi^- + p \rightarrow \pi^- + p, \qquad (\beta)$$

$$\pi^- + p \rightarrow \pi^0 + n. \qquad (\gamma)$$

There are known in literature various attempts at theoretical explanation of the data on scattering of mesons by nucleons[4]. In the article Biswas[5], the conclusion is reached that, by taking into account radiation damping, a picture is obtained for the scattering of mesons by protons that is close to experiment. We show below that this conclusion is wrong. The results of Biswas are essentially based on conclusions reached in the works of Cornaldesi and Field[6,7] where errors were comitted in performing the reduction over spin states (when deriving the differential cross section). As a result of these errors pseudoscalar coupling led to an increase of the total scattering cross section with an increase in the energy $E_\mu$ of the bombarding meson. This contradicts the results of numerous other investigations.

In the present work use is made of the theory of radiation damping in a covariant form. All calculations are performed according to the perfected methods of perturbation theory [8-11]; a pseudoscalar meson field is chosen and the coupling between the meson field and the nucleon is taken to be a linear combination of pseudoscalar and pseudovector couplings.

To calculate the radiation damping (which is the purpose of this work) one must solve the integral equation which defines the scattering matrix $R$:

$$R = K - (i/2)\, KR. \qquad (1)$$

Equation (1) follows from

$$S = 1 - iR, \;\; S = \left(1 - \tfrac{i}{2} K\right) \Big/ \left(1 + \tfrac{i}{2} K\right),$$

where $S = 1 + \sum_{n=1}^{\infty} S_n$ is the unitary collision matrix, $K = \sum_{n=0}^{\infty} K_n$ is a hermitian operator (see Schwinger [12]).

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We solve Eq. (1) in the lowest nonvanishing order of approximation for second order processes (scattering processes), i.e.,the equation

$$R_2 = K_2 - (i/2)\, K_2 R_2, \qquad (2)$$

where $K_2$ is the first nonvanishing term in the expansion of $K$. In the second term on the right hand side of Eq. (2) a sum over all intermediate states of the same energy and charge is understood.

To obtain the scattering amplitude one needs a clear picture of $K_2$. It is known[8] that $K_2 = iS_2$; thus, given the matrix elements of $S_2$, we can obtain those of $K_2$.

As is well known one obtains from Eq. (2) an integral equation containing a nonseparable kernel which leads to well known difficulties in obtaining a solution. However, taking advantage of the smallness of certain quantities (for example the quantity $x_1 = 2l^2/(2E\epsilon - \mu^2)$; for $E_\mu \lesssim 180$ MeV we have $x_1 \leq 1/8$), one can obtain an approximate integral equation with a separable kernel. For process ($\alpha$) this equation is:

$$X_{fi}(\theta) = \frac{f^2}{\mu^2} M \Big\{\Big[\Big(G_1' - x_1\cos\theta\Big)\Big(\frac{W}{M}\gamma_4 - 1\Big) \qquad (3)$$

$$- 2(\Gamma y + 1)(1 + x_1\cos\theta)\Big](1 - x_1\cos\theta)$$

$$- i\omega\int_0^\pi \Big[(G_1' - x_1\cos\theta\cos\theta')\Big(\frac{W}{M}\gamma_4 - 1\Big)$$

$$- 2(\Gamma y + 1)(1 + x_1\cos\theta\cos\theta')\Big]$$

$$(E\gamma_4 + M) X_{fi}(\theta')\sin\theta' d\theta'\Big\}$$

where $l$ = momentum of the meson or nucleon in the center of mass system; $\mu$, $M$ the masses of meson and nucleon; $\epsilon$, $E$ total energies of meson and nucleon; $g$, $f$ pseudoscalar and pseudovector coupling constants, respectively,

$$h = c = 1;\ G_1' = f(E, \varepsilon, l);$$

$$\Gamma = \frac{\mu}{M};\ y = -\frac{g}{f};$$

$$W = E + \varepsilon;\ \omega = \frac{l}{8\pi W}.$$

Equation (3) is reduced by standard methods to a system of two algebraic equations. The matrices $\gamma_\mu$ are assumed to be numbers close to unity. The solution of Eq. (3) has the following form:

$$X_{fi}(\theta) = \frac{f^2}{\mu^2} M\Big\{\Big[f_1(\theta) + \frac{A + iB}{L + iM} + \frac{A_1 + iB_1}{L_1 + iM_1}\cos\theta\Big]$$

$$+ \Big[f_2(\theta) + \frac{C + iD}{L + iM} + \frac{C_1 + iD_1}{L_1 + iM_1}\cos\theta\Big]\gamma_4\Big\}, \qquad (4)$$

where $f_1(\theta) = \alpha_3 + \beta_3\cos\theta + \bar{\gamma}_3\cos^2\theta$;

$f_2(\theta) = \alpha_4 + \beta_4\cos\theta + \bar{\gamma}_4\cos^2\theta$,

and $A$, $B$,.. $\bar{\gamma}_3$, $\bar{\gamma}_4$ are expressions depending on $E, \epsilon, l, y, \omega, f^2$.

The integral equations for processes ($\beta$) and ($\gamma$) reduce to a system of four algebraic equations; the solutions are similar to (4). The differential scattering cross section, for process ($\alpha$),for example,looks as follows in the center of mass system:

$$\frac{d\sigma^{(\alpha)}}{d\Omega} = \frac{1}{2}\frac{M^2}{W^2}|X_{fi}(\theta)|^2.$$

Since the resulting expression for the differential scattering cross section is very cumbersome, we do not write it out in full.

For purposes of numerical calculations we write the differential scattering cross sections as follows:

$$d\sigma^{(\alpha)}/d\Omega = \varkappa(\bar{A}_1 + \bar{B}_1\cos\theta + \bar{C}_1\cos^2\theta \qquad (5)$$

$$+ \bar{D}_1\cos^3\theta + \bar{E}_1\cos^4\theta + \bar{F}_1\cos^5\theta);$$

$$d\sigma^{(\beta)}/d\Omega = \varkappa(\bar{A}_2 + \bar{B}_2\cos\theta);$$

$$d\sigma^{(\gamma)}/d\Omega = \varkappa(\bar{A}_3 + \bar{B}_3\cos\theta + \bar{C}_3\cos^2\theta$$

$$+ \bar{D}_3\cos^3\theta + \bar{E}_3\cos^4\theta + \bar{F}_3\cos^5\theta),$$

where

$$\varkappa = 2\pi f^4\Big(\frac{h}{\mu c}\Big)^2\frac{M^2}{W^2}\frac{x\gamma}{2};\ x = \frac{E}{\mu};\ \gamma = \frac{M}{\mu},$$

and $\bar{A}_1 = A_1 + A_1^r, \ldots;\ \bar{F}_3 = F_3 + F_3^r$.

Here, the first term in $\bar{A}_1, \ldots, \bar{F}_3$ is obtained by neglecting radiation damping, the second being the correction due to the damping.

Differential cross sections were computed for two energies $E_\mu$ -- 42 and 112 MeV -- of the bombarding meson in the laboratory coordinate system. Numerical estimates have shown that the coefficients $C_1$, $D_1$, $E_1$, $F_1$, $B_2$, $E_3$, $F_3$ are small in comparison with $A_1$, $B_1$, $A_2$, $A_3$, $B_3$, $C_3$. Assuming that at low energies ($E_\mu = 42$ MeV) the influence of radiation damping is small, and comparing numerical data with those of other authors[1-3] we obtain in an unique way the constants $\gamma$ and $f^2$.

The numerical data indicate that values $\gamma < 0$ are unacceptable, because they do not lead to values of $\sigma^{(\alpha)}/\sigma^{(\beta)}$ as given by experiments[2,3]. The calculation gives $\sigma^{(\alpha)}/\sigma^{(\beta)} > 1$ for $E_\mu = 42$ MeV, whereas the experimental value is of the order of ½; in addition, $d\sigma^{(\alpha)}/d\Omega$ shows a sharp forward directionality for $\gamma < 0$. It also follows from the numerical data that $\gamma = 10$ and 3 are inadmissible values: for these values of $\gamma$, the total scattering cross sections increase with the energy of the bombarding mesons more slowly than the experiments indicate[1-3]. Using the value $\gamma = 1.5$, we get satisfactory agreement with the experimental value of $\sigma^{(\alpha)}/(\sigma^{(\beta)} + \sigma^{(\gamma)})$, namely : $\sigma^{(\alpha)}/(\sigma^{(\beta)} + \sigma^{(\gamma)}) \approx 1$; $\sigma^{(\beta)} \gtrsim \sigma^{(\gamma)}$.

As far as angular distributions are concerned, one finds that with $\gamma = 1.5$, $E_\mu = 42$ MeV, $d\sigma^{(\alpha)}/d\Omega$ indicates that the mesons are scattered predominantly in the backward direction, $d\sigma^{(\beta)}/d\Omega$ indicates an isotropic distribution, and $d\sigma^{(\gamma)}/d\Omega$ indicates that the mesons are scattered in a backward direction, in the center of mass system. At low energies angular distributions have not been directly measured and therefore it is not clear whether the above results contradict experiment or not. However the few investigations[2,3] (see review article, reference 4) indicate that in the energy region $E_\mu = 30$ MeV, and for small angles, the interference between nuclear and electromagnetic interactions must be considered.

The results obtained for angular distributions for $E_\mu = 42$ MeV are qualitatively valid also for $E_\mu = 112$ MeV (disregarding damping). In this case the coefficient $C_1$ in $d\sigma^{(\alpha)}/d\Omega$ is relatively small, although the angular distribution indicates a backward directionality.

We note that the relation between the coefficients $A_3$, $B_3$, $C_3$, in Eq. (5) for $d\sigma^{(\gamma)}/d\Omega$ ( $A_3 < B_3 < C_3$ ) is the opposite of that for $d\sigma^{(\alpha)}/d\Omega$, i.e., closer to the experimental results. Furthermore, the calculated $d\sigma^{(\beta)}/d\Omega$ disagrees with the angular distribution of $\pi$ - mesons as given by experiments: $\pi^-$-mesons are scattered mostly forward. Also, the calculated value of $\sigma^{(\alpha)}/(\sigma^{(\beta)} + \sigma^{(\gamma)})$ disagrees with the experimental value.

Using the best values of the constants: $\gamma = 1.5$, $f^2 = 0.51$, one can compute the coefficients $A_1^r$ ,..., $F_3^r$, which account for the damping. The corrections due to the damping turn out to be of the order of a few percent; hence it is clear that they cannot change the relation $\sigma^{(\alpha)}/(\sigma^{(\beta)} + \sigma^{(\gamma)}) < 1$ which we obtain for $E_\mu = 112$ MeV and $\gamma = 1.5$, and therefore cannot explain the experimental relation $\sigma^{(\alpha)}/(\sigma^{(\beta)} + \sigma^{(\gamma)}) \approx 3$ for $E_\mu = 120$ MeV. We therefore deduce that quantum theory of radiation damping in the above treated approximation does not lead to quantitative agreement with experiment, although it does predict certain qualitative features (for example, the passage of the total cross section through a maximum, etc. ).

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*Note added in proof*: Analogous results are obtained in the recently published paper by Zharkov[13], where the same problem is treated by different mathematical methods.

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