Meson Production at Energies Close to Threshold*

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The energy spectrum of mesons produced in the collisions of two nucleons is determined. The most probable events are those in which the nucleons are produced with very small relative energy. Consequently, the most probable energy of the mesons occurs close to the maximum energy permitted by conservation laws. The ratio between the cross section for the production of deuterons (in the reactions $p+p=n+\pi^+$; $n+p=p+n+\pi^0$; $n+n=n+p+\pi^-$) and the cross section for the production of free neutrons and protons with parallel spin is obtained.

1. THE cross section for the production of mesons with momentum $\mathbf{P}_\mu$ in the interval $d\mathbf{P}_\mu$, when the relative momentum of the nucleons, $\mathbf{P}_n$, lies in the interval of solid angle $d\omega_n$, is equal to

$$d\sigma = d\mathbf{P}_\mu P_n^2 (dP_n / dE_n)\, d\omega_n \left|\int \varphi_{1*} H' \varphi_0\, d\tau\right|^2. \quad (1)$$

Here ϕ_0 and ϕ_1 are the exact wave functions for the relative motion of the two nucleons in the initial and final states. Because the form of the operator H' will not be used, the expression (1) contains no hypothesis on the strength of the interaction between nucleon and meson field. The function ϕ_0 describes the state of larger relative momentum of the nucleons and changes sign in a distance of order of magnitude $\hbar / P_n^0 < r_0$, where r_0 is the interaction radius. Therefore, in the integration over the relative coordinates of the nucleons, distances less than r_0 play a role and the integral depends essentially on the behavior of the potential at short distances. This behavior is unknown at present. It is easily seen that the dependence of the cross section on the relative kinetic energy of the nucleons in the final state can be found without hypotheses on the form of interaction between the nucleons and the operator H'. In fact, if the functions ϕ in (1) are normalized per unit volume, then considered as functions of the complex variable P_n, they must have a pole for imaginary P_n corresponding to real or virtual binding of the system of two nucleons[1]. Therefore, the dependence of the integral in (1) on the relative energy of the nucleons in the final state, E_n, is given by the expression

$$\left|\int \varphi_{1*} H' \varphi_0 d\tau\right|^2 = \frac{U(\mathbf{P}_n, \mathbf{P}_n^0, \mathbf{P}_\mu)}{E_n + \varepsilon}. \quad (2)$$

where P_n^0 is the initial relative momentum of the nucleons, and ϵ is the absolute value of the binding energy. From the theory of neutron-proton scattering one easily derives that

$$U(\mathbf{P}_n, \mathbf{P}_n^0, E_\mu) = U(0, \mathbf{P}_n^0, E_\mu)\,[1 + O(E_n / V_0)],$$

where V_0 is the depth of the potential well. The resonance denominator in Eq. (2) coincides with that which is found in the theory of neutron-proton scattering, and the cross section does not depend on the direction of the vector $\mathbf{P}_n$. (The case of two neutrons and two protons is considered below.) The quantity $\epsilon = \epsilon_0 = 2.2$ MeV for parallel spin and $\epsilon = \epsilon_1 = 0.07$ MeV for anti-parallel spin of the neutron and proton.

Using the laws of conservation of energy and momentum and integrating (1) over $d\omega_\mu\, d\omega_n$, we get

$$d\sigma = C(E_n^0, E_\mu) \quad (3)$$

$$= \frac{\sqrt{(E_\mu^m - E_\mu)\, E_\mu}}{E_\mu^m - E_\mu + \varepsilon'} \sqrt{1 + \frac{E_\mu}{2\mu c^2}} \left(1 + \frac{E_\mu}{\mu c^2}\right) dE_\mu;$$

$$E_\mu^m = \frac{E_n^0 - \mu c^2}{1 + (\mu/2M)}; \quad \varepsilon' = \frac{\varepsilon}{1 + (\mu/2M)},$$

where E_μ^m is the maximum energy of the mesons. For energies E_μ, close to the maximum ($E_\mu^m -$

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Note in Proof: After this work was completed several papers appeared treating the same questions [see e.g., K. M. Watson, Phys. Rev. 88, 1163 (1952)].

[1] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*.

$E_\mu \ll E_\mu^m$) we can set $C(E_n^o, E_\mu) \approx C(E_n^o, E_\mu^m)$. As is seen from (3), the spectrum of mesons has a maximum at the energy $E_\mu \approx E_\mu^m - \epsilon'$.

2. We can get the ratio between the cross section (3) for parallel spin of the nucleons and the cross section corresponding to the production of a deuteron in the final state. Inside the range of nuclear forces ($r < r_0$) the functions ϕ for free and bound states of the neutron and proton differ only in a constant factor which is determined by the value of the functions at $r = r_0$ (if the energy of the nucleons can be neglected compared to the depth of the potential well V_0). Outside the range of forces the functions are known from the theory of the deuteron and the theory of neutron-proton scattering[2]. . It can be shown that

$$\left|\frac{\varphi_0(r_0)}{\varphi_{P_n}(r_0)}\right|^2 = \left(\frac{M}{\hbar^2}\right)^{3/2} \frac{\varepsilon_0^{1/2}}{2\pi}(E_n + \varepsilon_0)\left\{1 + O\left(\frac{\varepsilon}{V_0}\frac{E_n}{V_0}\right)\right\}. \quad (4)$$

Using (4), it is easy to obtain

$$d\sigma = \frac{\sigma_0}{2\pi\sqrt{[1 + (\mu/2M)]\varepsilon_0 E'_\mu}} \times \frac{\sqrt{(E_\mu^m - E_\mu)E_\mu}}{(E_\mu^m - E_\mu + \varepsilon'_0)} \frac{\sqrt{1 + (E_\mu/2\mu c^2)}}{\sqrt{1 + (E'_\mu/2\mu c^2)}} \times \frac{1 + (E_\mu/\mu c^2)}{1 + (E'_\mu/\mu c^2)} \frac{C(E_n^0 E_\mu)}{C(E_n^0 E'_\mu)} dE_\mu d\omega_\mu; \quad (5)$$

$$\varepsilon_0 = \frac{\varepsilon_0}{1 + (\mu/M)}, \quad E'_\mu = E_\mu^m + \varepsilon_0;$$

Here σ_0 is the cross section for the production of deuterons per unit solid angle of the mesons. Near the upper end of the spectrum of mesons $C(E_n^o E_\mu)/C(E_n^o E'_\mu) \approx 1$. The experimental spectrum of mesons represents the superposition of two curves: the distribution (5) and the analogous distribution for anti-parallel spin of neutron and proton. Integrating (5) over dE_μ and $d\omega_\mu$ and assuming $C(E_n^o E_\mu)$ to be an increasing function of E_μ we get the ratio

$$\int \sigma_0 d\omega_\mu \Big/ \int \sigma d\omega_\mu dE_\mu \geqslant 4\sqrt{\varepsilon_0 / E_\mu^m}. \quad (6)$$

Equality in (6) corresponds to H' being independent of meson energy (under the restriction $E_\mu \ll \mu c^2$). If $H' \sim \sqrt{E_\mu}$ then

$$\int \sigma_0 d\omega_\mu \Big/ \int \sigma d\omega_\mu dE_\mu = {}^{16}/_{3}\sqrt{\varepsilon_0 / E_\mu^m}. \quad (6')$$

3. When two neutrons appear in the final state (the reactions are then $p+n=n+n+\pi^+$ or $n+n=n+n+\pi^o$), then only the case of anti-parallel spin occurs with appreciable probability. Neutrons with parallel spin cannot, consistent with selection rules, appear in S states and, consequently, do not interact at low energy. Therefore, the cross section for the production of mesons with neutrons having parallel spin in the final state does not have a resonance factor (2) and is considerably smaller than the cross section for neutrons with anti-parallel spin. For the same reason as before this latter cross section does not depend on the direction of the vector $\mathbf{P}_n$. The distribution of mesons in energy is given by the expression (3). However, the quantity ϵ is unknown in this case.

4. In the case with two protons in the final state (the reactions are then $p+n=p+p+\pi^-$ or $p+p=p+p+\pi^o$), finding the energy dependence of the wave function at $r=r_0$ and using the results of the theory of proton scattering[3,4] it is easy to prove that :

$$d\sigma = C_1(E_n^0 E_\mu) \times \frac{F(\eta)\sqrt{(E_\mu^m - E_\mu)E_\mu}\sqrt{1+(E_\mu/2\mu c^2)}(1+E_\mu/\mu c^2)dE_\mu}{F^2(\eta)(E_\mu^m - E_\mu) - \frac{\hbar^2}{M+(\mu/2)}\left\{-\frac{1}{a} + \gamma E_n - (h(\eta)/R)\right\}^2} \quad (7)$$

$$F(\eta) = \frac{2\pi\eta}{e^{2\pi\eta} - 1}; \quad \eta = \frac{e^2}{\hbar v_n};$$

$$h(\eta) = \mathrm{Re}\frac{\Gamma'(-i\eta)}{\Gamma(-i\eta)} - \ln\eta;$$

$$R = \frac{\hbar^2}{Me^2} = 2.9 \cdot 10^{-12} \text{ cm}; \quad a = -7.7 \cdot 10^{-13} \text{ cm};$$

$$\gamma = 3.4 \cdot 10^{11} \text{ MeV}^{-1} \text{ cm}^{-1}.$$

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[2] Ia. A. Smorodinskii, J. Exper. Theoret. Phys. USSR 17, 941 (1947); Doklady Akad. Nauk SSSR 60, 217 (1948)

[3] L. D. Landau and Ia. Smorodinskii, J. Exper. Theoret. Phys. 14, 269 (1944).

[4] J. D. Jackson and J. M. Blatt, Revs. Mod. Phys. 22, 77 (1950).

The Coulomb repulsion of the protons, as can be seen from (7), significantly diminishes the spectrum of mesons near the upper limit as compared with the cases considered above. As in the case of two neutrons the spins of the protons are anti-parallel and the cross section is independent of the direction of the vector $\mathbf{P}_n$.

5. The differential cross section σ_1 in the center of the mass system for the absorption of mesons by the deuteron is easily connected with the cross section σ_0, using the principle of detailed balance. A simple calculation gives

$$\sigma_1 = \sigma_0 (E_n^0) \frac{E_n^0}{E_\mu} \frac{[M + (\mu / 2)]}{2\mu} \tag{8}$$

$$\times \frac{\sqrt{1 + (E_\mu^0 / \mu c^2)}}{\sqrt{1 + (E_\mu / 2\mu c^2)}} \frac{[1 + (2E_n^0 / \mu c^2)]}{[1 + (E_\mu / \mu c^2)]};$$

$$E_n^0 = \mu c^2 + \frac{P_\mu^2}{4M} + E'_\mu - \varepsilon_0.$$

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Remarks on the Theory of Fusion

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A generalization of De Broglie's theory of fusion is given, which holds for infinite-dimensional as well as finite-dimensional wave equations.

IN de Broglie's theory of fusion, which generalizes the idea of the neutrino theory of light [1], a method of constructing particles with higher spin from particles with spin 1/2 was indicated [2].

In the present work, a point of view somewhat more general than the theory of fusion is proposed. The problem is formulated as follows: Consider two relativistically invariant equations, infinite-dimensional in general,

$$\gamma_\mu^{(1)} \frac{\partial \psi^{(1)}}{\partial x_\mu} + \varkappa' \psi^{(1)} = 0, \quad \gamma_\mu^{(2)} \frac{\partial \psi^{(2)}}{\partial x_\mu} + \varkappa'' \psi^{(2)} = 0, \tag{1}$$

They will generate two reducible representations of the Lorentz group $\tau^{(1)}$ and $\tau^{(2)}$. It is required to find the invariant equation corresponding to the Kronecker product $(\tau^{(1)} \times \tau^{(2)})$. In our arguments, we will use the results and notation of the work of Gel'fand and Iaglom [3].

[1] A. A. Sokolov, J. Exper. Theoret. Phys. USSR 7, 1055 (1937)

[2] L. De Broglie, Théorie générale des particules a spin (méthode de fusion), Paris, 1943

[3] I. Gel'fand and A. Iaglom, J. Exper. Theoret. Phys. USSR 18, 703 (1948)

1. For the following, we need the reduction formula for the direct product of two irreducible representations of the Lorentz group. The infinitesimal representation of the group is determined by the operators F^+, F^-, F^0, H^+, H^-, H^0. Their form for irreducible representations is given by numbers k_0, k_1 [3]. If k_0, k_1 are simultaneously integral or half integral real numbers, and $k_0 < k_1$, then the representations are finite-dimensional.

The space of the representation is given by basis vectors ξ_p^k, where the total momentum k, appearing as the weight of the sub-group of rotations H^+, H^-, H^0, runs through the series of numbers ($k = k_0, k_0 + 1, k_0 + 2, \ldots$). In the finite-dimensional case, the sequence k breaks off at $k = k_0 - 1$ ($p = k, k-1, \ldots -k$). It is not difficult to prove that the infinitesimal representations of the group, τ_n, have two scalar operators Δ_1 and Δ_2 which commute with every operator of the representation and have the form

$$\Delta_1 = F^+F^- + F^{02} - H^+H^- - H^{02} - 2iH^0,$$
$$\Delta_2 = 2F^0H^0 + F^+H^- + F^-H^+. \tag{2}$$

These operators are given by the formulas